Assignments for CvxOpt-10 (CS-709)

1 Theory: Convex Analysis

1. Show that dot product between two vectors when represented in any orthogonal basis is the same. Does this happen in any basis?

2. Write down the outer description for $span(X)$ where $X = \{[1\ 1\ 1\ 1]^\top, [1\ 1\ -1\ -1]^\top\}$.

3. What is the dimension of the affine set given by the following system of equations: $\sum_{j=1}^{n}(i+j)x_j = i,\ \ i = 1,\ldots,m$ (Assume $2 \leq m \leq n$).

4. Is the set $S = \{\mathbf{x} \mid \min_i x_i \leq 1\}$ (here x_i are components of $\mathbf{x}$) convex ?

5. Problems 2.2, 2.8, 2.13 from Boyd's book.

6. Assume that $X = \{x_1,\ldots,x_k\}$ and $Y = \{y_1,\ldots,y_m\}$ are finite sets in $\mathbb{R}^n$ with $k+m \geq n+2$ and all points in $X \cup Y$ are distinct. Suppose that for any subset $S \subset X \cup Y$ comprised of $n+2$ points, the convex hulls of $X \cap S$ and $Y \cap S$ do not intersect, then show that the convex hulls of X and Y also do not intersect. (Hint: Try proof by contradiction).

7. Show that $ri(Cl(S)) = ri(S)$ where S is convex.

[**Due: 16-Aug**]

8. Prove Farka's Lemma (assuming separation theorem).

9. Problems 3.5, 3.7, 3.12, 3.31, 3.32, 3.35 from Boyd's book.

10. Re-write the following MP as an LP and hence write down a dual of it:

$$\begin{aligned} \min_{x,y,z} \quad & |x| + |y| + |z| \\ \text{s.t.} \quad & x + y \leq 1, \; 2x + z = 3 \end{aligned}$$

[Due: 09-Sep]

11. Find Legendre transform (f^*) of f where $f(\mathrm{x})$ is given by: i) $f(\mathrm{x}) = \max_i x_i$ ii) $f(\mathrm{x}) = -(\Pi_i x_i)^{\frac{1}{n}}, \mathrm{x} > 0$. What is $(f^*)^*$ in both cases ?

12. Generalizing the result we discussed in class (assuming differentiability wherever necessary) show that $\nabla f^*(\mathrm{y}) = \mathrm{x}$ if $\nabla f(\mathrm{x}) = y$.

13. Consider the LP in the primal form as discussed in lectures. Find necessary and sufficient conditions on A,b,c such that the problem is solvable.

14. Given $\rho > 0$ find $\inf_{\mathrm{v}} \rho \log(\sum_i \exp(v_i)) - \mu^\top \mathbf{v}$.

15. Let $a_1, \ldots, a_n$ be positive reals and let $0 < s < r$ be integers. Find $min_x \sum_i a_i x_i^{2r}$ s.t. $\sum_i x_i^{2s} = 1$. Can you find the maximum of the same ?

16. Of all possible rectangles inscribed in a given a circle, show that the rectangle of maximal area is a square.

17. Boyd: 2.26, 2.33, 4.3, 4.5, 4.22, 4.23, 5.17, 5.18

[Due: 11-Oct]

18. Consider the problem of maximizing the harmonic mean:

$$\max_{\mathbf{x} \in \mathbb{R}^n} H(\mathbf{A}\mathbf{x} + \mathbf{b}) \;\; \text{s.t.} \; \mathbf{A}\mathbf{x} + \mathbf{b} > 0$$

where $\mathbf{A}$ is a $m \times n$ matrix and $H(\mathrm{x}) \equiv \frac{1}{\frac{1}{x_1} + \ldots \frac{1}{x_n}}$. Pose this problem in standard form of any named optimization problem you encountered in the course.

19. Use cvx[1] to solve the above harmonic mean maximization problem; problems in (3) and (5) of Quiz-2.

20. Solve the following problem using Gradient descent (with "optimal" constant step size), Nesterov's algorithm (on page 80 in the book) and cvx: $\min_{\mathbf{x}\in\mathbb{R}^n} \frac{1}{2}\mathbf{x}^\top\mathbf{H}\mathbf{x} - \mathbf{b}^\top\mathbf{x}$ where $\mathbf{H}$ is a Hilbert matrix[2], $\mathbf{b}$ is a vector computed as $\mathbf{H}\mathbf{x}^*$ where $\mathbf{x}^*$ is a random vector generated by you apriori ($\mathbf{x}^*$ will be the optimal soln. of the problem). Wherever applicable take the initial guess for optimal soln. as the zero vector. For various values of $n = 2, 3, 4, 5, 10, 20, 50, 100$ try plotting graphs of no. iterations vs. $||\mathbf{x}^k - \mathbf{x}^*||$. Do the results agree with the theory ? For what values of n are Nesterov/gr.des. methods faster than cvx ? Also for what values of n is the solution faster than computing the solution using inversion of matrix ? Feel free to use any command of matlab.

[**Due: 12-Nov**]

[1] Available at http://cvxr.com/cvx/

[2] $(i,j)^{th}$ entry of a Hilbert matrix is $\frac{1}{i+j-1}$