CS 747, Autumn 2023: Lecture 19

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Decision-time Planning in MDPs

- Problem
- Rollout policies
- Monte Carlo tree search
- Evaluation functions
- Summary

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Decision-time planning: Problem

[1]

1. https://www.pexels.com/photo/a-woman-playing-billiards-10627127/.

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- So far we have assumed that an agent's learning algorithm produces π or Q as output. While acting on-line, the agent just needs a "look up" or associative "forward pass" from any state s to obtain its action.

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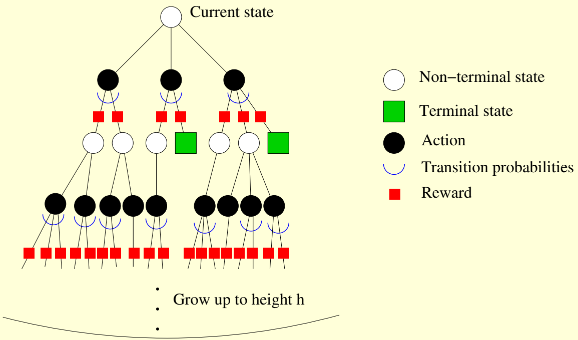
- How to rigorously do so?

Tree Search on MDPs



- Expectimax calculation. Set $Q^h \leftarrow \mathbf{0}$ //Leaves.
  For $d = h - 1, h - 2, \ldots, 0$://Bottom-up calculation.
  $V^d(s) \leftarrow \max_{a \in A} Q^{d+1}(s, a)$;
  $Q^d(s, a) \leftarrow \sum_{s' \in S} T(s, a, s')\{R(s, a, s') + \gamma V^d(s')\}$.

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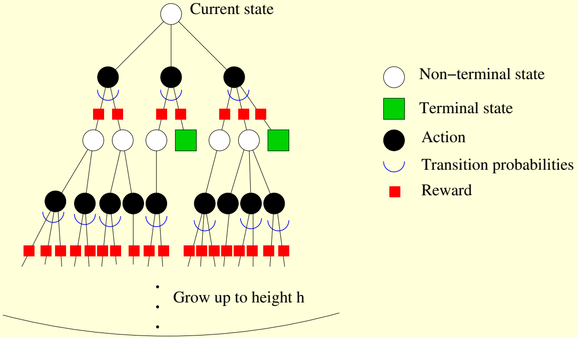

- Need $h = \Theta(\frac{1}{1-\gamma})$ (or $h =$ episode length) for sufficient accuracy.
- With branching factor b, tree size is $\Theta(b^h)$. Expensive!
- Often M is only a sampling model (not distribution model).
- Can we avoid expanding (clearly) inferior branches?

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- Let policy π' satisfy $\pi'(s) = \text{argmax}_{a \in A} Q^{\pi}(s, a)$ for $s \in S$.
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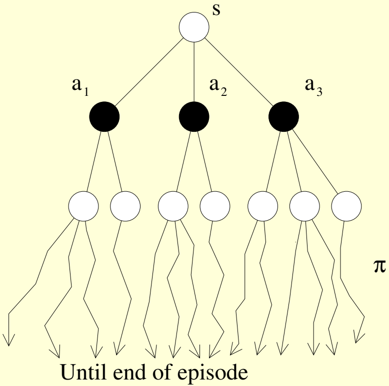

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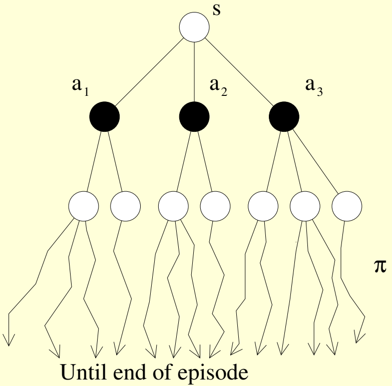

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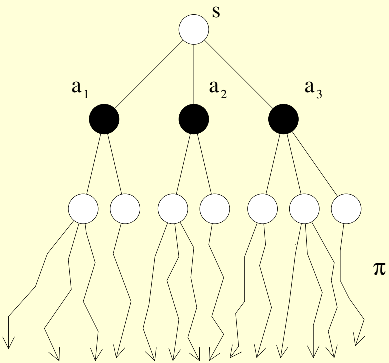

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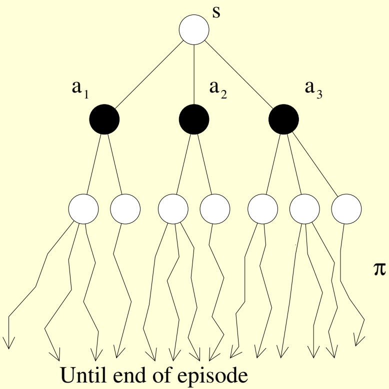

