Nonparametric Markov Random Field order estimation and its application in Texture Synthesis

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ABSTRACT

In this paper we propose a new theory for the order estimation of nonparametric Markov random field (N-MRF) model. Texture synthesis based on N-MRF model performs well visually for a wide range of natural textures, [9]. The result of texture synthesis is dependent upon the model order, and the computational complexity increases parabolicaly with the model order. Therefore, it is required to estimate the minimum model order for computationally efficient texture synthesis. In the proposed methodology, the basic definition of local conditional density is redefined. The proposed model order estimation (MOE) approach for N-MRF model has been tested with a number of stochastic and near-regular textures, collected from the Brodatz's standard database [3]. Results show the efficacy of the proposed approach in solving the MOE problem efficiently.

1. INTRODUCTION

Texture synthesis is an important topic in the field of computer vision and image understanding [9, 14, 1, 7, 6, 15, 13]. The basic problem is to synthesize an arbitrarily sized texture from a small sample of natural texture. It is difficult to find a unified algorithm for different kinds of natural textures, varying from stochastic to near-regular. Within this domain, nonparametric Markov random field (N-MRF) model approach has gained good confidence, [9, 14, 1, 7]. The algorithms based upon N-MRF model can be categorized broadly into two classes, 1) pixel-based algorithms (PIX-AL) [9, 14, 1], and 2) patch-based algorithms (PAT-AL) [7, 6, 15, 13]. In case of PIX-AL, one site is synthesized, whereas PAT-AL synthesizes a fixed/arbitrary size of patch within one iteration. Therefore, PAT-AL is faster than the PIX-AL and maintains the local structure within the synthesized texture. But, a significant drawback of PAT-AL is the probable presence of broken features at the boundary of adjacent patches. This problem have been attempted in [6, 15, 13].

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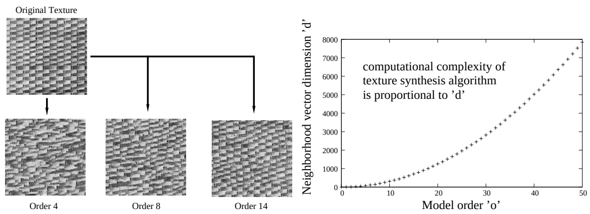
ICVGIP '10, December 12-15, 2010, Chennai, India



(a) Effect of N-MRF model order on texture synthesis result; The synthesis algorithm is taken from [9]

(b) The variation of dimension of neighborhood vector with respect to the model order

Figure 1: Motivation for the study of N-MRF model order estimation (MOE)

In figure, 1(a), the effect of order in PIX-AL for texture synthesis, is shown. From this figure it is clear that as the order is increased, the synthesized texture looks much more similar to the original one. From figure 1(b), it can be observed that, as we increase the order, the computational complexity increases. Therefore, it is required to estimate the minimum order for which the synthesis result will be visually similar to the original texture. We have attempted to solve this problem in this paper.

Liang et al. [7] has described the effect of patch-size on the synthesis results. They showed that as the size increases the synthesized texture resembles closely the original texture. A parameter referred to as boundary zone width is also used in patch-based algorithm, [7]. They stated that, this second parameter can not be too small or too large. Therefore, the estimation of these two parameters within PAT-AL is required, but more difficult than the order estimation problem of PIX-AL. We believe that, the proposed order estimation methodology will pave a new direction for solving more complex issues of parameter estimation within PAT-AL's.

1.1 A brief review of earlier methods

There have been several attempts in the past to estimate the MRF model order. These approaches can be classified into two groups, 1) maximization of pseudo-likelihood measure and 2) indirect approaches. Besag in [2], has proposed a pseudo-likelihood measure for parametric MRF model. The second approach given in [11], builds around the concept of combining the spatial periodicity and the order estimation problem. This second approach is restricted to near-regular textures, where one can find a spatial periodicity and can depend upon it for order estimation. Natural textures however, cover a wide range from near-regular to stochastic. Therefore, this approach described in [11], has both theoretical and practical limitations.

