Digital Signal Processing CS 407, 2004 edition.
Quiz Duration:75 minutes
OPEN NOTES. No photocopies. No texts.

- State and justify clearly, any assumptions you make.
- Write legibly. Write the question number very clearly.

1. Let $x(t)$ be an analog signal, band-limited to $10kHz$. This signal has been sampled at $20kHz$ to give us the sequence $x(n)$. However, downstream, some specialised hardware can support a rate of only $10kHz$. How would you manipulate the sequence $x(n)$ to obtain a suitable input for the given hardware. Demonstrate your answers by drawing fourier transforms.

2. Let (x_i) be a sequence and $X(\omega)$ its FT, and so be it with (y_i) and $Y(\omega)$. Define the new sequence z as the **interleaving** of x and y. In other words:
$$\begin{aligned} z_{2i} &= x_i \\ z_{2i+i} &= y_i \end{aligned}$$
Compute $Z(\omega)$ in terms of $X(\omega)$ and $Y(\omega)$.

3. Let $f(t) = \frac{\sin 1.25\pi t}{t}$ and $x_i = f(i)$ be the observations of $f(t)$ at integer points. Is the sequence (x_i) sufficient to recreate $f(t)$? Compute the unique function $g(t)$ such that (i) $g(i) = f(i)$ for all integer points i, and (ii) $G(\omega) = 0$ for all $|\omega| > \pi$.

4. Let $f(t)$ be a signal with $F(\omega) = 0$ for all $|\omega| > \pi$. Let (x_i) be obtained by sampling $f(t)$ at integer points. What should the impulse response h of a **differentiator** be? In other words, if $y = x * h$, then $y_i = f'(i)$, where f' is the derivative of f.