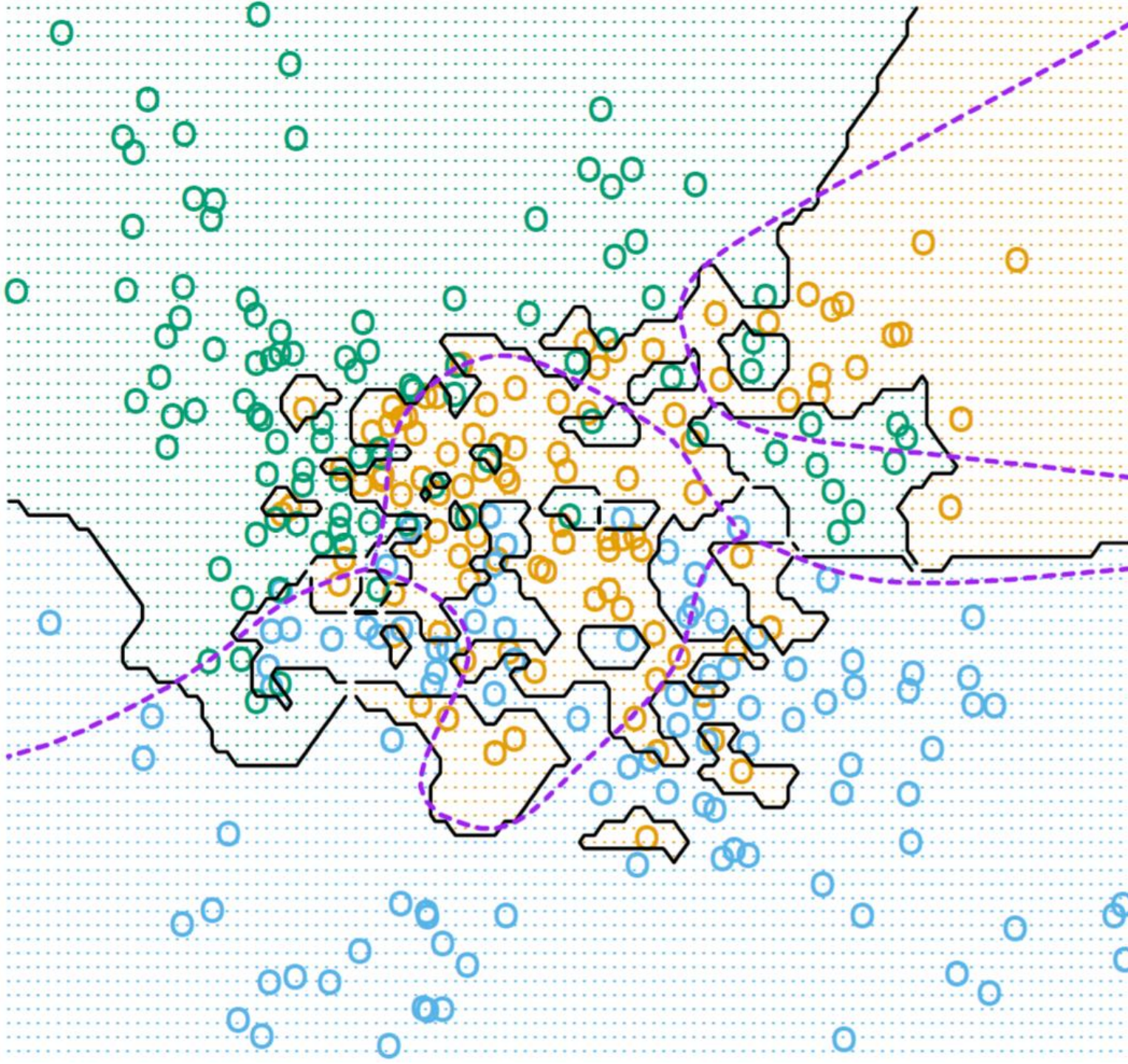


# NEAREST NEIGHBOUR CLASSIFIER

Lecture-4







## CLASSIFICATION BOUNDARY WITH NNC

- 2-d Euclidean example
- Dirichlet Tesselation or Voronoi Decomposition
- Bayes optimal in purple