In this paper, we will consider the only first approach, and concentrate on the maximization of pseudo-likelihood (PL) measure and the estimation of local conditional density, based on the work described in [2]. In section 2, we give a brief description of the N-MRF model and texture synthesis algorithm based upon N-MRF model. Section 3 discussed the PL measure. The limitations of this methodology is briefly given in this section. In section 4, we provide the solutions to overcome the limitations of the PL based order estimation problem. Results obtained for a wide range of natural textures are given in section 5. Section 6 concludes the paper.

2. NONPARAMETRIC MARKOV RANDOM FIELD AND TEXTURE SYNTHESIS

2.1 N-MRF model Definition

Let Y_s define a random variable , assuming values from a finite set of values $\mathbf{L}$, at site s on a lattice $S = \{s = (i,j) : 0 \leq i,j \leq M\}$.The neighborhood set of site s can be defined as, $\aleph_s = \{r \in S, s.t. ||r - s|| \leq ||o||\}$, where o is the order (the only parameter in the MRF) and $||.||$ is the L^2 norm. For simplicity let us define the neighborhood vector of random variable Y_s as, $X_s = \{Y_r; r \in \aleph_s\}$. The Markovian assumption in the given context is,

$$p\left(Y_s|\{Y_r; r \in S, r \neq s\}\right) = p\left(Y_s|\{Y_r; r \in \aleph_s\}\right) = p(Y_s|X_s) \quad (1)$$

this describes the fact that given the random variables at neighborhood sites, $\aleph_s$, the r.v. at s is independent of all other sites, $\{r; r \in S - \{s\} - \aleph_s\}$. This conditional probability is termed as local conditional pdf (LCPDF).

In [9] the nonparametric MRF is defined as the nonparametric estimation of the local conditional probability density function.

$$p(Y_q|X_q) = \frac{1}{h}\frac{\sum_{s \in S_{in}} K_h(Y_s - Y_q)K_h(X_s - X_q)}{\sum_{s \in S_{in}} K_h(X_s - X_q)} \quad (2)$$

, where $q \in S_{in}$, h (the window parameter) $= \sigma\{4/(n(2d+1))\}^{1/(d+4)}$; n is the number of data points, i.e., the number of pixels within the input texture; d is the dimension of the neighborhood vector, σ^2 is the average marginal variance of Y_s, and the kernel function is defined as $K_h(z) = \exp\{-\frac{z^T z}{2h^2}\}$, where $(.)^T$ defines transpose.

2.2 Brief description of synthesis algorithm

The sampling of new pixel ($q \in S_{out}$) at the output texture (S_{out}), is done according to ICM algorithm, as described in [2]. In the original code [8], the authors of [9] have implemented an approximate version of ICM. Rather than evaluating the true LCPDF, they considered the similarity metric (say $D_{s,q}$) between the X_s and X_q, where $s \in S_{in}$ the input texture and $q \in S_{out}$ the output texture, as shown below.

$$D_{s,q} = (X_s - X_q)^T \Phi_q (X_s - X_q) \quad (3)$$

Here, Φ_q corresponds to the temperature matrix at site $q \in S_{out}$ required for the local simulated annealing, [9]. If for a given $s \in S_{in}$, this distance is minimum, then replace the pixel at $q \in S_{out}$ with the pixel value at s, i.e., Y_s. Approximately, one can say that, if this distance is minimum then for $Y_q = Y_s$, the LCPDF described in equation 2 will be maximum. It can happen that for a set of sites $\{s \in S_{in}\}$ the metric $D_{s,q}$ is same. In that case, we pick any site from this set $\{s\}$ for the synthesis of site at $q \in S_{out}$. In our work, we have followed this approximate ICM methodology.

