CS 747, Autumn 2022: Lecture 25

Shivaram Kalyanakrishnan

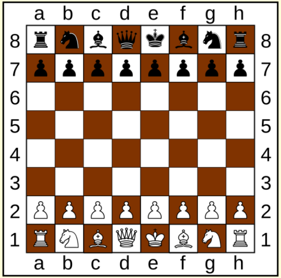
Department of Computer Science and Engineering
Indian Institute of Technology Bombay

Autumn 2022

Search in Games, Decision-time Planning in MDPs

- Game trees and minimax search
- Decision-time planning in MDPs
  - Problem
  - Rollout policies
  - Monte Carlo tree search
- Summary

Search and Games



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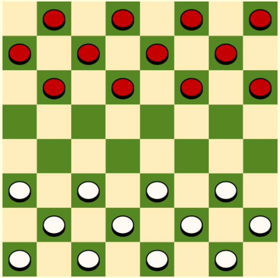

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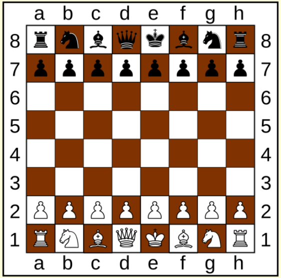

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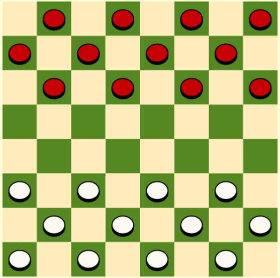

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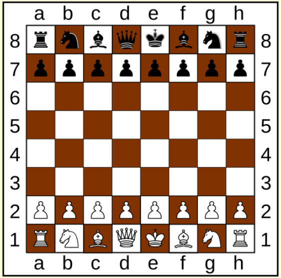

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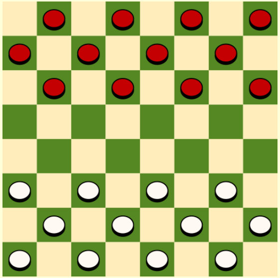

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- Winning at chess/checkers: a search problem?

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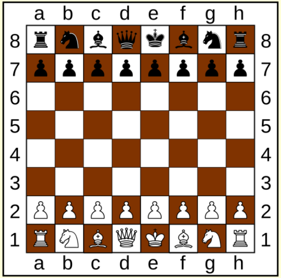

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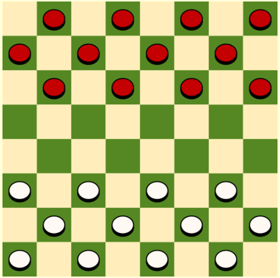

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- What's the main difference from previous examples?

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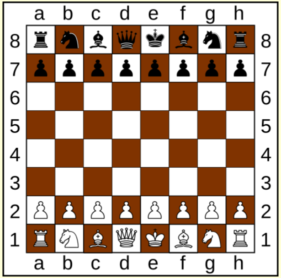

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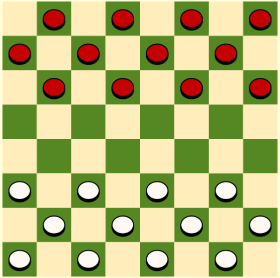

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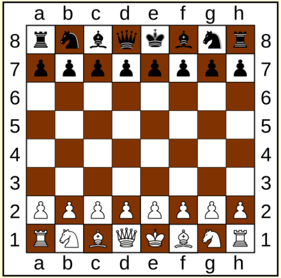

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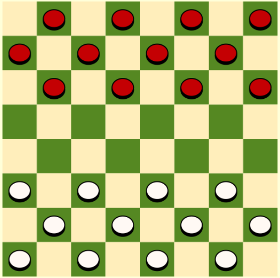

Checkers/Draughts [2]

- Winning at chess/checkers: a search problem?
- What's the main difference from previous examples? There's another player!
- Instances of turn-based two player zero-sum games.

