# A Cross Layer Congestion Control Algorithm in Wireless Networks for TCP Reno-2

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*Abstract*— In this paper, we discuss a cross layer congestion control technique of TCP Reno-2 in wireless networks. In this both TCP layer and PHY layer jointly control congestion. The PHY layer changes transmission power as per the channel condition, interference received and congestion in the network, whereas the TCP layer controls congestion using Reno-2 window based flow control. Our simulations show that the cross layer congestion control technique provides performance improvement in terms of throughput and window size variations.

Keywords: TCP, Reno, Congestion, Optimal, KKT, Lagrangian, Price

## I. INTRODUCTION

In a wired network, the links are assumed to be reliable and of fixed capacities. So, if there is any packet loss or delay in the link, then it is due to congestion in the link. There are various techniques used in the Internet to counter the congestion problem. They are either to avoid congestion (proactive) or to control congestion (reactive). These techniques are implemented in the Transport Control Protocol (TCP) of Internet. In TCP, congestion is said to have occurred when the sender receives three duplicate acknowledgments (*dupacks*) or when a timeout (packet loss) occurs. The TCP congestion control techniques are divided into three broad categories, viz., (i) window based, (ii) equation based, and (iii) rate based.

The window based congestion control technique is based on an *Adaptive Window Management* technique, in which increase and decrease of congestion window (*cwnd*) is based on packet drops and *dupacks*. The increase and decrease of *cwnds* are based on the principle of Adaptive Increase and Multiplicative Decrease (AIMD). TCP Tahoe [1], Reno [2], [3] and Vegas [4] are some of the commonly deployed variants of window based congestion control.

In equation based congestion control [5], the sender explicitly adjusts its sending rate as a function of the measured rate of loss events. Drop of one or more packets within a single round trip time (RTT) is considered as the loss event rate. The main aim of equation based congestion control is to maintain a relatively steady sending rate, while still being responsive to congestion. Equation based congestion control is appropriate for real-time traffic.

For a high speed network, the instantaneous rate of transmission  $x_i$  of an user *i* can be approximated by  $x_i = \frac{w_i}{\tau_i}$ , where  $w_i$  is the window size and  $\tau_i$  is the RTT of user *i* at that instant. So, for a high speed network, the window based congestion control can be modeled as a rate based congestion control. In this, instead of changing the window size in each RTT, instantaneous rates are regulated to avoid congestion.

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The congestion control/avoidance techniques of TCP are proven to be effective in the wired network of the Internet. But, this is not true in wireless networks, where packet loss or/and delay can result due to congestion as well as due to time varying nature of the wireless channel. Also, the link capacities in a wireless network are not fixed but depend upon the signal to interference and noise ratio (SINR) of the link. SINR depends upon the power transmission policy of the network. Hence, applying the congestion control of wired networks directly in wireless networks may not be suitable. A joint congestion and power control technique is proposed in [6] to address the congestion control problem of wireless networks. The authors of [6] applied their joint congestion and power control technique for TCP Vegas in a multi-hop wireless network. This is a cross layer approach involving TCP and physical (PHY) layer. The TCP layer performs the window based flow control and PHY layer varies the transmission power of wireless nodes depending on the channel condition and interference.

In this paper, we have adopted the approach of [6] for TCP Reno-2 for a wireless network. Reno-2 is more suitable for a wireless network. This is because, it is more robust than TCP Reno and Tahoe, for multiple packet loss in a window. In Reno-2, even if there is no congestion and multiple packet loss occurs in a window due to time varying nature of the channel, the window size (*cwnd*) reduces only to half and not further. Thus packet loss due to wireless channel can also be taken care of by the Reno-2 congestion control algorithm. Simulations are used to verify the cross layer congestion control technique for TCP Reno-2 in a wireless ad-hoc network.

This paper is organized as follows. In Section II, we discuss an optimization based congestion control technique for a wired network and extend that to a wireless network. Further, in Section III, we describe a cross layer congestion control mechanism for a wireless network. We discuss the utility function and shadow price of TCP Reno-2 in Section IV. We evaluate the cross layer congestion control algorithm for Reno-2 in Section V. We discuss our simulations, results and convergence analysis of the cross layer congestion control algorithm in Section V-A, V-B and V-C respectively. Finally, we present the future work and the limitations of the cross layer congestion control algorithm in Section VI.