3. MAXIMUM LOG-PSEUDO-LIKELIHOOD OF THE LCPDF AND ORDER ESTIMATION

In [2], Besag had proposed a pseudo-likelihood measure for parametric MRF model. The joint probability of a time sequence can be written as a multiplication of local conditional probabilities, according to the Markovian theory of conditional independence. But, in case of a spatial distribution, such as an image, the joint distribution of the pixel random variables cannot be written as a multiplication of LCPDF's found at the sites of the lattice. In such case, one needs a pseudo-likelihood (PL) measure, which is actually multiplication of the the local likelihoods, i.e., $PL = \prod_{q \in S_in} P(Y_q|X_q)$. CsiszÃąr and Talata in [4], Ji and Seymour in [5] have studied the consistency problem of parametric MRF neighborhood order estimation, using log of PL measure (LPL). This is given by,

$$LPL = \sum_{q \in S_{in}} log[P(Y_q|X_q)] \quad (4)$$

In case of N-MRF model, the estimation of LCPDF, i.e., $P(Y_q|X_q)$, has to be nonparametric, rather than parametric. Let us say this is $N - LPL$. By nonparametric we mean that, the likelihoods $P(Y_q|X_q)$ are evaluated according to the equation 2. However, there is a problem in using the $N - LPL$ for model order estimation. This is explained in the next section.

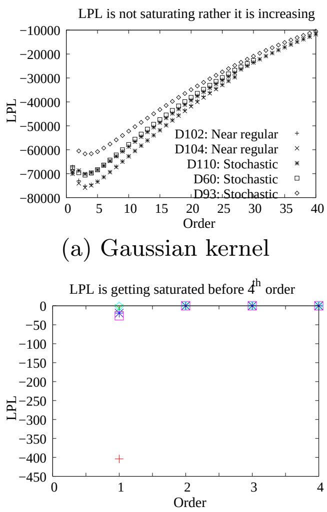
4. PROPOSED N-MRF MODEL ORDER ESTIMATION METHODOLOGY

To estimate the order, one has to evaluate the $N - LPL$ measure for each order $o = 1, 2, \ldots, K$, say $K = 40$. If the $N - LPL$ attains a maximum value and saturates beyond order $o = z$, then the minimum order for faithful texture synthesis according to N-MRF model is $o = z$.

4.1 Why the $N - LPL$ is not suitable ?

In figure 2(a), the variation of the log-pseudo-likelihood, i.e., $N - LPL$ with the model order is shown for a number of textures (both near-regular and stochastic). Figure 2(a) shows that if we measure the LCPDF using the equation 2, the measure $N - LPL$ does not saturate for any of the textures. In practice, it is known that beyond a certain order the synthesis results do not improve. Therefore, it can be concluded that, the earlier definition of LCPDF estimate, i.e., equation 2, cannot provide the solution for model order estimation (MOE) problem.

This measure $N - LPL$ is not saturating due to the normalizing parameter h as shown in the equation 2. According to Scott [10], $h = \sigma\{4/(n(2d+1))\}^{1/(d+4)}$. Therefore, the parameter varies with the dimension d of the neighborhood vector. This dimension d increases parabolically as the



(a) Gaussian kernel

(b) Dirac delta kernel

Figure 2: Limitation of Order estimation in the case of LCPDF evaluation

order o increases, as shown in figure 1. Again, this bandwidth parameter h is also used within the kernel argument, as $K_h(z) = \exp\{-\frac{z^T z}{2h^2}\}$. If one has to remove this effect of parameter h, one has to consider Dirac delta function as the kernel function.

4.2 Why the Dirac delta kernel $N - LPL$ is not suitable ?

In case Dirac delta is used as a kernel function the equation for the likelihood evaluation take the form,

$$p(Y_q|X_q) = \frac{\sum_{s\in S_{in}} \delta_h(Y_s - Y_q)\delta_h(X_s - X_q)}{\sum_{s\in S_{in}} \delta_h(X_s - X_q)} \qquad (5)$$

The variation of $N - LPL$ according to the likelihood defined in equation 5, is shown in figure 2(b). From the figure, we note that, the $N - LPL$ saturates for an order that is less than or equal to order $o = 4$, for all textures. From experience, it is known that in general, the natural textures cannot be synthesized with such low order model (with single level resolution).

