# EDGE ENVELOPE BASED RECONSTRUCTION OF TORN DOCUMENT 

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#### Abstract

The paper proposes a new algorithm for reconstruction of torn document ripped-up by hand using a unique feature, Edge Envelope. The essence of the paper is to match the edges using high curvature points and edge variations as human visual system does while reconstructing the document. Moreover the ambiguity created during the matching of edges due to shearing can be overcome by smoothing the edge envelopes using a filter. Further, a newly defined parameter Normalized Edge Envelope Difference can be used as an effective quantitative measure for matching the edges of torn pieces, which exploits property of human visual system. The effectiveness of the algorithm is demonstrated through preliminary results by taking limited amount of torn pieces.


## Keywords

Track-Map, Corner Points, Edge Envelope, Normalized Edge Envelope Difference.

## 1. INTRODUCTION

Recovery of ripped-up document is a problem that often arises in archival study, forensics, defence and investigation sciences. Document may be ripped-up by hand or shredded by a machine [14]. The reconstruction of a torn or shredded document is a puzzling, complex and painstaking task to be performed by a human operator, and often becomes intractable even for a small number of fragmented pages. In both cases, the automatic reconstruction of the original document is a challenging task. In this paper, we deal with automatic reconstruction of torn document ripped-up by hand.

In case of machine shredding of pages, each fragment piece of paper has straight edge only and hence the problem may be considered as a variety of jigsaw puzzle [13]. But the reconstruction of torn document by hand is considerably different

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## 2. PROPOSED ALGORITHM

The proposed algorithm has four phases as shown in Fig. 1. Initially, each piece of ripped-up document is pre-processed to obtain single pixel width contour. Then, a feature edge envelope is extracted from contour in order to carry out the matching and joining process. In the following section, we describe each of the phases in detail.

### 2.1. Preprocessing Phase

The aim of the preprocessing phase is to estimate outline/contour of the torn pieces. The application of conventional algorithms like canny, sobel, prewitt which work on gray level images results in unwanted details (like edges of the text, image etc.) present within the torn piece in addition to the required contour as in Fig. 2 (a). These unwanted details make the tracking of the outlier/contour difficult. Hence, the scanned image of torn piece is converted to binary image before applying conventional edge detection algorithm.

The torn piece is scanned such that it is well within the boundary of the scanned image. This makes sure that the background is well estimated and it is used for computation of a threshold value using histogram [2]. The computed threshold is used to binarize the image. The binary image is then passed through canny/sobel edge detector to get the single pixel width contour. From Fig. 2 (b) it is observed that, the proposed way of getting the contour is free from multiple and spurious contours compared with that of direct application of edge detection algorithms (Fig. 2 (a)).

The scanned image of each torn piece $P_{t}, t=1,2,3, \ldots, p$, where $p$


Fig. 1 Overview of the Proposed Algorithm.


Fig. 2 (a) Contour extracted using Canny Edge Detector (b) Contour extracted after Binarization
is total number of torn pieces, is converted to binary image. The binary image is then passed through canny/sobel edge detector to get the single pixel width contour $C_{t}$. The coordinate locations of points on a contour $C_{t}$ are very important information in pattern recognition and other applications. They can be obtained by using a contour tracking algorithm. It scans an image from left to right, and up to down to locate the first point on a given $C_{t}$ as the tracking point. Once the first point has been located, then 8connected neighbors are checked in clockwise to find the next tracking point. This procedure is repeated until all the points on $C_{t}$ have been traced. The contour $C_{t}$ can then be expressed in terms of track-map $T_{t}$ which has coordinate locations of points on a contour $C_{t}$.

To match the torn pieces it is required to extract each edge of the torn pieces. The extraction of edges is based on points with high curvature values or with discontinuity of the slopes, because of their significance in visual perception [5]. These high curvature points are extracted using corner detection algorithms. Among the different corner detection algorithms like Bending value [11], Harris [3], SUSAN [9], Bending value method is used in this approach. Because, the calculation of bending value only requires (integer) addition, subtraction, and comparison operations, thus it is simple in computation.

Corner point detection using bending value involves break point detection and it's bending value computation. The point under consideration is a break point if it doesn't form a straight line with its two immediate neighboring points. By this, points which form a straight line are eliminated thereby reducing the competitors for corner points. Hence, it is more efficient than those algorithms evaluating all the points on a given curve and involving more complex computations, such as division and tangent operations [11].

In this approach, instead of computing break points after extraction of $T_{t}$, it is computed during extraction of $T_{t}$. The extracted break points are used to find the bending value as in [11]. Now the break point with maximum bending value is considered as the probable candidate for corner point.