## II. OPTIMIZATION BASED CONGESTION CONTROL

Modeling congestion control as an optimization flow control problem has been addressed in [7], [8], [9], [10]. The authors of these papers, have modeled an optimization based flow control in which the sources adjust their transmission rates in response to the congestion in the network. The authors of these papers try to model the flow control problem as a constraint based optimization problem. The solution to this constraint based optimization problem gives the solution to congestion control or flow control in networks. The authors differ in defining the objective functions of the constraint based optimization problem and solution methodologies. [9] is similar to [7] in defining the objective functions, but differs in solving the objective function. In [9], the users decide their rates based on the charges received from the network, whereas in [7], the users decide the payments they can make to the network and receive the rates allocated by the network.

Efforts were made by various authors to use the congestion control techniques of TCP in a wireless network. [6] has used power control along with congestion control in a wireless network and analyzed it for TCP Vegas. In [11], the author has modeled the power control and utility maximization of a wireless network as a sum product problem. This approach is used to design a new Joint Optimal Congestion control and Power control (JOCP) algorithm for a wireless network. In JOCP, the authors have proposed a distributed power control algorithm which works jointly with the existing TCP congestion control algorithm to increase the end-to-end throughput and energy efficiency of a multi-hop wireless network. This JOCP algorithm does not change the original TCP and the layered structure of the network. Rather, it is a cross layer approach, where the interactions among the physical and transport layer is used to increase the end-to-end throughput.

In the following subsection we discuss the optimization based congestion control for a wired network. We then discuss its extension to a wireless network as discussed in [6].

## A. Wired Network

We consider that a set I of source-sink pairs share a network of L unidirectional links. The capacity of the individual links l are  $c_l$ ,  $l \in L$ . Each source-sink pair  $i \in I$  has a utility function  $U_i(x_i)$  associated with its transmission rate  $x_i$ . The utility function  $U_i(x_i)$  is concave, increasing and double differentiable [12] for elastic<sup>1</sup> traffic. Let the route matrix R consists of the route information of all possible links and source-sink pairs. Any element  $R_{li}$  of R is defined as:  $R_{li} = 1$ , if, source-sink pair i uses link l and  $R_{li} =$ 0, otherwise. The aggregate flow  $y_l$  at each link is defined as:  $y_l = \sum_i R_{li} x_i$ 

If each link l is associated with price  $\lambda_l$ , as a congestion measure, then the aggregate price<sup>2</sup> of all links for a sourcesink pair i in the route is defined as:  $q_i = \sum_l R_{li}\lambda_l$ . At equilibrium, a source chooses its maximum profit by choosing local parameters as:

$$\max_{x_i} \left[ U_i(x_i) - q_i x_i \right],\tag{1}$$

Since at equilibrium, each source tries to maximize its profit (individual optimality) by choosing an appropriate rate, the individual optimality (Eqn. (1)) can be re-written as a social optimality equation [7] as follows:

$$\max_{X \ge 0} \sum_{i} U_i(x_i),$$
  
s. t.,  $RX \le C;$   $X = \{x_i\}$  and  $C = \{c_l\}$  (2)

A primal-dual distributed algorithm of the maximization equation (Eqn. (2)) signifies that the price  $\lambda_l$  is updated as a congestion parameter and is a dual variable, whereas the rate of transmission  $x_i$  is updated as a primal variable.