Given the facts that, neither Gaussian nor Dirac delta kernel is useful to provide a practical solution for the MOE problem, we now look into the texture synthesis algorithm more closely and try to infer what should be the form of equation 2 so that MOE problem can be solved with respect to faithful texture synthesis.

4.3 New Definition for LCPDF

Let us define a new LCPDF evaluation, and the corresponding $N - LPL$ measure, as given in equation 6. In this new definition, we have used Dirac delta kernel for the center pixel random variable Y_q, and Gaussian kernel for the neighborhood vector X_q. In figure 3, we show the variation of the $N - LPL$ measure according to the new LCPDF evaluation described by equation 6. It can be observed that, this new $N - LPL$ attains saturation at different model orders depending upon the texture. A more detailed analysis of these results is given in section 5. The next sub-section provides intuitive reasons for using this new definition of LCPDF.

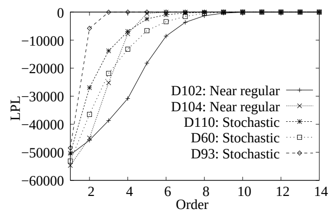


Figure 3: Proposed LPL variation with respect to the model order

$$p(Y_q|X_q) = \frac{\sum_{s\in S_{in}} \delta(Y_s - Y_q)K_h(X_s - X_q)}{\sum_{s\in S_{in}} K_h(X_s - X_q)} \qquad (6)$$

4.4 An intuitive reason for the new LCPDF definition

If one considers the texture synthesis algorithm as described in section 2.2, it becomes clear that, the algorithm replaces the current value of $Y_q, q \in S_{out}$ with the value Y_s, $s \in S_{in}$, for which $D_{q,s}$ is minimum. In other words, it samples from the set $\{Y_s : s \in S_{in,q} \subset S_{in}\}$, where $\forall s_1, s_2 \in S_{in,q}$, $D_{s1,q} = D_{s2,q}$ and for any $u \in S_{in}$ $D_{u,q} \geq D_{s1,q}$. This implies that the value of Y_q is sampled from the histogram of the data set $\{Y_s : s \in S_{in,q}\}$. From a probabilistic point of view, one does not require a kernel function (which has a finite bandwidth) for sampling of Y_q.

Again, the set $\{Y_s : s \in S_{in,q}\}$ is formed according to function $D_{s,q}$, given in equation 3. Now in the synthesis algorithm, we start with a random variation of pixel values, and therefore, if this function $D_{s,q}$ is considered as $\delta(X_s - X_q)$, the texture synthesis algorithm will fail. Since, at any stage (when the synthesis algorithm has not converged to the global solution) the neighborhood vector X_q will not be equal to any of the neighborhood vector X_s, $s \in S_{in}$. Therefore, one can conclude that, with respect to the synthesis algorithm, the kernel function for the neighborhood vector should not be the Dirac delta.

Intuitively we have reasoned out that, according to the texture synthesis algorithm, why one should use Dirac delta kernel for the center pixels Y_q and continuous finite bandwidth kernel for neighborhood vector X_q. In the following section, we will analyze the results.

5. RESULTS

Figure 4 provides $D104$ texture synthesis results for a number of orders, $o = 2, 4, 8, \ldots, 24$. (Here we have chosen only even number of orders not because of any particular reason, it is just to show the variation of synthesis results in a regular interval of orders.) The estimated order according to the proposed LCPDF is $o = 12$. This is a near-regular texture. Again, figure 5 provides results for the texture $D9$, with orders $o = 1, 3, 5, \ldots, 23$. Here the estimated order is $o = 9$. This is a stochastic texture. As can be seen from these two examples that, the estimated order produces visually good result and moreover, the synthesis results change marginally in terms of a preattentive perception. In this paper, we have shown these results with the variation of order for only two textures. The rest of the results are considered with respect to the estimated order only.