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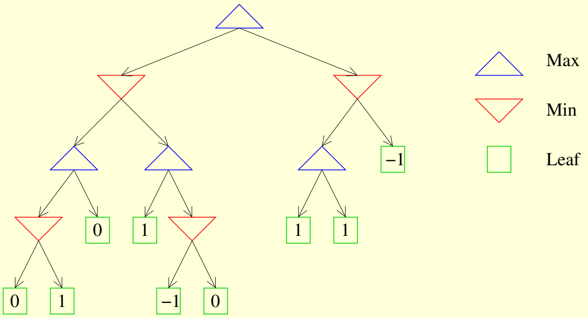
Game Tree

- Assume turn-taking zero sum game played by Max and Min.
- Action costs usually taken as 0, but leaves have value
  -1 (Max loses), 0 (draw), 1 (Max wins).
- Value of Max node is maximum of values of children.
  Value of Min node is minimum of values of children.



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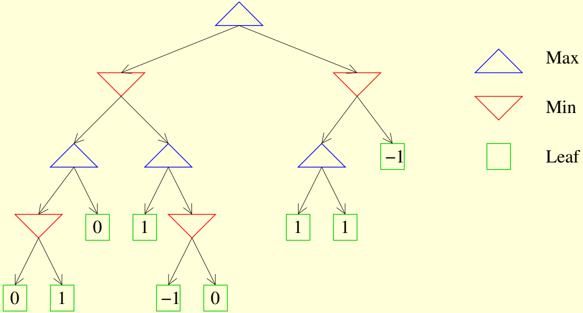

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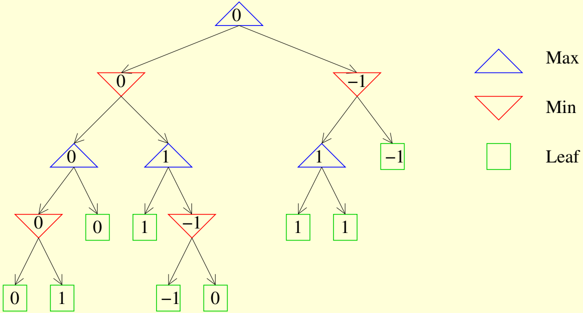

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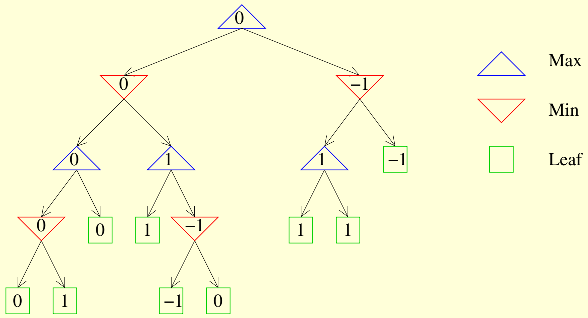
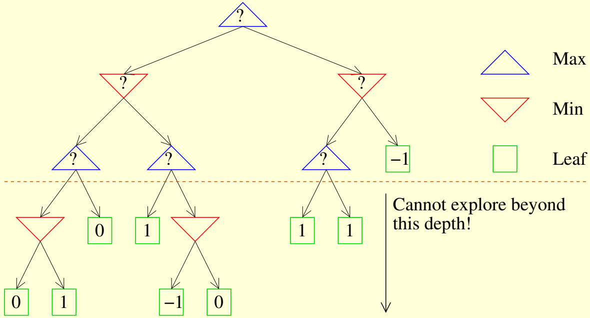


- In 2007, a massive, long-running computation concluded that the value of the root node for Checkers is 0 (draw).

Evaluation Function

- The Checkers tree has $\approx 10^{40}$ nodes; Chess has $\approx 10^{120}$. Infeasible to solve!