## B. Wireless Network

The capacity of a link in a wireless network is considered as a function of transmission power. Hence, for a wireless network, we modify the social optimization of Eqn. (2) as:

$$\max_{X \ge 0} \sum_{i} U_{i}(x_{i}),$$
s.t.,  $RX \le C(P); \quad P = \{P_{l}\},$ 
 $P_{l} \le P_{l_{Max}}, \quad \forall l; \quad P, X \ge 0,$ 

$$(3)$$

where  $P_l$  is the transmission power in  $l^{th}$  link. Here, the link capacity  $c_l$  is a function of transmission power  $P_l$ . Hence, the capacity of a congested link can be increased by increasing power in that link. For a CDMA based network, using Shannon's capacity theorem, we determine the maximum capacity attainable in link l as:  $c_l = \frac{1}{T}log(1 + M.SINR_l)$  packets/sec, where T is the symbol period and M is a constant that depends on the modulation scheme used by the node for a successful transmission. The  $SINR_l$  of link l is expressed as:

$$SINR_l = \frac{P_l G_{ll}}{\sum_{k \neq l} P_k G_{lk} + n_l},\tag{4}$$

where  $G_{ll}$  is the path gain from the transmitter of link l to the receiver of the link l and  $G_{lk}$  is the path gain from the transmitter on link k to the receiver on link l.  $n_l$  is the thermal noise on the link l.

## **III. CROSS LAYER CONGESTION CONTROL**

We solve Eqn. (3) using KKT [13] optimality conditions by solving the complementary slackness conditions at equilibrium. For this, we associate a Lagrangian Multiplier  $\lambda_l$  for the first constraint in Eqn. (3). Then we determine the stationary points of the Lagrangian as:

$$\phi_{system}(X, P, \lambda) = \sum_{i} U_i(x_i) - \sum_{l} \lambda_l \left(\sum_{i} y_l - c_l(P)\right)$$
(5)

Maximization of  $\phi_{system}$  is decomposed<sup>3</sup> as in [6]:

$$\max \quad I(X,\lambda) = \sum_{i} U_{i}(x_{i}) - \sum_{l} \lambda_{l} \sum_{i} R_{li}x_{i},$$
$$\max \quad I(P,\lambda) = \sum_{l} \lambda_{l}c_{l}(P), \quad (6)$$
s.t.,  $X \ge 0; \quad 0 \le P_{l} \le P_{l_{Max}},$ 

The first maximization equation involving  $I(X, \lambda)$  in Eqn. (6) is solved by the congestion control mechanism of TCP by increasing/decreasing the window size in each RTT for each

<sup>&</sup>lt;sup>1</sup>Elastic traffic consists of traffic for which users do not necessarily have an minimum requirements, but would get as much as data through to their respective destinations as quickly as possible.

 $<sup>^{2}</sup>$ Sources are assumed to have an access to the aggregate price of all links in their route

<sup>&</sup>lt;sup>3</sup>Distributed solution is possible as along as there is an interaction between the two decomposed equations through some information passing (message passing in our case). This is known as sum product algorithm [11]

flow. The second maximization equation involving  $I(P, \lambda)$ in Eqn. (6) is solved by choosing appropriate transmission power of wireless nodes. Both  $I(X, \lambda)$  and  $I(P, \lambda)$  are related by a common variable  $\lambda$ , which plays a significant role in determining the equilibrium window size and transmission power. Any change in  $\lambda$  results in change in throughput and transmission power. Without loss of generality, we assume that the symbol period T and the modulation index M as unity and re-write  $c_l = log(SINR_l)$ . Hence, we re-write  $I(P, \lambda) = \sum_l \lambda_l \log(SINR_l(P))$ . By differentiating  $I(P, \lambda)$ with respect to  $P_l$ , we evaluate the  $l^{th}$  component of the gradient  $\nabla I(P, \lambda)$  and solve the maximization problem by the Steepest Descent method as:

$$P_{l}(t+1) = P_{l}(t) + \Delta \left(\nabla I(P,\lambda)\right)$$
$$= P_{l}(t) + \Delta \left(\frac{\lambda_{l}(t)}{P_{l}(t)} - \sum_{j \neq l} \frac{\lambda_{j}(t)G_{jl}}{\sum_{k \neq j} P_{k}(t)G_{jk} + n_{j}}\right)$$
$$= P_{l}(t) + \Delta \frac{\lambda_{l}(t)}{P_{l}(t)} - \Delta \sum_{j \neq l} m_{j}(t)G_{lj},$$
(7)

where  $\Delta$  is a constant, called the step size in the direction of the gradient and  $m_j(t)$  is the message received from node *j* to the link *l* and is defined as:

$$m_j(t) = \frac{\lambda_j(t)SINR_j(t)}{P_j(t)G_{jj}}$$
(8)

