Water and Development

Part 3d: Transient and Unsaturated Systems: Water Table

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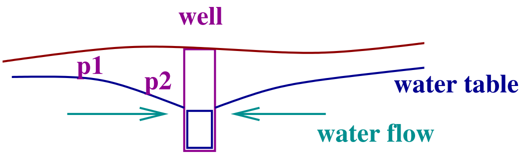
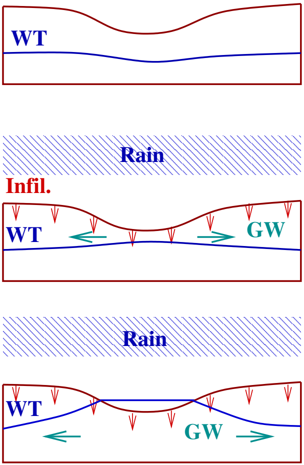
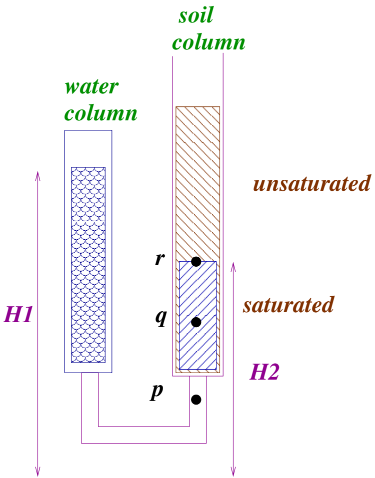
Issues with the earlier approach

- Saturated condition never occurs in isolation. In fact the water-table is an unknown boundary.
- Most phenomena are *transient*, i.e., change with time. Thus the conservation equation is more complex.





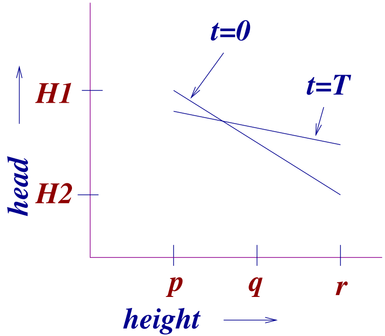
The Unsaturated water column



What is likely to happen?

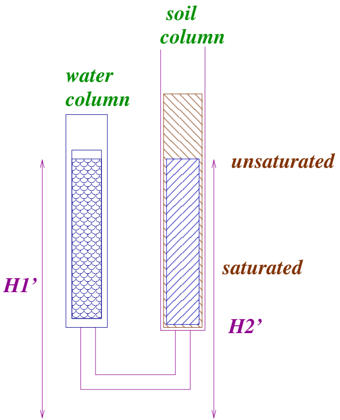
- Firstly, $h(p) = H_1, h(r) = H_2$ and $h(p) > h(q) > h(r)$.
- Thus water will seep from the water column into the soil column and the saturated part will increase in height.
- However, for every Δx drop in the water column, there will be a rise of $\Delta x / S_y$ in the saturated soil column!
- If $S_y = 5\%$, then a 1mm drop in water column $\Rightarrow$ a 20mm rise in saturated part.

The heads within the soil column



- The head at p is exactly the height in the water column.
- As we go up from p to q and r, total heads *drops linearly* from H_1 to H_2.
- As time progresses, the height in the water column drops marginally but the height in the soil column increases substantially, thanks to $S_y << 1$.
- All the same, the linearity is maintained.

The Unsaturated water column: steady state



- The governing equation:

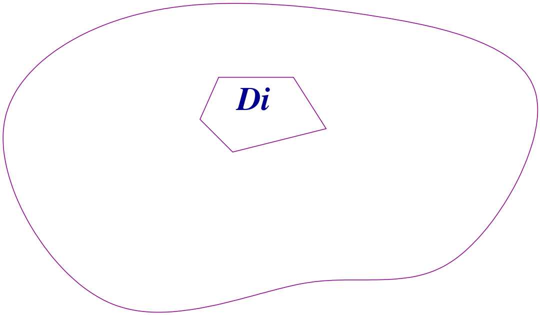
$$H_1' = H_1 - \Delta x = H_2 + \Delta x / S_y = H_2'$$

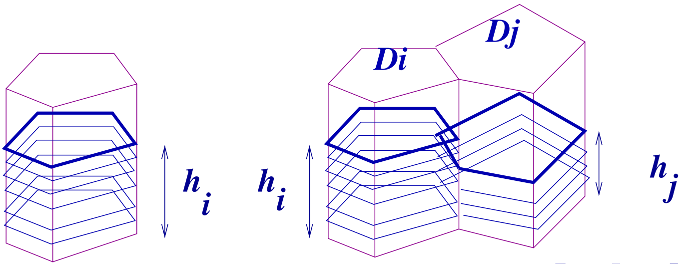
- This gives $\Delta x = (H_1 - H_2) * S_y / (1 + S_y)$.
- If $S_y = 2\%$, $\Delta x = \Delta H * 0.0196$.

