Market Games

An analysis of efficiency and strategy

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Talk Outline

- Markets -Brief history and modern notions
- The supply and demand curve
- Fisher Market games
  - Equilibrium–Efficiency and strategic behaviour
  - Key lemmas and features of equilibria
- The cowherds of Gokul
- The engineering placement game

History

- Exchange, Trade and Mone and Emergence of money
  - gifts and presents, totemic money, account-keeping
  - coinage–taxes, armies, levies, mercantile money
  - fiat money–monetary theories
- Markets in India -taxes, unmonetised commodity exchanges through *jatras*, services through *balutedari*, the *kirana*, the *savkar*, the SHG, the Mall!
- Markets-models and mechanisms
  - Walras and tatonnement
  - Condorcet and other markets–rudiments of strategy
  - Efficiency–Fisher
- Modern markets
  - price as the signal–producer and consumer surplus
  - believed to be efficient in allocation of resources
  - Arrow and Debreu and the social welfare theorems

Motivation

- Markets-buyers with money and preferences, sellers with goods and quantities-agents
  - Equilibrium-Price discovery and allocations.
  - Various models: Fisher, Arrow-Debreu etc.
- Expectation: The markets are efficient.
  - allocate resources, require very little interference, predictable
  - rationality is sufficient, punish irrationality
- Our findings: Under strategic (non-cooperative) behaviour by agents (buyers).
  - highly unpredictable
  - induce outside-market relationships

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- Our findings: Under strategic (non-cooperative) behaviour by agents (buyers).
  - highly unpredictable
  - induce outside-market relationships
- A model for markets : Single-commodity, Fisher market,
- A model for strategic behaviour : Games, Nash Equilibrium Gokul, engineering placements.

Games, Nash Equilibrium

- Players: $\{1, \ldots, N\}$
- Strategy spaces: $\mathbb{S}_1, \ldots, \mathbb{S}_N$
- Set of strategy profiles: $\mathbb{S} = \mathbb{S}_1 \times \mathbb{S}_2 \times \cdots \times \mathbb{S}_N$
- Payoff functions: $u_i : \mathbb{S} \to \mathbb{R}$

Nash Equilibrium (NE):

- is the solution concept of a game, where no player benefits by changing her strategy unilaterally.
- $S = (s_1, \ldots, s_N)$ is a strategy profile, where each $s_i \in \mathbb{S}_i$.
- S is a NE iff $\forall i$ and $\forall s_i' \in \mathbb{S}_i$, $u_i(s_i', S_{-i}) \leq u_i(s_i, S_{-i})$.
- $S_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N)$.

Example

Prisoner's Dilemma

- $N = 2, \mathbb{S}_1 = \mathbb{S}_2 = \{\text{confess (C), don't confess (D)}\}$.
- Payoff matrix:

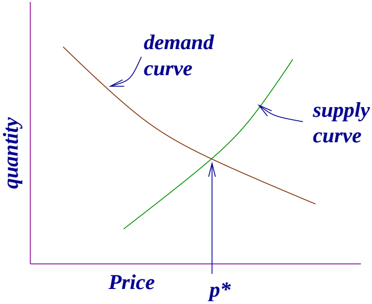
	C	D
C	1,1	6,0
D	0,6	5,5

- (C, C) is the only NE with payoff $(1, 1)$.
- (D, D) has payoff $(5, 5)$, but not stable.

Though it has drawbacks, the Nash equilibrium and its extensions (repeated, correlated) are generally considered acceptable.

The Supply-Demand curve and price discovery

- There is a supply curve $Supp(p)$, amount of goods which will be supplied at a price p.
- There is a demand curve $Dem(p)$, i.e., the amount demanded at the market price p.
- The market price p^* is given by the intersection of the supply curve and the demand curve, i.e., the price at which supply equals demand.



Implementation

Ability to pay	1	2	2	2	3	4
Ability to produce	1	1	2	2	2	3

The market price is $p^* = 2$ and at that price $S(p^*) = D(p^*) = 5$. But how does it really work?

- Sellers as agents with p_i as offer price and c_i as cost price.
- If $(p_1, \ldots, p_k)$ are offer prices then if $|D(p_i)| \geq |\{j | p_j \leq p_i\}|$, sale is guaranteed. Payoff 1 and 0 otherwise.
- $p^* = p_i$ when $c_i \leq p^*$ and $p_i = c_i$ otherwise is Nash equilibrium.
- Thus, the NE startegy *discovers* the optimum price p^*.
- Many implicit assumptions: Why not just a matching of a supplier with a demander?

