CS 747, Autumn 2023: Lecture 1

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Autumn 2023

Multi-armed Bandits

1. The exploration-exploitation dilemma
2. Definitions: Bandit, Algorithm
3. ϵ-greedy algorithms

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A Game

Coin 1

$\mathbb{P}\{\text{heads}\} = p_1$

Coin 2

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Coin 3

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- p_1, p_2, and p_3 are unknown.
- You are given a total of 20 tosses.
- Maximise the total number of heads!

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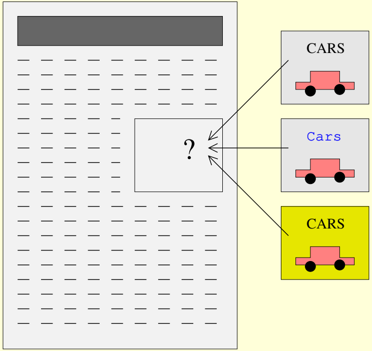
- If you knew p_1, p_2, p_3 beforehand, how would you have played? How many heads would you have got in 20 tosses?

To Explore or to Exploit?

- On-line advertising: Template optimisation



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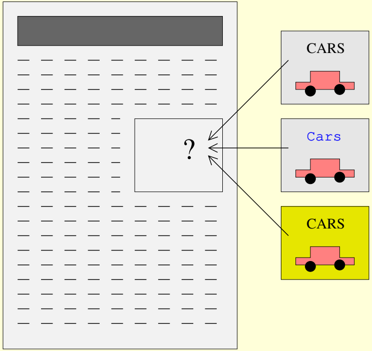

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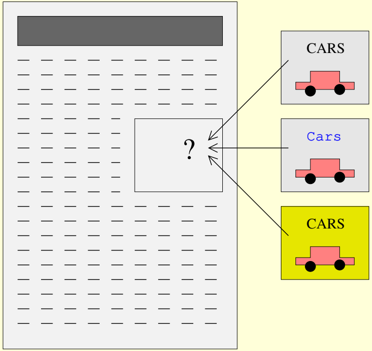

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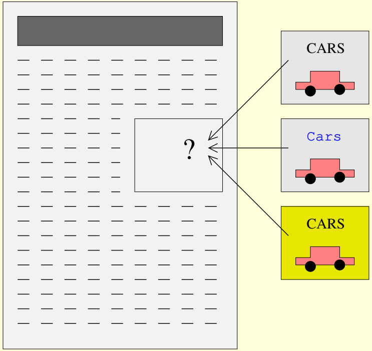

- Clinical trials
- Packet routing in communication networks
- Game playing and reinforcement learning

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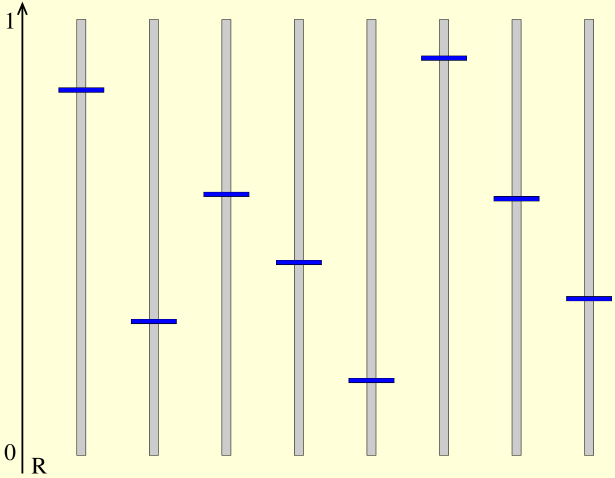
Stochastic Multi-armed Bandits



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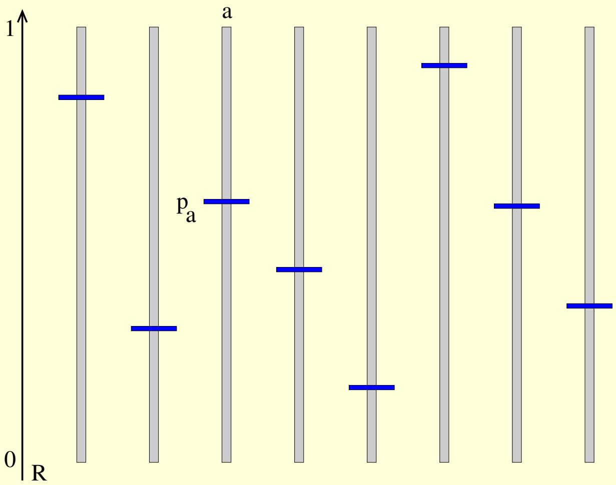