2 points

- Movement in the water table requires S_y.
- Hydraulic heads in steady state depend on position of water table.

Modeling Unsaturated Domains





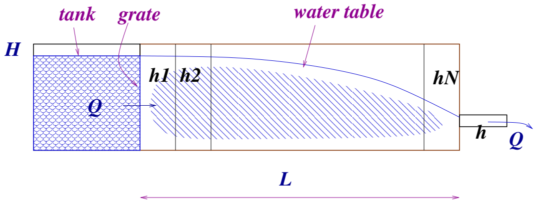
Modeling Unsaturated Domains : Principles

- The domain decomposition is as before.
- The variable q_i is unchanged: net recharge (vol./day) into D_i.
- Variable h_i signifies the height of the saturate column in D_i.
- Darcy's law is multiplied by Δt to get a volume equation:

$$\sum_j \Delta t \cdot \frac{(h_i - h_j) A_{ij} K}{L_{ij}} + S_y \Delta h_i A_i = q_i \Delta t$$

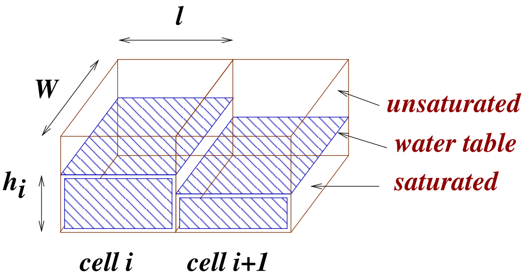
- Δh_i: change in saturation height, A_i: area of domain D_i, A_{ij}: interaction area between D_i and D_j.

Soilbox 1



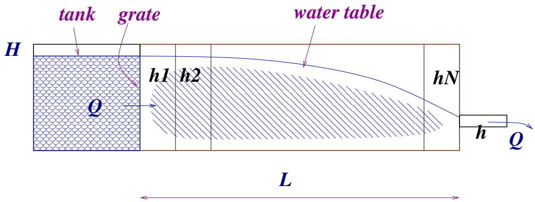
- Constant head H on the left and h on the right.
- Possibly implemented by a grate on the right. Then both h and Q are unknown. Lets assume h is known.
- Or, there is a well with known discharge Q and h is unknown.
- Water-table is formed, with saturated portion below the WT and unsaturated above. *However, height of saturated h_i part not known before-hand.*

Two adjoining cells



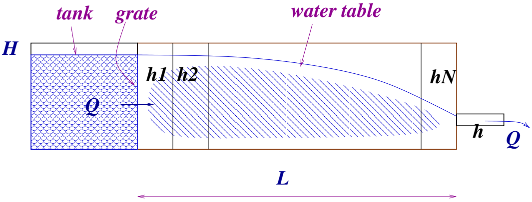
- What is the flow from cell i to cell $i+1$?
- Heads equal heights of water-table, i.e., h_i, h_{i+1}.
- Boundary surface: $h_{i+1} \times W$.
- Flow approximated by Darcy: $\Delta h_i \times h_{i+1} \times W/L$.

The Unsaturated Soil Box 1



- Constant head H on the left and h on the right. Unknown discharge Q on the right.
- Height of saturated part, say h_i not known before-hand and yet:
- $(h_i - h_{i-1}) * K * (h_i * W)/L + (h_i - h_{i+1}) * K * (h_{i+1} * W)/L = 0$
- $N = 3, H = 4, h = 2$ gives us $h_2^2 - 2h_2 - 4 = 0$, i.e., $h_2 = 3.71$.