The Fisher Market (Linear Case)

Input: A set of buyers ($\mathcal{B}$), a set of goods ($\mathcal{G}$), and $\forall i \in \mathcal{B}, \forall j \in \mathcal{G}$:

u_{ij} : payoff (*i.e.*, happiness) of buyer i for a unit amount of good j

m_i : money possessed by buyer i

q_j : quantity of good j

Goal: Computation of equilibrium prices $(p_j)_{j \in \mathcal{G}}$ and an equilibrium allocation $X = [x_{ij}]_{i \in \mathcal{B}, j \in \mathcal{G}}$ such that

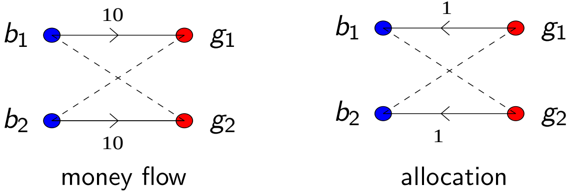
- Market Clearing:
$$\forall j \in \mathcal{G}, \ \sum_{i \in \mathcal{B}} x_{ij} = q_j \text{ and } \forall i \in \mathcal{B}, \ \sum_{j \in \mathcal{G}} x_{ij} p_j = m_i$$
- Optimal Goods: $x_{ij} > 0 \implies \dfrac{u_{ij}}{p_j} = \max_{k \in \mathcal{G}} \dfrac{u_{ik}}{p_k}$.

Payoff (happiness) of buyer i w.r.t. X is $u_i(X) = \sum_{j \in \mathcal{G}} u_{ij} x_{ij}$

Example

Input: $U = \begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix}$, $\mathbf{m} = \langle 10, 10 \rangle$, $\mathbf{q} = \langle 1, 1 \rangle$.

Output: $\langle p_1, p_2 \rangle = \langle 10, 10 \rangle$, $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.



$$u_1(X) = 10, u_2(X) = 10.$$

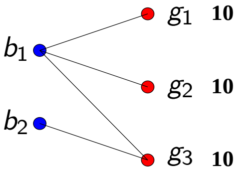
Somewhat like a flow problem. Much attention and recent solution in strongly polynomial time by Orlin.

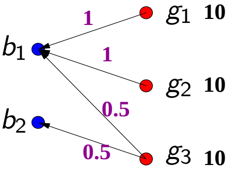
Basic Steps

- Guess the prices.
- Set up the Tight Graph
  - (b_i, g_j) is an edge iff it is most efficient.
- See if the flow problem is solvable.

$U_1 = [1\ 1\ 1]$
$U_2 = [1\ 2\ 3]$

$m_1 = 25 \quad m_2 = 5$



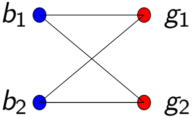


Observations

- Equilibrium prices are unique and equilibrium allocations form a convex set.
- Solution is independent of scaling u_{ij}'s.
- Quantity of goods may be assumed to be unit.
- All equilibrium allocations give the same payoff to a buyer.

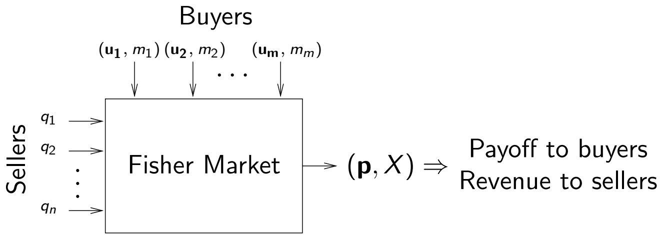
Example: $U = \begin{bmatrix} 1 & 19 \\ 1 & 19 \end{bmatrix}, \quad \mathbf{m} = \langle 10, 10 \rangle.$

$\langle p_1, p_2 \rangle = \langle 1, 19 \rangle,$



- Payoff tuple is $(10, 10)$ from any equilibrium allocation.

The Fisher Market Game



Question: How does the market work with strategic buyers?

- We take utility tuples as the strategies.
- naive hope: honestly posting utilities is the best strategy

Better Payoff?

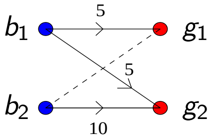
Question: Does a buyer have a strategy to achieve a better payoff?
Answer: Yes!

