CS 747, Autumn 2023: Lecture 16

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Reinforcement Learning

1. Multi-step returns
2. TD(λ)
3. Control with TD learning

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Yes. It uses a 2-step return as target.

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- For each $n \geq 1$, we have $\lim_{t \to \infty} V^t = V^\pi$.
- What is the effect of n on bootstrapping? Small n means more bootstrapping.

Combining Returns

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- Can use any convex combination of the applicable G's.

The λ-return

- A particular convex combination is the λ-return, $\lambda \in [0, 1]$:

$$\boxed{G_t^\lambda \stackrel{\text{def}}{=} (1-\lambda) \sum_{n=1}^{T-t-1} \lambda^{n-1} G_{t:t+n} + \lambda^{T-t-1} G_{t:T}}$$

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- In general, λ controls the amount of bootstrapping.

- If $\lambda > 0$, transition (s^t, r^t, s^{t+1}) contributes to the update of every previously-visited state: that is, $s^0, s^1, s^2, \ldots, s^t$.
- The amount of contribution falls of geometrically.
- Updating with the λ-return as target can be implemented elegantly by keeping track of the "eligibility" of each previous state to be updated.

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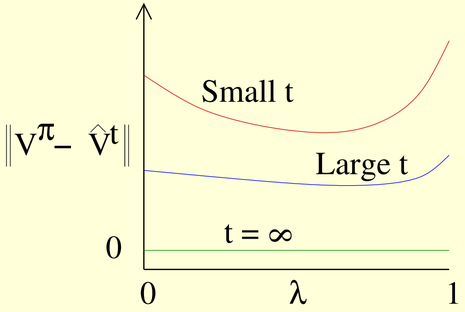
Initialise $V : S \to \mathbb{R}$ arbitrarily.
Repeat for each episode:
    Set $z \to \mathbf{0}$.//Eligibility trace vector.
    Assume the agent is born in state s.
    Repeat for each step of episode:
        Take action a; obtain reward r, next state s'.
        $\delta \leftarrow r + \gamma V(s') - V(s)$.
        $z(s) \leftarrow z(s) + 1$.
        For all s:
            $V(s) \leftarrow V(s) + \alpha \delta z(s)$.
            $z(s) \leftarrow \gamma \lambda z(s)$.
        $s \leftarrow s'$.

Effect of λ



- Lower λ: more bootstrapping, more bias (less variance).
- Higher λ: more dependence on empirical rewards, more variance (less bias).
- For finite t, error is usually lowest for intermediate λ value.

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   We consider three different update rules.

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- If $\pi^t = \pi$ (time-invariant) and it still visits every state-action pair infinitely often, then $\lim_{t \to \infty} \hat{Q}^t$ is Q^π for Sarsa and Expected Sarsa, but is $Q^\star$ for Q-learning!

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Temporal Difference Learning: Review

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- TD learning applies to both prediction and control.
- Q-learning, Sarsa, Expected Sarsa are all model-free (use $\Theta(|S||A|)$-sized memory); can still be optimal in the limit.
- Sarsa(λ) commonly used in practice.