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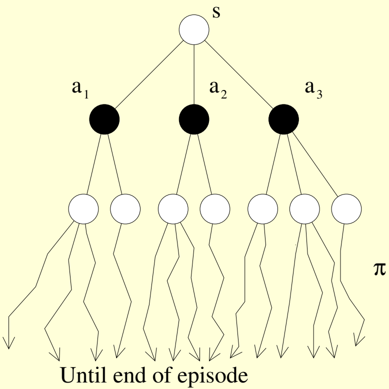

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- Take action $\pi'(s) = \text{argmax}_{a \in A} \hat{Q}^{\pi}(s, a)$.
- Repeat same process from next state s'.

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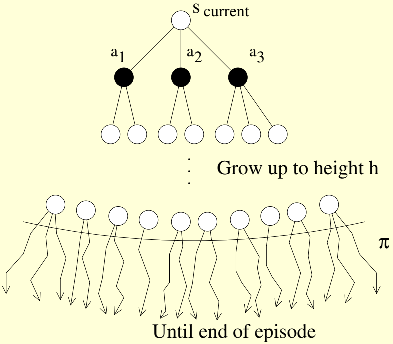
Monte Carlo Tree Search (UCT Algorithm)

- Build out a tree up to height h (say 5–10) from current state s_{current}. "Data" for the tree are samples returned by M.
- For (s, a) pairs reachable from s_{current} in $\leq h$ steps, maintain
  - $Q(s, a)$: average of returns of rollouts passing through (s, a).
  - $ucb(s, a) = Q(s, a) + C_p\sqrt{\frac{\ln(t)}{\text{visits}(s,a)}}$.



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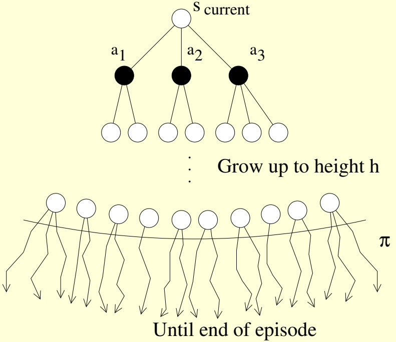

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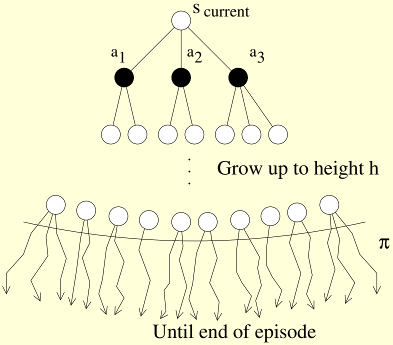

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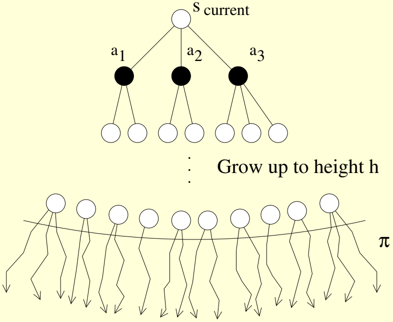

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- UCT focuses attention on rewarding regions of state space.
- Rollouts can easily be parallelised.
- Extremely successful algorithm in practice.

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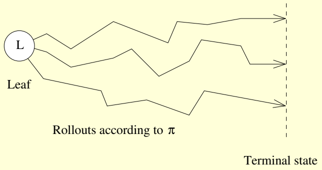
Evaluation Function

- With **rollouts**, value estimate of L = average rollout return.



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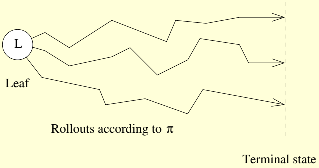

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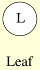

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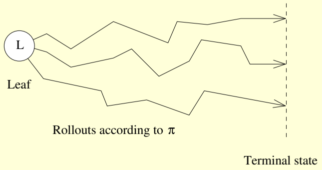

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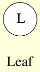

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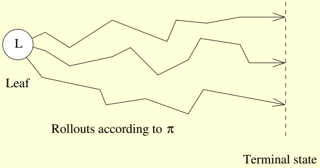

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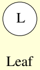

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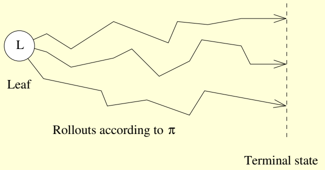

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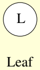

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- Evaluation functions save compute time. Can be combined with rollouts.

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- Key requirement: simulator (model).
- More computationally expensive than lookup of π or Q.
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