From Eqn. (7), it is evident that the transmission power of a node in the next time slot  $P_l(t+1)$  in a link l depends on three parameters, viz., (i) transmission power in the present time slot  $P_l(t)$ , (ii) shadow price  $\lambda(t)$  and (iii) the weighted sum of message received from all neighboring nodes. The third factor is responsible for decreasing the transmission power of the concerned node in the next time slot, i.e., the transmission power of the concerned node should be such that the interference resulted at some other nodes is below some threshold. This is known as the co-operation principle in power control of wireless network.  $\lambda(t)$  is responsible for increasing power in the next time slot. Intuitively, more the shadow price, more the congestion, more the transmission power in the next time slot, i.e., transmission power increases with respect to shadow price. However, this increase is not linear. If the transmission power in the congested link is already high, then, the increase in power in the congested link will increase interference in other links and hence should not be increased. This is reflected through message passing in this framework.

In power control techniques, each wireless node needs to advertise its  $SINR_l$  requirement either on a separate channel or on the same channel. These nodes update their path gains, noise levels, interference causes by other nodes etc., either after receiving the advertised signal or in a periodic manner.

# IV. TCP RENO-2

In TCP Reno-2, *cwnd* is decreased by half for one or more mark or increased by one for no mark in one RTT. We assume the marking probability to be a measure of congestion. In this section, we discuss the utility function and the shadow price of Reno-2. The utility function of Reno-2 is derived in [14] and is given by:

$$U_i(x_i) = \frac{1}{\tau_i} \log \left[ \frac{x_i \tau_i}{2x_i \tau_i + 3} \right]$$
(9)

Since the utility function of TCP Reno-2 is concave for  $x_i, \tau_i \ge 0$ , our problem formulation in Eqn. (1) holds good.

The concept of pricing is different for different schemes of TCP, viz., queuing delay in TCP Vegas and loss probability in TCP Reno and Reno-2. In TCP Reno-2, the probability of dropping of packets can be modeled as the buffer over flow in a M/M/1/B queue, where, B is the buffer size at the link. A closed form expression of the packet loss probability for this kind of model involving Reno-2 is derived in [15]. Since the loss probability in TCP Reno-2 is considered as the price, the price  $\lambda_l$  in a link l with aggregate traffic of  $y_l$  and capacity  $c_l$  is expressed as:

$$\lambda_{l} = \begin{cases} \frac{\max\left(0, (y_{l} - c_{l})\right)}{y_{l}} & \text{if } y_{l} > 0, \\ 0 & \text{if } y_{l} = 0. \end{cases}$$
(10)

# V. EXPERIMENTAL EVALUATION OF CROSS LAYER CONGESTION CONTROL ALGORITHM FOR RENO-2

The cross layer congestion control algorithm for Reno-2 is based on the joint power control of PHY layer and the congestion control of TCP layer. We discuss the cross layer congestion control algorithm for Reno-2 in Algorithm 1. In our implementation, the initial window size  $w_{initial}$ , initial power  $P_{l_{Min}}$ , minimum  $SINR_l$  and  $\delta$  are taken as configuration parameters. We take  $w_{initial} = 3$ ,  $\delta = 0.5$  in our simulations. The value of  $P_{l_{Min}}$  and  $P_{l_{Max}}$  in our simulation is 3 and 15 units respectively. The frequency of  $SINR_l$  update is also a configuration parameter (usually this is a multiple of RTTs). We calculate the data rate  $x_i$  as:  $x_i = \frac{w_i}{\tau_i}$ , whereas  $w_i$  and  $\tau_i$ are decided using Reno-2 congestion control principle.