# ASYMPTOTIC BOUND

- **Theorem:** Let  $R^{NN}$ , and  $R^*$  denote the misclassification rates with NNC ( $m \rightarrow \infty$ ) and Bayes optimal respectively, then, under mild conditions, we have

$$0 \leq R^* \leq R^{NN} \leq 2R^*$$





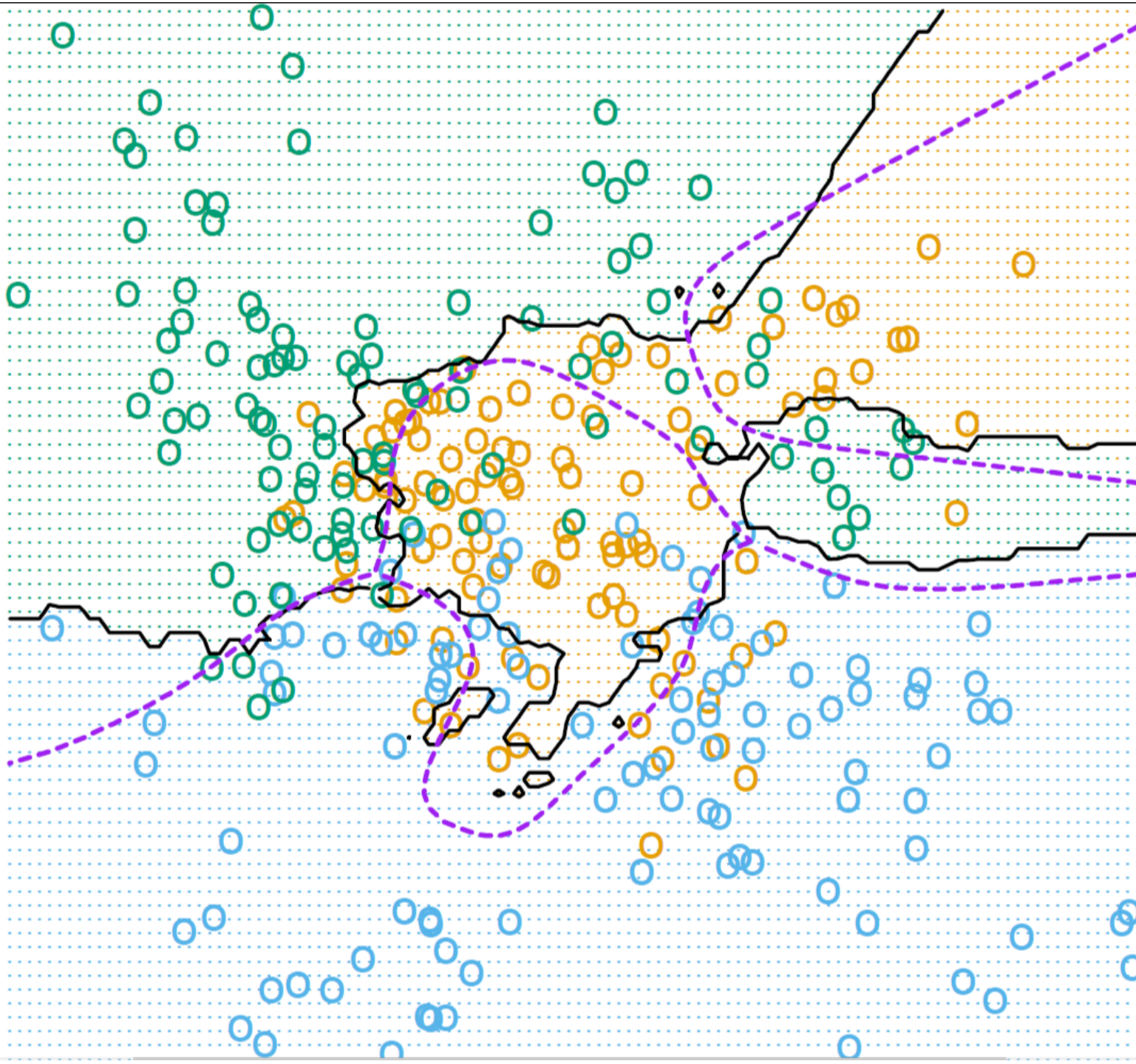
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- Bayes consistency, if  $R^* = 0$ ; need not be otherwise





## CLASSIFICATION BOUNDARY WITH KNNC

- Smoother boundary
- Closer to Bayes optimal



# ASYMPTOTIC BOUND

- **Theorem:** Let  $R^{kNN}$ , and  $R^*$  denote the misclassification rates with NNC ( $m \rightarrow \infty$ ) and Bayes optimal respectively, then, under mild conditions, we have

$$0 \leq R^* \leq \dots \leq R^{3NN} \leq R^{NN} \leq 2R^*$$

- Bayes consistency, if  $R^* = 0$ ; need not be otherwise



# BAYES CONSISTENCY [STONE77]

- **Theorem:** If  $m \rightarrow \infty, k \rightarrow \infty, \frac{k}{m} \rightarrow 0$ , then with probability at least  $1 - \delta$ , we have:

$$R_m^{NN} - R^* \leq \sqrt{\frac{72\gamma^2 \log 2/\delta}{m}}$$
$$\gamma \leq \left(1 + \frac{2}{\sqrt{2 - \sqrt{3}}}\right)^n - 1$$



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Curse of dimensionality!