The above procedure of corner point detection results in many unwanted corners (Fig. 3), which creates a lot of ambiguity at matching phase of the torn pieces. These spurious corners are removed if the angle at the corner point between the corners


Fig. 3 Removal of spurious corner points.
exceeds a predefined threshold. From Fig. 3, the angle at the corner point $B$ is calculated using the following equation,

$$
\begin{equation*}
\theta=\cos ^{-1}\left(\frac{u \cdot v}{|u||v|}\right) \tag{1}
\end{equation*}
$$

where, $u$ is a distance vector between present corner point $B$ under consideration and its immediate previous corner point $A$. $v$ is a distance vector between present corner point $B$ under consideration and its immediately following corner point $C$.
In conventional methods, a polygonal approximation [7] is applied to each contour as a preprocessing step. It is a nonparametric and a recursive method to get the best representation of the contour $[10,14]$. This recursive nature of polygonal approximation makes the contour tracking complex compared to the method discussed in present section, which is a non recursive.

### 2.2. Feature Extraction

Human visual system is one of the most sophisticated and versatile systems in nature. This complicated system uses the edge variations of torn pieces for matching with its counterpart. This concept of human visual system is groundwork for the proposed feature extraction.

In the feature extraction phase, the edge variation between the two corner points is captured and is referred as Edge Envelope. From


Fig. 4 Edge Envelope Extraction.

Fig. 4, $C P_{1}^{1}$ and $C P_{1}^{2}$ are the two corner points of piece $P_{1}, L$ is the line joining these two corner points, $E_{1}$ is the edge between these two corner points and the points along the edge $E_{1}$ are denoted by $E_{1}\left(x_{i}, y_{i}\right)$, where $i$ varies from 1 to total number of points $t_{p}$ defining the edge. The mapping of the two-dimensional spatial co-ordinate points $E_{1}\left(x_{i}, y_{i}\right)$ to its one-dimensional Edge Envelope is as follows:

1) Consider a straight line $L$ between two corner points $C P_{1}^{1}(x, y)$ and $C P_{1}^{2}(x, y)$. Further, $L_{i}$ is a straight line between two points $C P_{1}^{2}(x, y)$ and $E_{1}\left(x_{i}, y_{i}\right)$.
2) In order to capture the edge variations, the perpendicular distances $A_{i}$ between straight line $L$ and the points $E_{1}\left(x_{i}, y_{i}\right)$ corresponding to edge $E_{1}$ are computed using following Eqns. (2) and (3).

$$
\begin{align*}
\alpha_{i} & =\tan ^{-1}\left(\frac{C P_{1}^{2}(y)-C P_{1}^{1}(y)}{C P_{1}^{2}(x)-C P_{1}^{1}(x)}\right)-\tan ^{-1}\left(\frac{C P_{1}^{2}(y)-E_{1}\left(y_{i}\right)}{C P_{1}^{2}(x)-E_{1}\left(x_{i}\right)}\right) \\
A_{i} & =\operatorname{sqrt}\left(\left[C P_{1}^{2}(y)-E_{1}\left(y_{i}\right)\right]^{2}+\left[C P_{1}^{2}(x)-E_{1}\left(x_{i}\right)\right]^{2}\right) * \sin \left(\alpha_{i}\right) \tag{3}
\end{align*}
$$

where, $i=1, \ldots, t_{p}$

$$
\alpha_{i} \text { is the angle between the lines } L \text { and } L_{i},
$$

3) The procedure of Edge Envelope extraction is repeated for all the edges.

### 2.3. Matching Phase

The human visual system finds the one-to-one similarity by comparing the edge patterns of the torn pieces. In a similar way, the degree of similarity between the edge patterns of torn pieces is computed by comparing the extracted Edge Envelopes. To find the one-to-one similarity like human visual system does, a new parameter Normalized Edge Envelope Difference (NEED) is introduced and is given by,

$$
\begin{equation*}
N E E D=\frac{1}{M} \sum_{j=1}^{M}\left|S_{1}^{a}(j)-S_{2}^{b}(j)\right| \tag{4}
\end{equation*}
$$

where, $M$ is $\max \left(\right.$ length $\left[S_{1}^{a}(j)\right]$, length $\left.\left[S_{2}^{b}(j)\right]\right)$,
$S_{1}^{a}(j)$ is the $a^{\text {th }}$ edge envelope signal of the torn piece $P_{1}$, $S_{2}^{b}(j)$ is the $b^{t h}$ edge envelope signal of the torn piece $P_{2}$. $P_{1}, P_{2} \in\left\{P_{t}\right\}$, and $P_{1}, P_{2}$ are mutually exclusive.

The value of NEED thus computed is used as the measure of one-to-one matching between the Edge Envelopes of the torn pieces and lower the value of $N E E D$, the better is the degree of similarity between the Edge Envelope under consideration. The corners for a torn piece may be topologically different from those of its counterparts, depending on the nature of hand movements during torn process and behavior of the corner detection process. Because
of this, total number of points $t_{p}$ of edge envelope under consideration and its counterpart will be different. To take care of this, the edge envelope with the shorter $t_{p}$ is interpolated to the length of the longer edge envelope. The outcome of the matching phase is the matched edges of the torn pieces that are to be joined to get a single reconstructed document.

Due to limitations in the imaging mechanism, the torn piece having a shear, after being scanned, may contain digital imperfections and unpredictable rigid transformations. This creates a lot of ambiguity in matching phase and it can be overcome by smoothing the edge envelope by filtering.