For testing the proposed definition of LCPDF, we have considered three sets of textures, 1) Near-regular (NR), 2) Stochastic (ST), and 3) NR+ST. The last category of textures, i.e., NR+ST, has a regular pattern with a stochastic perturbation, or vice versa. The test sets are chosen from the Brodatz's standard database, [3].

Earlier we have shown the proposed LPL variation with respect to a number of different textures, in figure 3. We have implemented two texture synthesis algorithms to test the performance of the proposed order estimation methodology in terms of visual similarity with the original texture sample. The first algorithm is based on the original texture synthesis paper [9], and the other algorithm implemented is based on [12]. In figure 6, the synthesis results for near-regular textures are shown with the estimated order. Figure 7 shows the results for Stochastic textures, and figure 8 provides the result for NR+ST set of textures. With an un-optimized code (c^{++}), the time taken to get the results is around 1hour to 3 hours, depending upon the nature of texture.

Within the supplementary result (**ICVGIP_order_estimation_NMRF_sup_mat·pdf**) we have shown the results for each texture for all orders. From the wide spectrum of results shown, it can be easily observed that, the estimated order can synthesize the input texture faithfully, along with minimum computational complexity.

6. CONCLUSION AND FUTURE WORK

To the best of our knowledge, the proposed work is the first methodology that has handled the issue of N-MRF model order estimation from a theoretical as well as practical point of view. We have redefined the estimation of LCPDF according to the need. The results obtained for a wide range of natural textures taken from a standard database, [3], establish the efficacy of proposed methodology in solving the MOE problem. There are several challenges that need to be addressed, 1) estimation of the order for in-homogeneous or structural textures, 2) extension of the idea of order-estimation for PAT-AL's, 3) defining texture similarity measures based upon the proposed methodology of order estimation, 4) application in unsupervised texture classification and segmentation with N-MRF model, etc.

We very much appreciate the reviewers' suggestions and questions. As it has been pointed out that this research is not complete from both theoretical and practical point of views. We would definitely like to experiment inclusion of the estimation of bandwidth parameter within this experimental setup, in near future. Although we belief that by estimating the bandwidth parameter will not solve the order estimation problem, and this proposed change of kernel function is one of the practical choice for the model order estimation problem. Moreover, at this juncture, we are not able to provide any theoretical justification of getting results with respect to our proposed defination of LCPDF. In near future, we will definitely try to do so, from the theoretical perspective. One possible pathway for theoretical justification is described in the following.

We know that as number of dimension increases, the required number of samples for the kernel density estimation also increases (almost exponentially). But, in our case we lack that many number of sample points. This practical limitation has to be taken into account during the calculation