The Unsaturated Soil Box 1

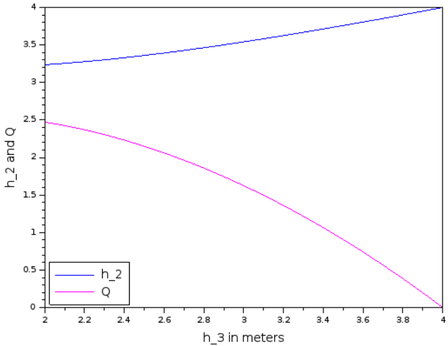


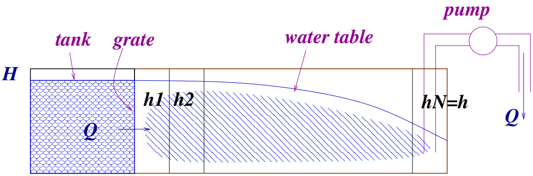
Lets keep $N = 3$ and see the dependance between h and Q.

- $(h_2 - 4) * h_2 * KW/L + (h_2 - h) * h * KW/L = 0$, i.e.,
- $h_2 = \frac{4-h+\sqrt{5h^2-8h+16}}{2}$ and $Q = (h_2 - h) * h * KW/L$
- What does this mean?

Soilbox 2

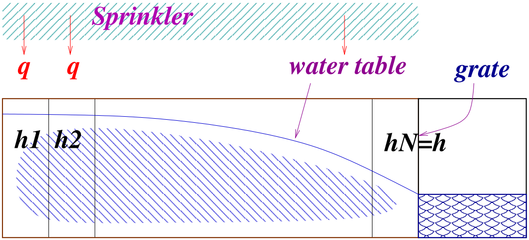
- Lets plot Q and h_2 w.r.t $h_3 = h$.
- This connects the flow Q out of the last cell and the head there.
- As h increases Q decreases.





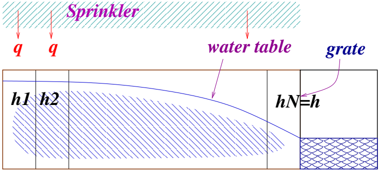
Equivalent Situation!

Steady state with rains



- $\Delta h_i, f_i$ variables, $f_1 = f_2 = \ldots = f_{N-1} = q$ known. Assume h known and constant, implemented by an overflow level.
- Again, $N-1$ flow conservation equations and $N-1$ unknowns $\Delta h_1, \ldots, \Delta h_{N-1}$. Note that $q_N = q - Nq = (1-N)q$, just by conservation.
- Interesting to compute h_is.

Steady state with rains

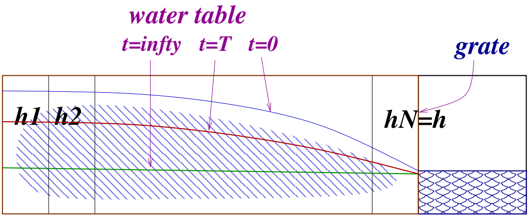


Say $N = 3$.

$$
\begin{aligned}
(h_1 - h_2) * h_2\alpha &= q \\
(h_1 - h_2) * h_2\alpha + q &= (h_2 - h) * h\alpha
\end{aligned}
$$

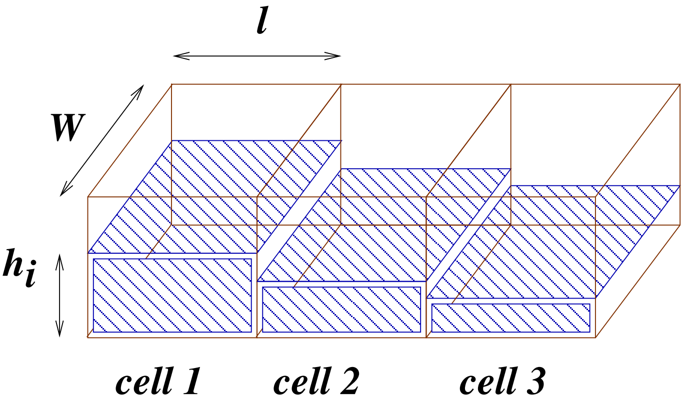
- Putting $h = 2, q = \alpha = 1$, we have $2q = (h_2 - h) * h$ which gives $h_2 = 3$.
- Next, $(h_1 - 3) * 3 = 1$ gives $h_1 = 3.33$.
- What happens if the soilbox height was only $3.2m$?
- What if K is increased?

What happens when the rain stops?