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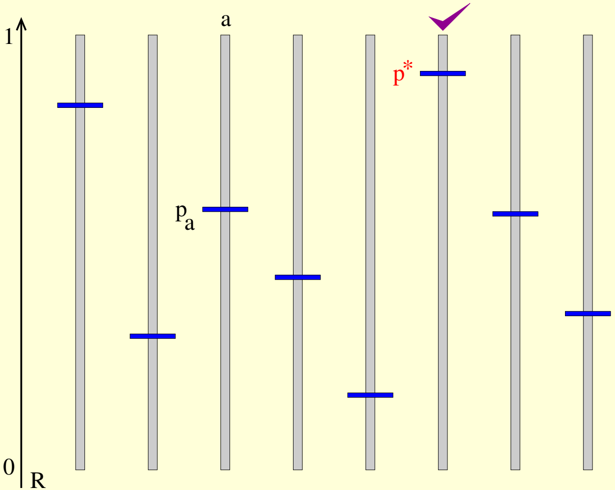

- *n* arms, each associated with a Bernoulli distribution (rewards are 0 or 1).
- Let A be the set of arms. Arm $a \in A$ has mean reward p_a.
- Highest mean is $p^\star$.

One-armed Bandits

[1]

1. https://pxhere.com/en/photo/942387.

Algorithm

- Here is what an algorithm does—

  For $t = 0, 1, 2, \ldots, T - 1$:

  - Given the history $h^t = (a^0, r^0, a^1, r^1, a^2, r^2, \ldots, a^{t-1}, r^{t-1})$,
  - Pick an arm a^t to sample (or "pull"), and
  - Obtain a reward r^t drawn from the distribution corresponding to arm a^t.

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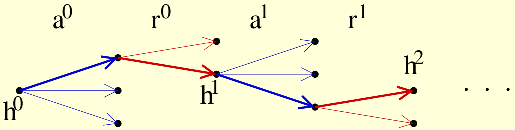
- The algorithm picks the arm to pull; the bandit instance returns the reward.

Illustration



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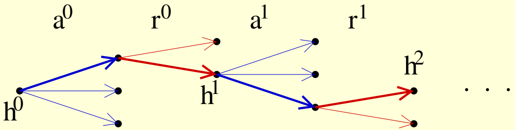

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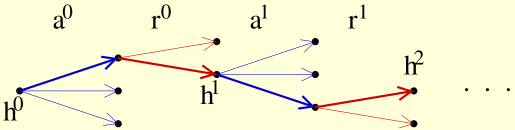

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  $\mathbb{P}\{h^T\} = \prod_{t=0}^{T-1} \mathbb{P}\{a^t|h^t\}\mathbb{P}\{r^t|a^t\}$, where
  $\mathbb{P}\{a^t|h^t\}$ is decided by the algorithm,
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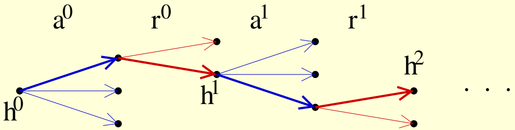
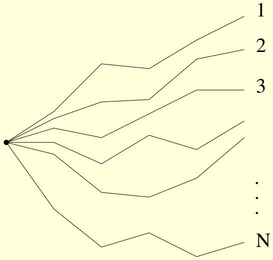


- An algorithm, bandit instance pair can generate many possible T-length histories.



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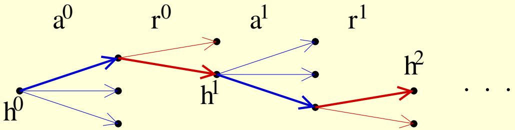

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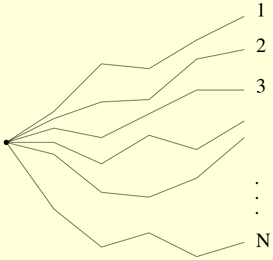

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How many histories possible if the algorithm is deterministic and rewards 0–1?

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  - If $t \leq \epsilon T$, sample an arm uniformly at random.
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- ϵG2
  - If $t \leq \epsilon T$, sample an arm uniformly at random.
  - If $t > \epsilon T$, sample an arm with the highest empirical mean.

- ϵG3
  - With probability ϵ, sample an arm uniformly at random; with probability $1 - \epsilon$, sample an arm with the highest empirical mean.

Questions

- Are ϵG1, ϵG2, ϵG3 deterministic or randomised algorithms?

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- Fix a 4-armed bandit instance with means $p_1 > p_2 > p_3 > p_4$.
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- Fix a 4-armed bandit instance with means $p_1 > p_2 > p_3 > p_4$.
  - If $\epsilon = 1$, what is the expected reward of ϵG1?
  - If $\epsilon = 0.8$ and T is relatively large, what is the expected reward of ϵG1?

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- Are ϵG1, ϵG2, ϵG3 deterministic or randomised algorithms?

- Fix a 4-armed bandit instance with means $p_1 > p_2 > p_3 > p_4$.
  - If $\epsilon = 1$, what is the expected reward of ϵG1?
  - If $\epsilon = 0.8$ and T is relatively large, what is the expected reward of ϵG1?

- Does ϵG1 perform worse than ϵG2 on each run?

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Next class: What is a "good" algorithm? What is the "best" algorithm?