Practice Problems on Kernel Methods

February 23, 2015

Note: Please do not copy answers from your friends. Please do not submit your solutions to us, as we do not plan to evaluate them. Please feel free to discuss solutions with course instructor.

1. Prove the representer theorem for Fisher linear classifier. More specifically, show that the optimal solution w^* of equation (3), in the practice problems for SVM, satisfies the following property: $\exists\ \alpha \in \mathbb{R}^m \ \ni\ w^* = \sum_{i=1}^m \alpha_i \phi(x_i)$. Hence neither for training, nor for inference, ϕ is explicitly needed and it is enough to specify a kernel over $\mathcal{X}$. Now write down an optimization formulation equivalent to (3) involving only kernel values and NOT involving any ϕ term (explicitly).

2. Exercises 5.2 and 5.4 in "Foundations of Machine Learning" by Mohri et.al.

3. Let $x \in \mathcal{X}$ denote an image stored on a computer. Assume that all images in $\mathcal{X}$ have the same resolution i.e., same number of pixels. Let x_i denote the number of pixels in x that fall in a color bin i. Assume that there are n color bins i.e., $i = 1, \ldots, n$. The function $k : \mathcal{X} \times \mathcal{X} \mapsto \mathbb{R}$, given by $k(x, y) \equiv \sum_{i=1}^n \min(x_i, y_i)\ \ \forall\ x, y \in \mathcal{X}$, is a valid kernel. Hint: First consider $n = 1$. Then try to guess a $\phi : \mathcal{X} \mapsto \mathbb{R}^m$ such that $k(x, y) = \phi(x)^\top \phi(y)$.