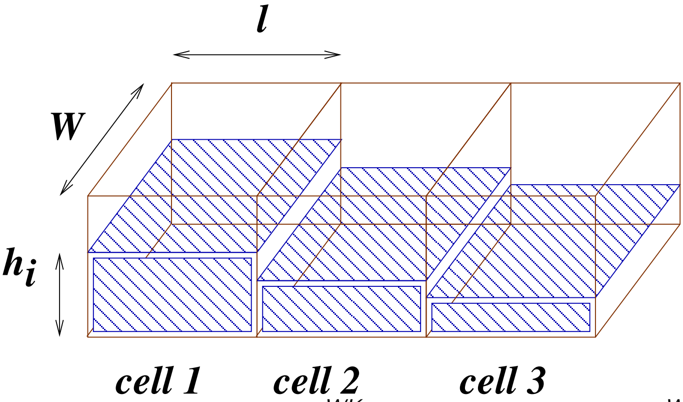
- The water-table slowly starts moving down. At $t = \infty$, the water-table is exactly at height h all through the soil box.
- The initial quantity of water in the soil-box was $\sum_i (A/n)h_i \times S_y$, where A is the area of the soil-box and S_y is the specific yield.
- The final quantity of water in the soil reduces to $Ah \times S_y$.

Cell and its neighbours



- For any (h_{i-1}, h_i, h_{i+1}) and a flow f_i into the cell, we have: $\Delta q = f_i - (h_i - h_{i-1}) * h_i * \frac{WK}{l} + (h_i - h_{i+1}) * h_{i+1} * \frac{WK}{l}$
- This Δq is the excess/deficit flow into a cell.
- Saturation: Δq is always zero.
- Unsaturated condition: This causes a rise or fall in the height h_i!

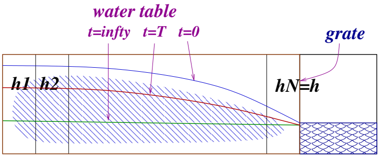
Rise and Fall



- $\Delta q = f_i - (h_i - h_{i-1}) * h_i * \frac{WK}{l} - (h_i - h_{i+1}) * h_{i+1} * \frac{WK}{l}$
- Unsaturated condition: This causes a rise or fall in the height h_i!
- The change in saturation after time Δt is given by:

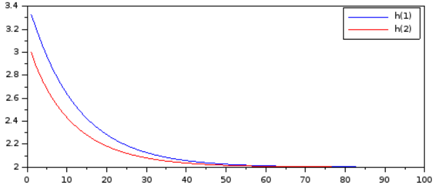
$$S_y * \Delta h_i * Wl = \Delta q * \Delta t$$

Lets take the discharge example



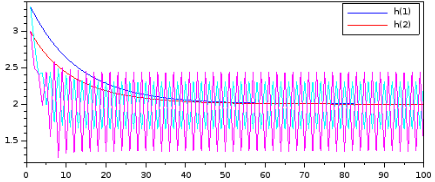
	h_1	h_2	h_3
$t = 0$	3.33	3	2
Δh	$-0.99\Delta t$	$-1.01\Delta t$	0
$t = 0.1$	3.2	2.9	2
Δh	$-0.96\Delta t$	$-0.84\Delta t$	0

- As the rain stops $f_i = 0$ for all i.
- $\Delta h = -\Delta t[(h_i - h_{i-1}) * h_i + (h_i - h_{i+1}) * h_{i+1}] * \frac{K}{S_y l^2}$
- Let us assume that $\frac{K}{S_y l^2} = 1$.

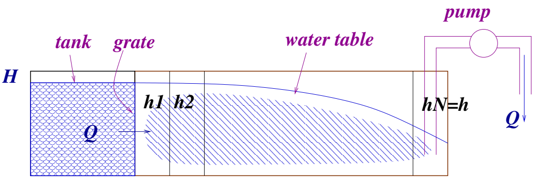


Delicate matters

- Thus, we are *discretizing* in both space and time.
- If the discretization is coarse in space, we are unlikely to get accurate answers.
- If coarse in time: instability ($\Delta t = 0.5$)



Another Situation: what happens when the pump turns off?