Algorithm 1 : Cross Layer Congestion Control Algorithm for Reno-2

- 1: Set initial window size  $w_i = w_{initial}$
- 2: Initialize  $P_l = P_{l_{Min}}$
- 3: Advertise the minimum  $SINR_l$  required
- 4: Update  $G_{lj}$  and  $G_{jj}$  periodically or after receiving the advertised signals
- 5: Determine maximum capacity of the link
- 6: Determine  $\lambda_l(t)$  using Eqn. (10)
- 7: Determine  $m_j(t)$  using Eqn. (8)
- 8: Calculate  $P_l(t+1)$  using Eqn. (7)
- 9: if  $|P_l(t+1) P_l(t)| \le \delta$  then
- 10: Continue transmission at  $P_l(t)$
- 11: else
- 12: Transmit at  $\min(P_l(t+1), P_{l_{Max}})$
- 13: end if
- 14: Change  $w_i$  according to the congestion control algorithm of Reno-2
- 15: Update  $SINR_l$  at each node and go to Step 3

## A. Simulations

We consider a topology with 6 wireless nodes and two pairs of TCP transmitters and receivers as shown in Fig.

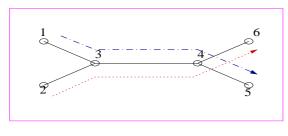


Fig. 1. System Model/Topology

1 for our simulations. The two pairs of transmitters and receivers in our simulation are (1-5) and (2-6). All nodes in our simulation are TCP Reno-2 agents. Depending upon the shadow price of the links and the message received, we update the transmission power of the participating nodes. We consider the TCP retransmission to be multiple of four RTTs. We update RTT by using the exponential averaging technique as:  $RTT = \alpha RTT_{estimated} + (1-\alpha)RTT_{measured}$ . The value of  $\alpha$  is taken as 0.85 for our simulation. Also, for simplicity, we assume that the time required for transmission in each of the segments 1-3, 2-3, 3-4, 4-5 and 4-6 is same and both forward and reverse channel characteristics are same. The channel gains (shadowing) are assumed to be log-normally distributed with variance  $\sigma = 8dB$ . The path loss factor  $\gamma$  is assumed to be 4. We use Matlab [16] for our simulations.

# B. Results

We simulate TCP Reno-2 congestion control mechanism both with power control based on the SINR values and without power control techniques. In the latter case, the transmission power of nodes are fixed at the maximum value. In the former case, depending upon the congestion and interference, the nodes transmit at some optimal power level. Fig. 2 shows the cwnd variation of joint power and congestion control mechanism, whereas Fig. 3 shows cwnd variation without power control. We observe that the fluctuations in *cwnd* with power control mechanism is lower as compared to that of without power control mechanism. Also, the average window size of joint power and congestion control scheme is larger than that of congestion control without power control. Hence, power control provides stabilized and better throughput. Intuitively, this occurs because the maximization of utility function with power control (Eqn. (3)) is done over a larger set of constraints than without power control (Eqn. (2)).

The transmission powers of all Reno-2 agents are shown in Fig. 6 (with power control). The power consumption of nodes in our cross layer scheme is less as compared to the fixed power scheme. Further, we analyze the pricing mechanism for both fixed and power control schemes in Fig. 4 and Fig. 5. The price in Reno-2 is a function of packet loss and hence is a function of congestion window. Price rises at the point of congestion (e.g., in Fig. 3 and Fig. 5 at t = 220, 250, 400, 550, 650, 900 and 950, cwnd is at the peak and hence the price is also at the peak, whereas immediately after the peaks, the cwnd decreases by half of previous cwnd and hence the price also decreases) and falls after congestion control (by decreasing cwnd).

