1. Let M, N be matchings in a graph G, with $|M| > |N|$. Show that there must exist matchings M', N' such that $|M'| = |M| - 1$, $|N'| = |N| + 1$ and the union and intersection of M, N (as edge sets) is the same as that of M', N'.

2. The *girth* of a graph is defined as the length of the shortest cycle in it. Let C, C' be cycles of length k in a graph of girth k. Show that the symmetric difference $C \oplus C'$ is a single cycle if and only if $C \cup C'$ is a single path. What will not work if the graph has smaller girth?

3. Suppose you are given a matrix of real numbers in which the last row and last column give the sums of the columns and rows respectively. You wish to round the numbers in the matrix to integers. Up or down rounding is allowed; no matter what the fractional part is, i.e. 18.1 may be rounded to 18 or 19. However, However, it is essential that even after rounding, the row and column sums remain row and column sums. Use network flow to decide if such rounding is possible.