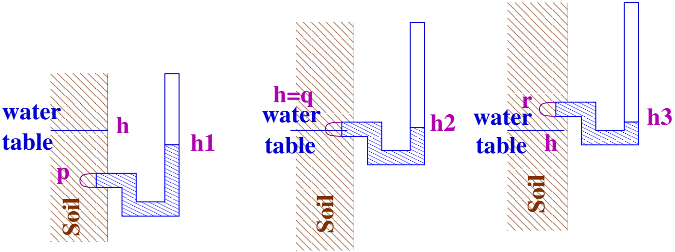
Predicting Rise and Fall in the WT

- Let h=ht. of water table and h' be the point at which the piezometer is inserted. Let h_i be the readings.
- (i) If $p < h$ then $p < h_1$. (ii) If $q = h$ then $q = h_2$. (iii) If $r > h$ then $r > h_3$.
- Darcy's law: GW flows from higher head to lower head.

p	$q = h$	r
8	10	11

h_1	h_2	h_3
9	10	10.5

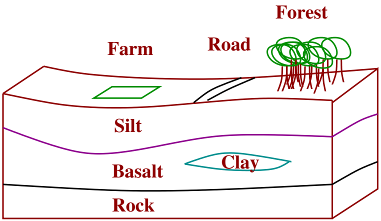
So, will the water-table rise in the near future?



Larger Picture

In general, we would like to

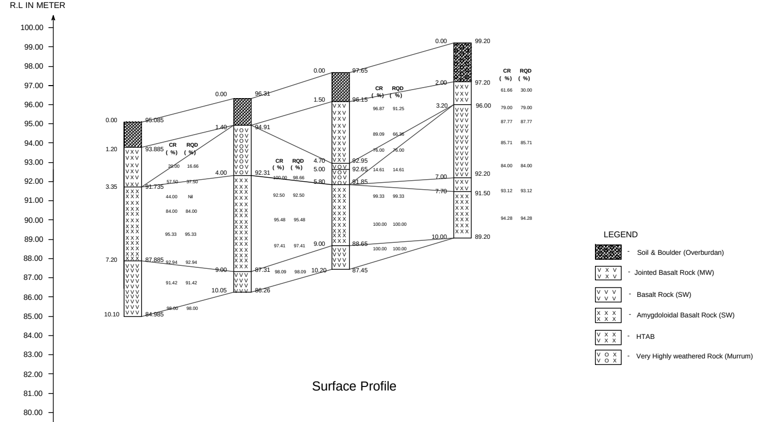
- analyse groundwater and surface water
- prescribe corrective measures
- understand sustainable use



A real-life scenario

- Various surface features such as farmslands, forests, built-up areas, which affect infiltration.
- Similar soils appearing as layers, and their geological properties, such as porosity, conductivity etc.
- climatic data such as rainfall, evaporation, etc.
- Water requirements and usage, such as for irrigation, domestic use, and so on.

Bore Logs-Under the ground



BH. NO	BH -1	BH- 2	BH -3	BH- 4
Depth	10.10 m	10.05 m	10.20 m	10.00 m
RL. M.	95.085 m	96.31 m	97.65 m	99.20 m
Chainage	15.00 m	35.00 m	55.00m	75.00 m

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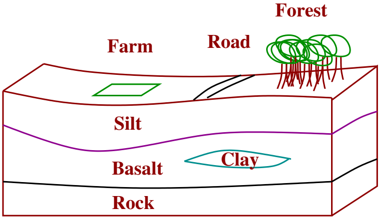
Project :- Soil Investigation at Gudvanwadi Check Dam.

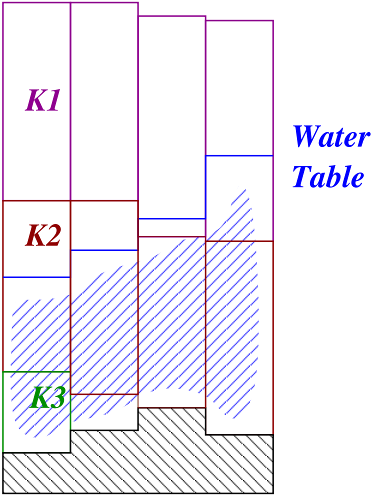
Subject :- Surface Profile

DATE : 11 / 05 / 2009 | DRN. BY : Ulhas | APPROVED BY:

Multi-layer model

- analyse groundwater and surface water
- prescribe corrective measures
- understand sustainable use

