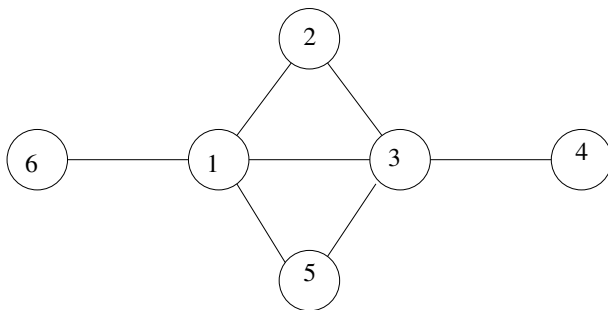


Problem 1:[20 marks] A company needs to allocate its employees for the projects it is to execute. For each employee e we are given an attribute Y_e which can be either *senior* or *junior*, and the location L_e of the office in which the employee works. We are also given A_e which for each employee is the set of projects for which he/she has abilities, and can be assigned.

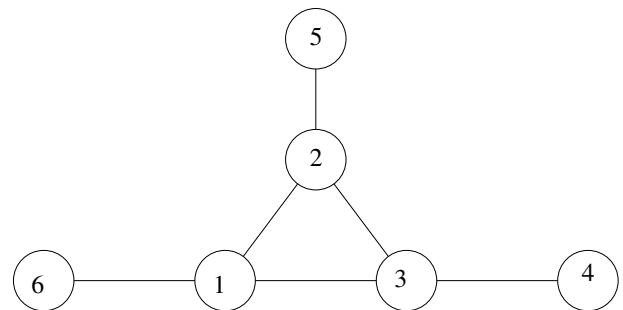
For each project p , we are to allocate N_p employees, of whom at least S_p need to be senior. The values S_p, N_p are known. Further, we must allocate at most 15 employee from each location (overall to all projects).

Give an algorithm to find an employee allocation (whenever possible) by expressing this as an instance of an appropriate version of the max flow/circulation problem. Clearly describe the correspondence between the vertices in your graph and the entities in the problem, e.g. “there is one vertex for every employee”, and also where the edges are with appropriate capacities and lower bounds as necessary. Please write these very clearly. Write one line on why each constraint is being satisfied.

Problem 2:(a)[4 marks] State Tutte’s theorem. (b)[4 marks] In the graphs below, show a perfect matching, or a suitable vertex set that explains why there is no perfect matching. (c)[7 marks] Run the general matching algorithm on the graph (A) below. Assume that vertices are considered in increasing order. Show the graph after each augmentation, and at the point when a blossom is discovered.



(A)



(B)

Problem 3:(a)[5 marks] State the walk matrix for the bidirected cycle on 4 nodes. Guess its stationary distribution and verify it by showing the multiplication etc.

(b) Consider an $n \times n$ stochastic matrix W in which each $w_{ij} > \epsilon > 0$, for some constant ϵ . Assume that $w_{ii} > \epsilon$ as well. Let x be a (row) vector of average value 0, i.e. $x \cdot \mathbf{1} = 0$. For any vector x , Let $P(x) = \sum_{i|x_i > 0} x_i$. Let $x(t) = xW^t$.

(i)[5 marks] Show that $P(x(t+1)) \leq P(x(t))(1 - \epsilon)$. Hint: Show that some node must have at least $P(x(t))/n$ “positive charge”, and some similar negative charge. How much of this might get annihilated?

(ii)[5 marks] Suppose y is a probability (row) vector. Show that yW^t tends to z where z is the stationary distribution of W . Hint: Consider $x = y - z$.