Practice Problems on Nearest Neighbours

January 20, 2015

Note: Please do not copy answers from your friends. Please do not submit your solutions to us, as we do not plan to evaluate them. Please feel free to discuss solutions with course instructor.

1. Let the probability of turning head with a coin be $p < 0.5$. Consider the function f defined over the set of odd numbers as follows. $f(k)$ is the probability that majority of k tosses of this coin are heads. Show that the function strictly decreases with increasing k.

2. Using the above, prove that $R^{2k+1} \leq R^{2k-1}$, $\forall\, k \in \mathbb{N}$. In particular, show that $R^{3NN} \leq 1.32R^*$ and $R^{5NN} \leq 1.22R^*$.

3. Consider a classification problem with p classes i.e., $|\mathcal{Y}| = p$. Let R^* denote the expected misclassification error with Bayes optimal and R^{NN} denote the limiting value of the same, as $m \to \infty$, incurred with a nearest neighbour classifier. Prove the following asymptotic bound:

$$R^{NN} \leq 2R^* - \frac{p}{p-1}\left(R^*\right)^2.$$

4. Mr. *Subuddhi* claims the following:"give me billions and billions and billions of examples of rational and irrational numbers. Then simply by employing a nearest neighbour algorithm, I can recognize rationality of any new real (unseen before)". Can he be true? In particular, will NN achieve a very low error?