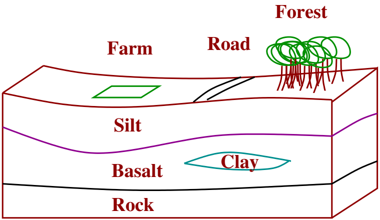




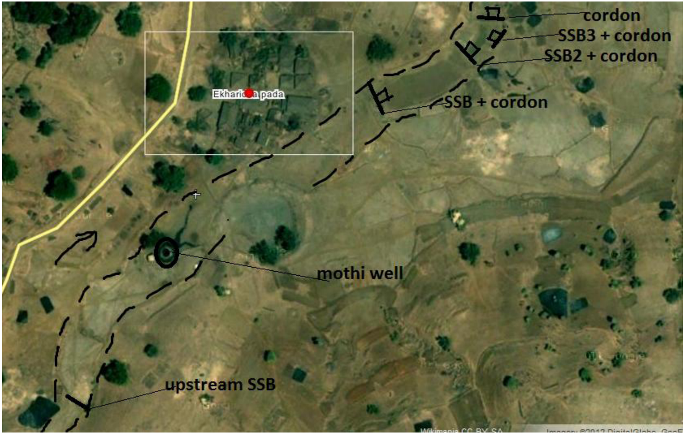
Groundwater Models

- The domain and its division into N multiple connected cells.
- Conductivity for each layer/cell.
- Two variables per cell. N conservation equations which depend on the geometry of saturated and un-saturated regions.
- Objective: Steady state/transient flows and heads for all cells.
- Boundary conditions. Either known flow or known head per cell. Climatological data.

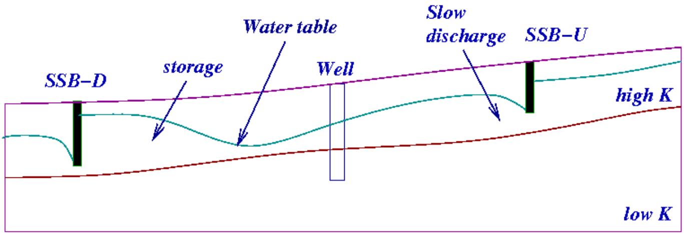
- Known head: May be varying in time, but known. Largely from groundwater data from wells or lake-levels.
- Known flows. Extraction, infiltration.



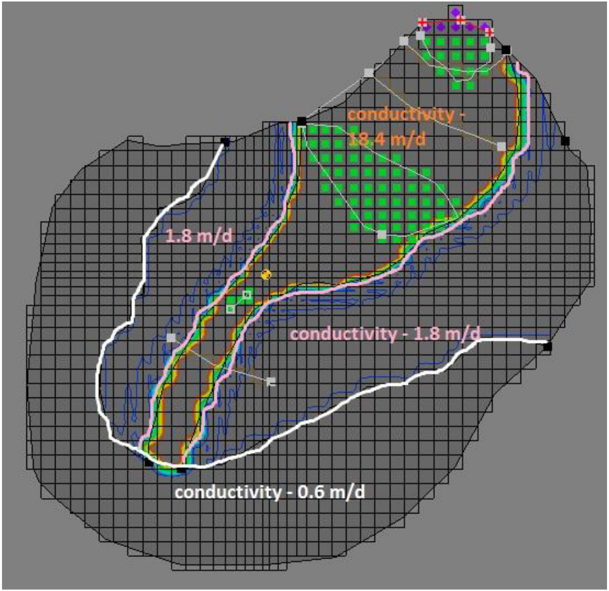
Ikharicha pada



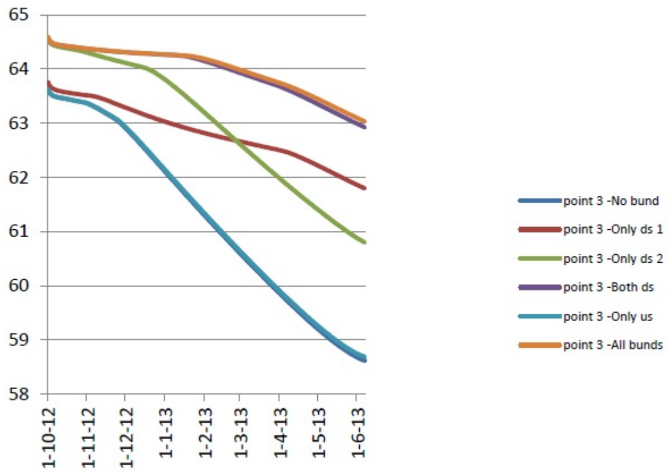
Problem


Water table
Slow
discharge
SSB−U
SSB−D
storage
Well
high K
low K

Model


conductivity -
18.4 m/d
1.8 m/d
conductivity - 1.8 m/d
conductivity - 0.6 m/d

Conclusions



Thanks