### 2.4. Joining Phase

The matched edges of the torn pieces may have different orientations because of the positioning of the torn pieces during scanning. Now in order to join the pieces, it is required to rotate and translate the pieces under consideration. This requires the computation of rotation and translation parameters.

Let $C P_{1}^{1}(x, y)$ and $C P_{1}^{2}(x, y)$ be the corner points of the edge $E_{1}$ for the torn piece $P_{1}$ which has its matching counterpart in torn piece $P_{3}$, whose edge is represented by $E_{2}$ with the corner points $C P_{3}^{2}(x, y)$ and $C P_{3}^{3}(x, y)$. The rotation angle $R A$ between the edges $E_{1}$ and $E_{2}$ is calculated using Eqn. (5).
$R A=\tan ^{-1}\left(\frac{C P_{1}^{2}(y)-C P_{1}^{1}(y)}{C P_{1}^{2}(x)-C P_{1}^{1}(x)}\right)-\tan ^{-1}\left(\frac{C P_{3}^{3}(y)-C P_{3}^{2}(y)}{C P_{3}^{3}(x)-C P_{3}^{2}(x)}\right)$
The corner points $C P_{3}^{2}(x, y), C P_{3}^{3}(x, y)$ of $P_{3}$ are updated by an angle $R A$ to $R C P_{3}^{2}(x, y), R C P_{3}^{3}(x, y)$ respectively. These new corner points are used to find the translation parameters $T_{x}, T_{y}$ as in Eqns. (6), (7)
$T_{x}=\frac{1}{2}\left(\left[C P_{1}^{2}(x)+C P_{1}^{1}(x)\right]-\left[R C P_{3}^{3}(x)-R C P_{3}^{2}(x)\right]\right)$
$T_{y}=\frac{1}{2}\left(\left[C P_{1}^{2}(y)+C P_{1}^{1}(y)\right]-\left[R C P_{3}^{3}(y)-R C P_{3}^{2}(y)\right]\right)$
The torn piece $P_{3}$ is rotated by an angle $R A$ and then translated by $T_{x}, T_{y}$. The updated torn piece $P_{3}$ is joined with its matched counterpart $P_{1}$ to make a larger piece $P_{13}$. This new piece $P_{13}$ with its updated corner points replaces $P_{1}, P_{3}$ and the above procedure is repeated for the remaining torn pieces $P_{t}$.

## 3. RESULTS AND DISCUSSION

For our experiments, we have used torn pieces of document and book. Two set of such scanned images of torn pieces are shown in Fig. 5 (a) and 6 (a) for illustration. Fig. 5 (a) shows scanned images of four torn pieces of a page taken from a book and Fig. 6 (a) shows six torn pieces of a document. For background estimation of all the scanned images of the torn pieces, border pixels of width four are used. The computed histogram from these border pixels is used to find the pixel value which has resulted in
maximum count. This pixel value with a deviation of three is used for Binarization of the scanned image of torn pieces. The canny edge detector with a threshold on 0.6 is used to get single pixel width contour. In corner detection phase, the points with an angle greater than 135 degrees are considered as spurious corners and are not considered for further analysis.

The proposed unique feature Edge Envelope is extracted for all the edges of the torn pieces. It is observed that, the scanned torn pieces of the document shown in Fig. 5 (a) and 6 (a) have reasonable shear associated with them. This creates a lot of ambiguity in matching phase and it is overcome by smoothing the edge envelopes using 5 -point moving average filter. The typical edge envelope $S^{l}$, with its matched $S^{2}$ and unmatched $S^{3}$ counterpart are shown in Fig. 7. From Fig. 7, it has been found that the degree of similarity between $S^{I}, S^{2}$ is more compared with that of $S^{I}, S^{3}$. Also, the newly defined $N E E D$ value between $S^{I}, S^{2}$ is lower compared to that of $S^{I}, S^{3}$. Thus the NEED can be used as an effective quantitative measure for matching the edges of torn pieces, which exploits the property of human visual system. The reconstructed documents of scanned images are shown in Fig. 5 (b) and 6 (b) respectively. It is observed from Fig. 5 and 6 that, the reconstructed documents from the torn pieces matches with that of the original documents.

## 4. CONCLUSIONS AND FUTURE WORK

A new novel technique for reconstruction of document ripped-up by hand is proposed, which uses a unique feature, edge envelope that helps in successful matching of torn pieces. As human visual system uses high curvature points and edge variations for finding match between the edges, a feature that captures these edge variations using high curvature corner points is proposed. The ambiguity created during the matching of edges due to shearing is overcome by smoothing the edge envelopes through filtering. From the results, a newly defined parameter NEED is indeed an effective quantitative measure for matching the edges of torn pieces. Even in-spite of using a single feature, the proposed algorithm is able to reconstruct the document ripped-up by hand very effectively.

Automatic reconstruction of a document from lager number of torn pieces from single and multiple pages is the scope of future work.

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Fig. 5 (a) Scanned images of torn pieces of a page from a book and (b) its reconstructed page


Fig. 6 (a) Scanned images of torn pieces of a document and (b) its reconstructed document


Fig. 7 Edge Envelopes of torn pieces.
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