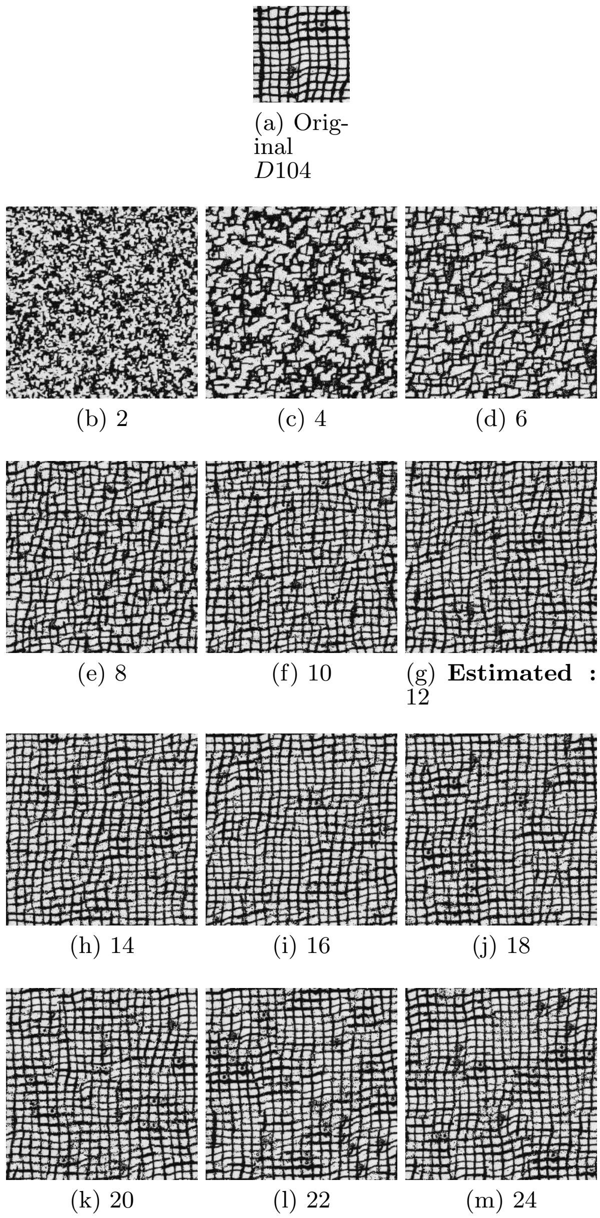
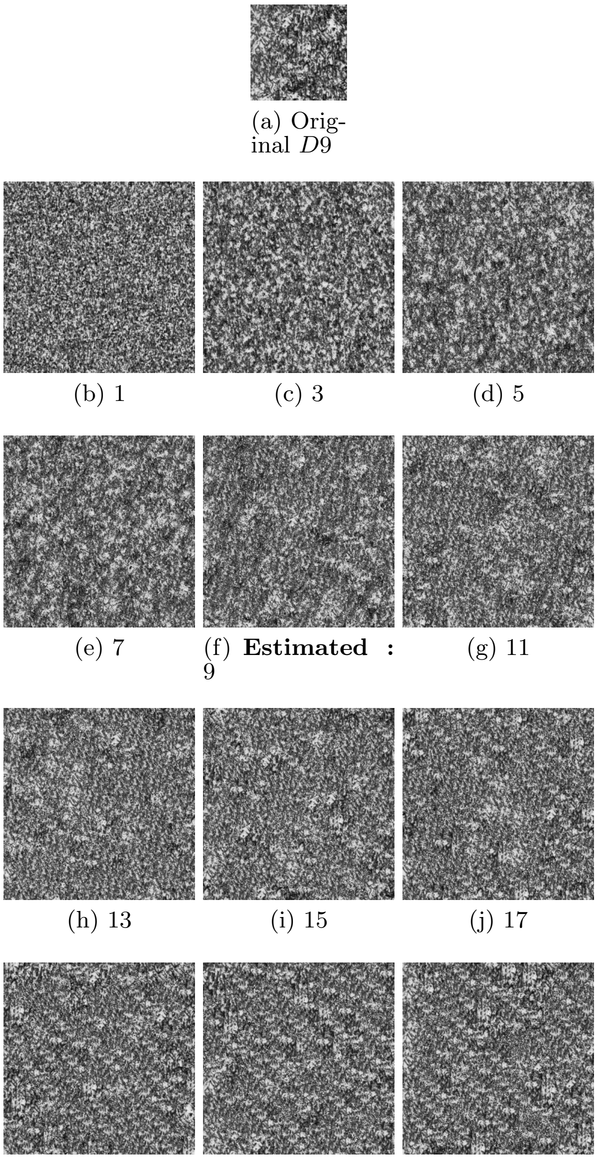


