CS725 Jan-Apr, 2015
Quiz 1, 11-Feb-2015

Roll No: CS725 INSTRUCTOR

Consider a ternary classification problem with classes 'positive', 'negative' and 'in-between'. For example, a kid trying to classify a bucket of water into hot (positive), cold (negative), and luke-warm (in-between). **The important point here is that the 'in-between' class has a relation with both the other classes in the sense that it is indeed between them.** Now, mimic the way we dealt with the problem of binary classification and think about adapting/extending i) nearest neighbour ii) decision tree and iii) maximum-margin linear classifier methodologies. In each case, clearly write/illustrate/identify the following: i) model and its parameters (if any) ii) Training algorithm iii) Inference/prediction algorithm. In case you pose ii) and/or iii) as an optimization problem, then presenting the formal optimization problem is enough. While $\mathcal{X}, \mathcal{Y} = \{-1, 0, 1\}$ denote the input, output spaces respectively; $\phi_1, \ldots, \phi_n$ denote the input features and $\mathcal{D} = \{(x_1, y_1), \ldots, (x_m, y_m)\}$ denotes the training set.

1. (*1 point*) Nearest Neighbour

MODEL: k-NN with modified inference instead of majority vote (see below)

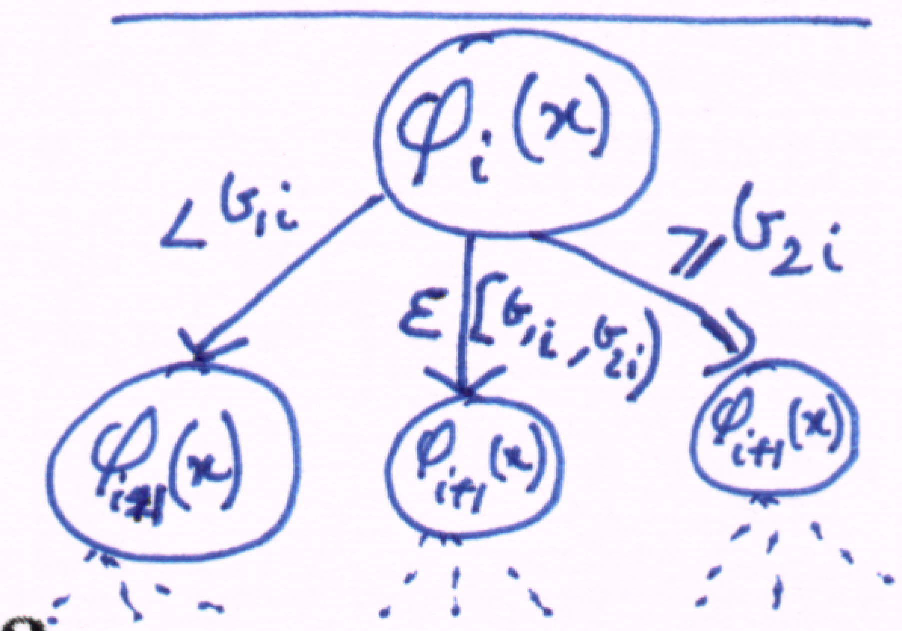
TRAINING: Store $\mathcal{D}$.

INFERENCE: Label of $x = \begin{cases} 1 & \text{if average of labels of k-NN} \geq 1/3 \\ 0 & \text{if average of labels of k-NN} \in [-1/3, 1/3) \\ -1 & \text{if average of labels of k-NN} < -1/3 \end{cases}$

2. (*1 point*) Decision Tree

At stage i

TERNARY TREE MODEL



Training for b_{2j}, b_{1j}:

$$\min_{b_{2j} \in \mathbb{R}} \sum_{i=1}^{m} \mathbb{1}\{(y_i - 1/2)\varphi_j(x_i) > (y_i - 1/2) b_{2j}\}$$

$$\min_{b_{1j} \in \mathbb{R}} \sum_{i=1}^{m} \mathbb{1}\{(y_i + 1/2)\varphi_j(x_i) \geq (y_i + 1/2) b_{1j}\}$$

Inference

Run through the tree and take average label of points in the leaf node. Similar to question ①

3. (*1 point*) Maximum-Margin Linear Classifier

Model: $\mathcal{F} = \left\{ f \;\middle|\; \exists w \in \mathbb{R}^n, b_1 < b_2 \in \mathbb{R} \ni f(x) = \begin{cases} 1 & \text{if } w^T\varphi(x) \geq b_2 \\ 0 & \text{if } b_1 \leq w^T\varphi(x) < b_2 \\ -1 & \text{if } w^T\varphi(x) < b_1 \end{cases} \forall x \in \mathcal{X} \right\}$

TRAINING:

$$\min_{w \in \mathbb{R}^n, b_1, b_2} \frac{1}{2}\|w\|^2 + \frac{C_1}{m}\sum_{i=1}^{m} \xi_i^1 + \frac{C_2}{m}\sum_{i=1}^{m} \xi_i^2$$

s.t.

$$(y_i - 1/2)(w^T\varphi(x_i) - b_2) \geq 1 - \xi_i^1, \quad \xi_i^1 \geq 0, \ \forall i,$$

$$(y_i + 1/2)(w^T\varphi(x_i) - b_1) \geq 1 - \xi_i^2, \quad \xi_i^2 \geq 0, \ \forall i,$$

$$b_1 \leq b_2.$$

INFERENCE: Already given in model.