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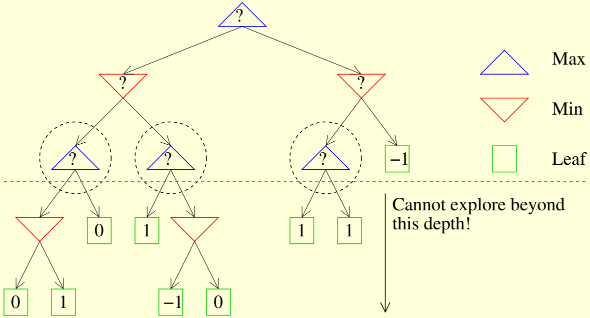

- At some depth d from current node, estimate node value using features.
- For example, in Chess, set evaluation as

$$w_1 \times \text{Material diff.} + w_2 \times \text{King safety} + w_3 \times \text{pawn strength} + \ldots.$$

- Weights $w_1, w_2, w_3, \ldots$ are tuned or learned.

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Decision-time planning: Problem

- So far we have assumed that an agent's learning algorithm produces π or Q as output. While acting on-line, the agent just needs a "look up" or associative "forward pass" from any state s to obtain its action.

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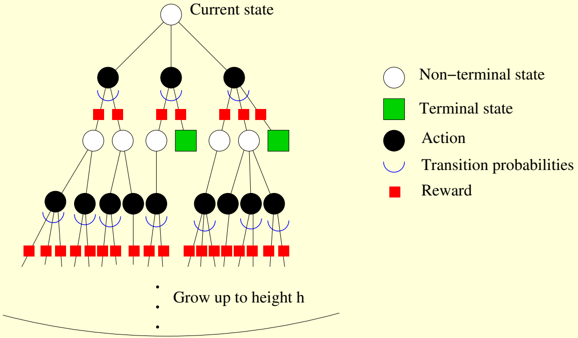
- How to rigorously do so?

Tree Search on MDPs



- Expectimax calculation. Set $Q^h \leftarrow \mathbf{0}$ //Leaves.
  For $d = h-1, h-2, \dots, 0$://Bottom-up calculation.
  $V^d(s) \leftarrow \max_{a \in A} Q^{d+1}(s, a)$;
  $Q^d(s, a) \leftarrow \sum_{s' \in S} T(s, a, s')\{R(s, a, s') + \gamma V^d(s')\}$.

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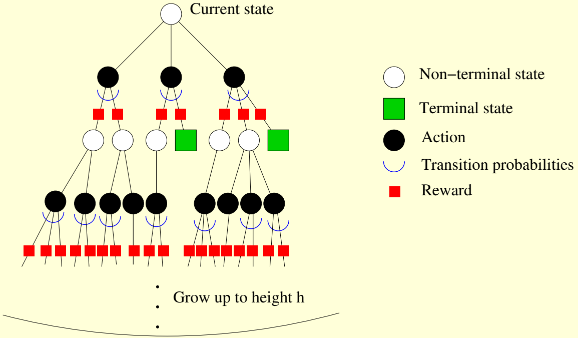

- Need $h = \Theta(\frac{1}{1-\gamma})$ for sufficient accuracy.
- With branching factor b, tree size is $\Theta(b^h)$. Expensive!
- Often M is only a sampling model (not distribution model).
- Can we avoid expanding (clearly) inferior branches?

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- Let policy π' satisfy $\pi'(s) = \text{argmax}_{a \in A} Q^{\pi}(s, a)$ for $s \in S$.
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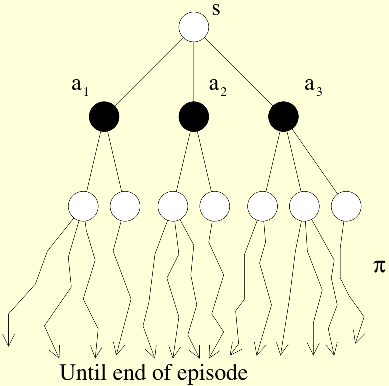

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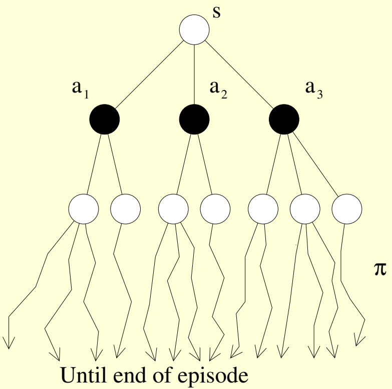