Figure 4: Texture $D104$: synthesis results with the variation of order



(a) Original $D9$

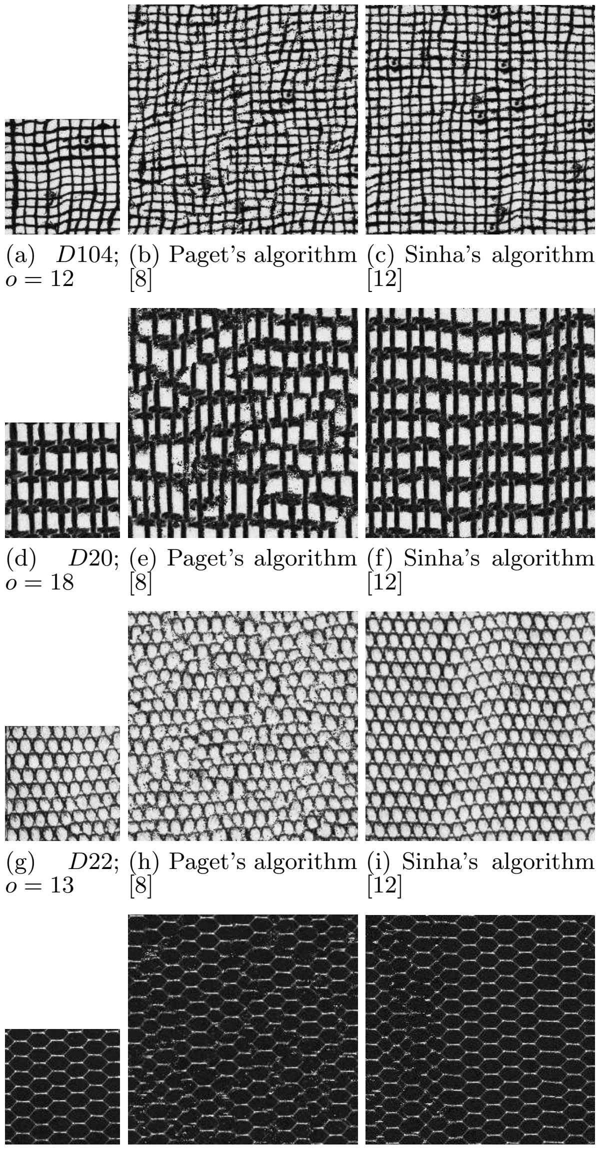
(b) 1 (c) 3 (d) 5

(e) 7 (f) **Estimated : 9** (g) 11

(h) 13 (i) 15 (j) 17

(k) 19 (l) 21 (m) 23

Figure 5: Texture $D9$: synthesis results with the variation of order



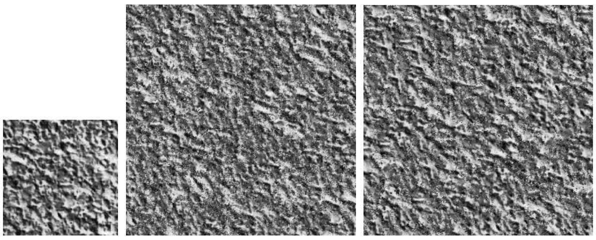
(a) $D104$; $o = 12$ (b) Paget's algorithm [8] (c) Sinha's algorithm [12]

(d) $D20$; $o = 18$ (e) Paget's algorithm [8] (f) Sinha's algorithm [12]

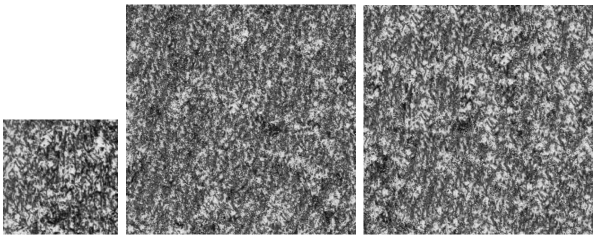
(g) $D22$; $o = 13$ (h) Paget's algorithm [8] (i) Sinha's algorithm [12]

(j) $D34$; $o = 17$ (k) Paget's algorithm [8] (l) Sinha's algorithm [12]

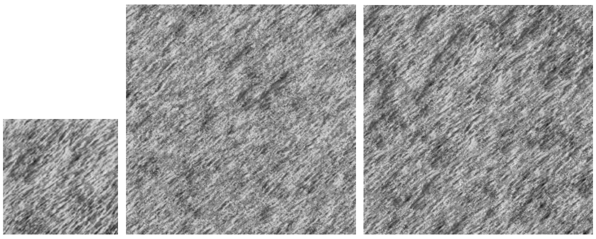
Figure 6: Near-regular (NR) texture synthesis results; estimated order is o



