CS 747, Autumn 2022: Lecture 7

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Markov Decision Problems

1. Alternative formulations of MDPs
2. Some applications of MDPs

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It is relatively straightforward to handle all these variations.

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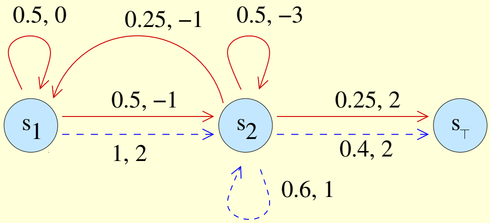

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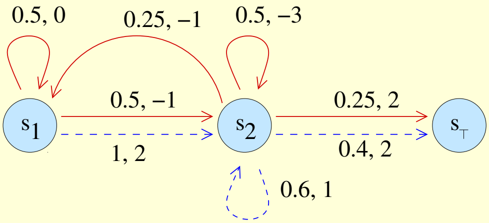

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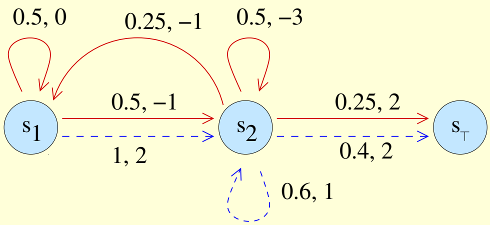

- Additionally, from every non-terminal state and for every policy, there is a non-zero probability of reaching the terminal state in a finite number of steps.
- Hence, trajectories or episodes almost surely terminate after a finite number of steps.

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- **Average reward** (withholding some technical details):
  $V^\pi(s) \stackrel{\text{def}}{=} \mathbb{E}_\pi[\lim_{m \to \infty} \frac{r^0 + r^1 + \cdots + r^{m-1}}{m} | s^0 = s]$.

Markov Decision Problems

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Controlling a Helicopter (Ng *et al.*, 2003)

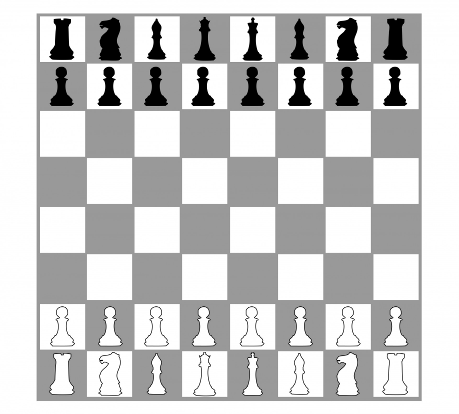
- Episodic or continuing task? What are S, A, T, R, γ?

[1]

1. `https://www.publicdomainpictures.net/pictures/20000/velka/police-helicopter-8712919948643Mk.jpg`.

Winning at Chess

- Episodic or continuing task? What are S, A, T, R, γ?



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1. https://www.publicdomainpictures.net/pictures/80000/velka/chess-board-and-pieces.jpg.

Preventing Forest Fires (Lauer *et al.*, 2017)

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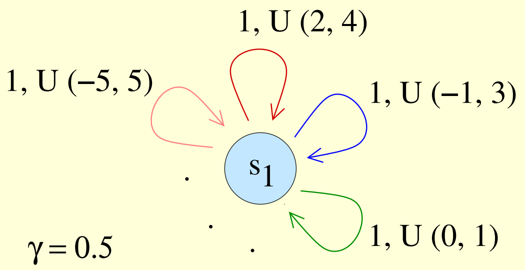
A Familiar MDP?

- Single state. k actions.
- For $a \in A$, treat reward of (s, a, s') as a random variable.



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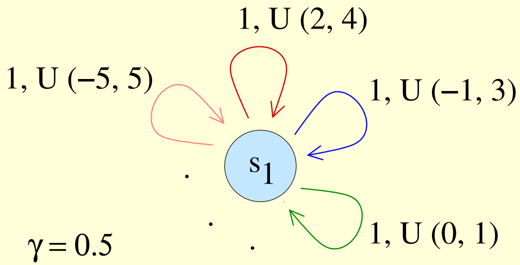

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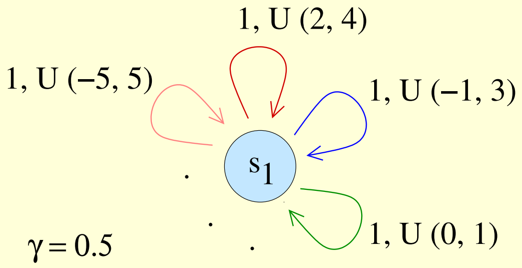

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- Such an MDP is called a multi-armed bandit!

Markov Decision Problems

- MDP, policy, value function
- MDP planning problem
- Policy evaluation

- Alternative formulations of MDPs
- Some applications of MDPs

- Banach's fixed point theorem
- Bellman optimality operator
- Value iteration
- Linear Programming
- Policy iteration