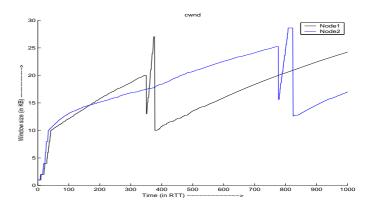


Fig. 2. Variation of Congestion window - with Power Control

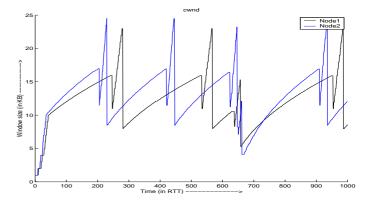


Fig. 3. Variation of Congestion window - without Power Control

# C. Convergence Analysis

In TCP Reno-2, we consider the probability of marking  $(\lambda_l)$ to be a measure of congestion. Indirectly this is controlled by the power control mechanism in the wireless channel as  $\lambda_l$ is used as the common variable between the two separate optimization problems of (Eqn. (6)). Since the transmitted power is bounded by minimum and maximum power levels, and the shadow price is zero at equilibrium state (total incoming traffic and capacity of a link are same, resulting no packet loss) a complimentary slackness condition between the primal variable  $x_i$  and dual variable  $\lambda_l$  of Eqn. (6) can be achieved. Now, going along the line of the proof given in [17] (see Theorem 1 of [17]) one can prove that the cross layer congestion control algorithm derived from Eqn. (6) will converge to a global optimum point. This requires the minimum SINR should be greater than one, else,  $I(P, \lambda) =$  $\sum_{l} \lambda_l \log(SINR_l(P))$  can not be approximated by the second optimization equation of (Eqn. (6)). Also, it is clear from [17], that  $I(P, \lambda)$  is a strictly concave function of logarithm of power transmission vector. Hence, the Lagrangian Multiplier  $\lambda_l$  facilitates a global maximization of Eqn. (5) and ensures convergence towards that. The step size ( $\Delta$ ) of Steepest Descent method of optimization in Eqn. (7) decides the rate of converges towards a global optimum point. The convergence is guaranteed as long as no new users enter or old users leave the network. For any addition and deletion of nodes/users, this algorithm will again take some iterations to converge.

We perform simulations with three different values of  $\Delta$  (0.1, 0.2 and 0.5) and observe the number of iterations to

converge to an optimum transmission power value in all three cases at different SINR level. For,  $\Delta = 0.1$ , it takes about 150 iterations to converge, (converges to a stable transmission power level), which becomes constant over time for a particular SINR. For higher values of  $\Delta$  though it takes fewer number of iterations to converge initially, but does not converge to a stable transmission power level. From simulations we have observed that our algorithm converges even for fading in wireless channel. This shows the robustness of our algorithm. Intuitively, one can establish the fact that, though the message passing can be erroneous due to fading, the convergence of cross layer algorithm is achieved and hence is robust.

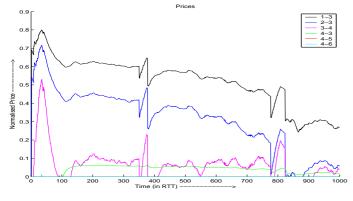


Fig. 4. Variation of Price - with Power Control

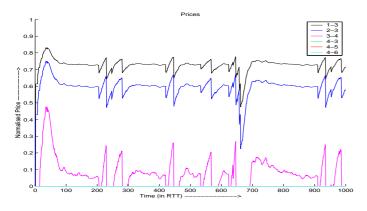


Fig. 5. Variation of Price - without Power Control

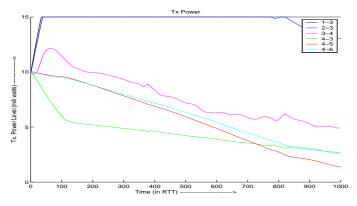


Fig. 6. Transmission Power - with Power Control

#### VI. DISCUSSION AND CONCLUSION

The proposed cross layer congestion control technique for Reno-2 converges very fast. This is due to the small step size  $(\Delta = 0.1)$  of Steepest Descent method Eqn. (7), and fixed maximum and minimum transmitting powers for each node. Our simulations verify the theoretical models that we have discussed, which is a maximization problem of an utility function. As expected, the cross layer congestion control technique provides stabilized throughput at low power transmission. But, if the channel conditions are very bad, then there would be more losses due to bad channel resulting in a significant increase in  $\lambda$ , which in turn results in an increase in power transmission. In that case, our power control algorithm does not converge. This is a drawback of the joint power and congestion control algorithm. This algorithm holds good as long as the minimum *SINR* is maintained at the nodes.

We have considered a simple topology for our simulation. A complex topology can be used to study other issues. Also, use of joint power and congestion control algorithm in bad channel condition needs some modification in the definition of packet loss and congestion. This modification can significantly increase the throughput.

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