Input: $n \times n$ matrix D giving distances between n points $c_1, \ldots, c_n$. The distances form a metric, i.e. $d_{ii} = 0$, $d_{ij} = d_{ji}$, $d_{ij} \leq d_{ik} + d_{kj}$ for all i, j, k. The integer p.
Output: Subset $h_1, \ldots, h_p$ of p points from the n, such that $\max_i d(c_i, \{h_1, \ldots, h_p\})$ is as small as possible, where $d(x, Y)$ is the distance from x to the closest point in Y. We will use $d(x, y)$ to denote the distance between points x, y.

We think of the points c_i as cities, and h_i as where to locate hospitals. We want to build hospitals at p of the locations so as to minimize the distance to the closest hospital for each city. We will say that a hospital at h_j serves c_i if it is the closest hospital to c_i. We may regard each h_j as a center of the area it serves, and $\max_i d(c_i, \{h_1, \ldots, h_p\})$ is the radius of the largest area. We want to minimize this maximum radius.

A simple algorithm is

1. Pick h_1 arbitrarily.
2. For $j = 2$ to k, set h_j to be that c_i that is farthest from $h_1, \ldots, h_{j-1}$, i.e. $d(c_i, \{h_1, \ldots, h_{j-1}\})$ is largest.

Let R denote the maximum radius for this choice of h_js. Let the choice made by the optimal algorithm be h_j^* for $j = 1$ to p, and the optimal radius be R^*. We will prove that $R \leq 2R^*$.

For each h_i, let B_i denote the set of cities within a distance R^* from h_i. Since each h_i is within distance R^* from some h_j^* we know that each B_i must contain at least one h_j^*.

Suppose each B_i contains exactly one h_j^*. Let us rename so that B_i contains h_i^*. Now consider any c_k. We know that some h_i^* is within distance R^* from c_k. But then $d(c_k, h_i) \leq d(c_k, h_i^*) + d(h_i^*, h_i) \leq 2R^*$.

Otherwise Suppose some h_j^* belongs to both B_i, B_k. Then

$$d(h_i, h_k) \leq d(h_i, h_j^*) + d(h_j^*, h_k) \leq 2R^*$$

Suppose $i < k$. Consider now the situation when h_k got added during the execution of the algorithm. By definition we know that $d(h_k, \{h_1, \ldots, h_{k-1}\}) = \max_i d(c_i, \{h_1, \ldots, h_{k-1}\})$. Thus

$$\max_i d(c_i, \{h_1, \ldots, h_{k-1}\}) \leq d(h_k, h_i) \leq 2R^*$$

Thus all cities c_i were already at distance at most $2R^*$ even with hospitals $h_1, \ldots h_{k-1}$. Thus they cannot be at a longer distance when all the hospitals are used.