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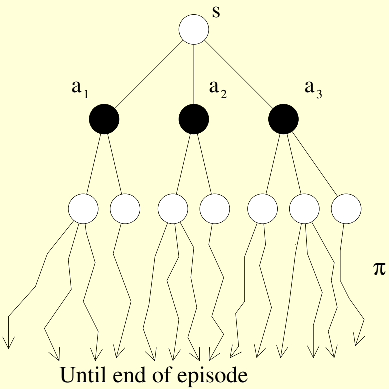

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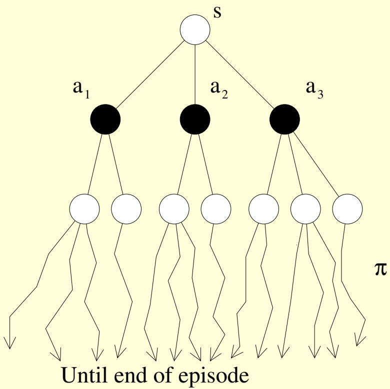

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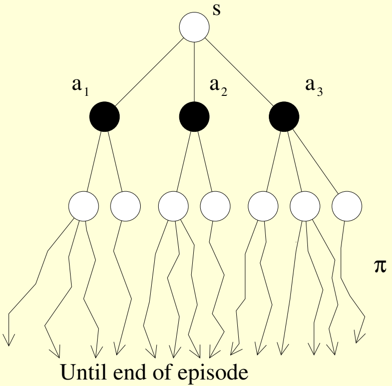

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- Repeat same process from next state s'.

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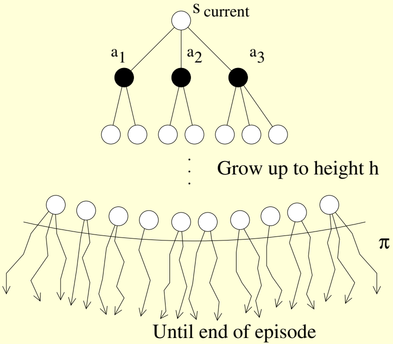
Monte Carlo Tree Search (UCT Algorithm)

- Build out a tree up to height h (say 5–10) from current state s_{current}. "Data" for the tree are samples returned by M.
- For (s, a) pairs reachable from s_{current} in $\leq h$ steps, maintain
  - $Q(s, a)$: average of returns of rollouts passing through (s, a).
  - $ucb(s, a) = Q(s, a) + C_p \sqrt{\frac{\ln(t)}{\text{visits}(s,a)}}$.



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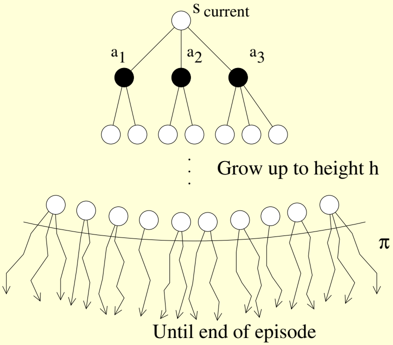

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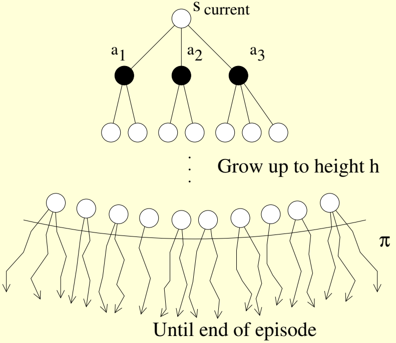

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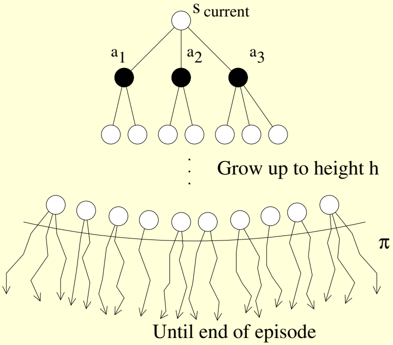

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- UCT focuses attention on rewarding regions of state space.
- Rollouts can easily be parallelised.
- Extremely successful algorithm in practice.

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