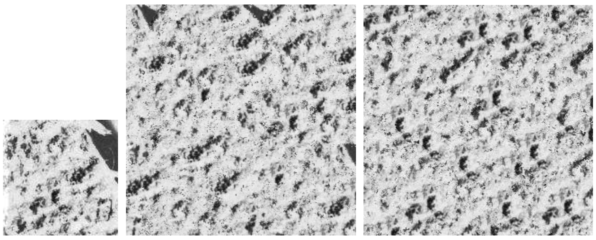
(a) $D4$; $o = 10$ (b) Paget's algorithm [8] (c) Sinha's algorithm [12]



(d) $D9$; $o = 9$ (e) Paget's algorithm [8] (f) Sinha's algorithm [12]

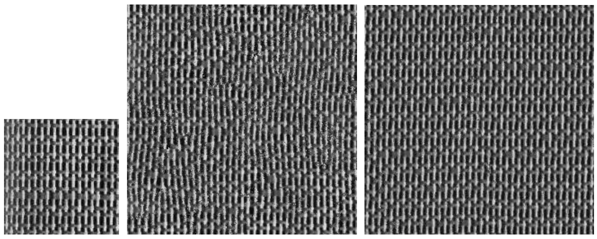


(g) $D93$; $o = 9$ (h) Paget's algorithm [8] (i) Sinha's algorithm [12]

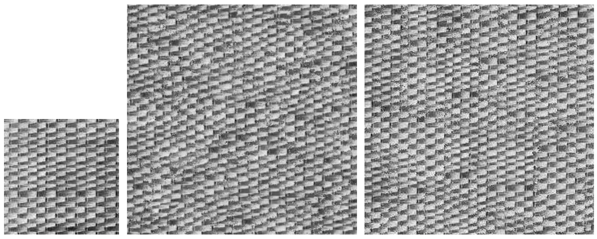


(j) $D97$; $o = 16$ (k) Paget's algorithm [8] (l) Sinha's algorithm [12]

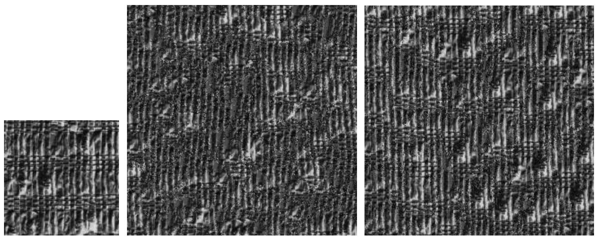
Figure 7: Stochastic (ST) texture synthesis results; estimated order is o



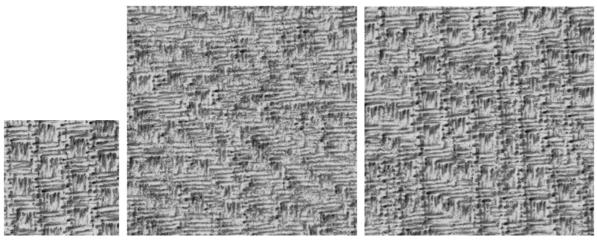
(a) $D53$; $o = 17$ (b) Paget's algorithm [8] (c) Sinha's algorithm [12]



(d) $D55$; $o = 14$ (e) Paget's algorithm [8] (f) Sinha's algorithm [12]



(g) $D80$; $o = 14$ (h) Paget's algorithm [8] (i) Sinha's algorithm [12]



(j) $D82$; $o = 12$ (k) Paget's algorithm [8] (l) Sinha's algorithm [12]

Figure 8: Near-regular + Stochastic (NR+ST) texture synthesis results; estimated order is o

of density and corresponding LPL. We belief that this practical limitation has forced the choice of kernel for Y_s within LCPDF function.

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