Midsemester Exam

CS709

23-Feb-2017 (2pm-5pm)

Note: Please answer the questions using rigorous and succinct mathematical arguments. Simplify expressions as much as possible. Always clearly justify your Yes/No answers.
Total Marks 45. Total time 3hrs.

Problem 1. Is the function $f_r : \mathbb{R}^n \mapsto \mathbb{R}$, defined below, convex for any given natural number r between 1 and n?

$$f_r(x) \equiv \text{sum of } r \text{ largest components/entries of } x.$$

[2 Marks]

(*Appeared 15-Sep-2012 Midsem*)

Problem 2. Consider the set of all symmetric matrices of size n (denoted by S_n) endowed with the usual $+, \cdot, \langle \cdot, \cdot \rangle_F$ to form an inner-product space. Let $C \subset S_n$ be the following:

$$C \equiv \{M \in S_n \mid M \succeq 0, trace(M) = 1\}.$$

In the context of this space associated with S_n, what is the dimensionality of C and C^o? Provide a simplified expression for C^o.

[8 Marks]

(*Appeared 19-Aug-2011 Quiz-1*)

Problem 3. Is the Log-sum-exp function f, defined below, convex?

$$f : \mathbb{R}^n \mapsto \mathbb{R},\ f(x_1, \ldots, x_n) \equiv \log\left(\sum_{i=1}^{n} e^{x_i}\right).$$

[5 Marks]

(*Solved example in Boyd pg74*)

Problem 4. Is the function $f : \mathbb{R}^{m\times n} \mapsto \mathbb{R}$, defined below, convex for any $p, q \in [1, \infty]$?

$$f(X) \equiv \begin{array}{ll} & \max_{u\in\mathbb{R}^m, v\in\mathbb{R}^n} e^{u^\top X v} \\ \text{s.t.} & \|u\|_p \le 1, \|v\|_q \le 1, \end{array}$$

[5 Marks]

Appeared 16-Sep-2011 Midsem

Problem 5. Prove the following statement: If a cone admits a finite primal description then it will also admit a finite dual description.

[5 Marks]

Discussed in Tutorial

Problem 6. Is the negative harmonic mean function $h : \mathbb{R}^n \mapsto \mathbb{R}$, defined below, convex?

$$h(x_1, \ldots, x_n) \equiv \frac{-n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}},\ \forall\ x_i > 0.$$

[5 Marks]

Appeared 15-Sep-2012 Midsem

Problem 7. Consider $K = \{(u, v) \in V \times \mathbb{R} \mid \|u\| \le v\}$, where V is a given (but arbitrary) set of vectors forming an inner-product space $\mathbb{V}$. Note that $\|\cdot\|$ used in defining K may NOT be the norm induced by the inner-product in $\mathbb{V}$. Write down a simplified expression for the dual cone K^* (in the space that is direct-sum of spaces $\mathbb{V}$ and $\mathbb{R}$).

[5 Marks]

Solved example in Boyd E.g. 2.25

Problem 8.

Let $\mathcal{P}$ be an arbitrary (but given) polyhedron in $\mathbb{R}^n$. Pose the problem of finding the largest $\|\cdot\|_p$-norm ball[1] (here, $p \ge 1$) lying inside the polyhedron as a convex problem. Further, do you think this problem can be posed as a convex program with finite number of linear inequality constraints[2]?

[10 Marks]

Appeared 16-Sep-2011 MidSem

[1] Needless to say, the center of the p-norm ball need not be the origin.

[2] In this case the domain of the convex program you write needs to be a vector space. Else it is trivial to answer this question.