In the previous example ($U = \begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix}$, $\mathbf{m} = \langle 10, 10 \rangle$),
if buyer 1 poses utility tuple as $\langle 5, 15 \rangle$ instead of $\langle 10, 3 \rangle$, then

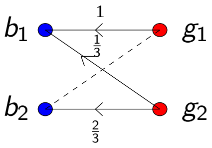
Output:

$$\langle p_1, p_2 \rangle = \langle 5, 15 \rangle$$

$$X = \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & \frac{2}{3} \end{bmatrix}$$



money flow



allocation

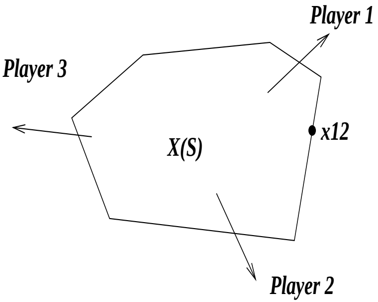
- $u_1(X) = 11, u_2(X) = \frac{20}{3}$.
- Note that payoff is calculated w.r.t. the true utility tuples.

The Fisher Market Game

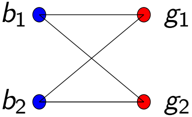
- Buyers are the players with $m = |\mathcal{B}|$ and $n = |\mathcal{G}|$.
- Strategy Set of buyer i: All possible utility tuples, *i.e.*, $\mathbb{S}_i = \{\langle s_{i1}, \ldots, s_{in} \rangle \mid s_{ij} \geq 0, \sum_{j \in \mathcal{G}} s_{ij} \neq 0\}$.
- Set of strategy profiles $\mathbb{S} = \mathbb{S}_1 \times \cdots \times \mathbb{S}_m$.

When a strategy profile $S \in \mathbb{S}$ is played,

- equilibrium prices $\mathbf{p}(S)$ and a set of equilibrium allocations $\mathbb{X}(S)$ are computed w.r.t. S and $\mathbf{m}$.
- Different allocations $X \in X(S)$ may give different happinesses to a buyer forcing a conflict resolution!



Example - Different Payoffs

- Consider previous example ($U = \begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix}$, $\mathbf{m} = \langle 10, 10 \rangle$), and the strategy profile $S = (\langle 1, 19 \rangle, \langle 1, 19 \rangle)$.
- $G(S) =$  , $\mathbf{p}(S) = \langle 1, 19 \rangle$.

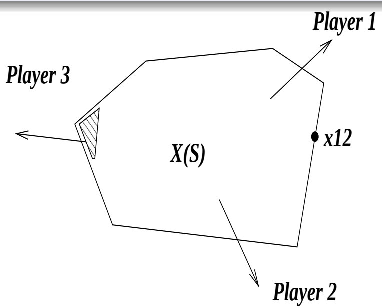
- $X_1 = \begin{bmatrix} 1 & \frac{9}{19} \\ 0 & \frac{10}{19} \end{bmatrix}$, $X_2 = \begin{bmatrix} 0 & \frac{10}{19} \\ 1 & \frac{9}{19} \end{bmatrix}$, $X_1, X_2 \in \mathbb{X}(S)$.
- Among all allocations in $\mathbb{X}(S)$, the highest payoff
  - for buyer 1 is from X_1; $u_1(X_1) = 11.42$, $u_2(X_1) = 5.26$.
  - for buyer 2 is from X_2; $u_1(X_2) = 1.58$, $u_2(X_2) = 7.74$.
- No allocation gives the highest payoff to both the buyers.

Definition

- A strategy profile S is said to be conflict-free if $\exists X \in \mathbb{X}(S)$, such that $u_i(X) = w_i(S),\ \forall i \in \mathcal{B}$.
- Such an X is called a conflict-free allocation.

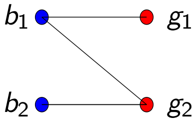
Lemma

For a strategy profile $S = (\mathbf{s_1}, \ldots, \mathbf{s_m})$, if $u_k(X) < w_k(S)$ for some $X \in \mathbb{X}(S)$ and $k \in \mathcal{B}$, then $\forall \delta > 0$, $\exists S' = (\mathbf{s'_1}, \ldots, \mathbf{s'_m})$, where $\mathbf{s'_i} = \mathbf{s_i},\ \forall i \neq k$, such that $u_k(X') > w_k(S) - \delta, \forall X' \in \mathbb{X}(S')$.



This paves the way for a suitable pay-off function and allows for the notion of Nash equilibrium.

Example - Conflict Removal

- Consider previous example ($U = \begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix}$, $\mathbf{m} = \langle 10, 10 \rangle$, $S = (\langle 1, 19 \rangle, \langle 1, 19 \rangle))$.
- For $\delta = 0.1$ and buyer 1, consider $S' = (\langle 1.1, 18.9 \rangle, \langle 1, 19 \rangle)$.
- $G(S') =$ 

, $\mathbf{p}(S') = \langle 1.1, 18.9 \rangle$.
- Unique equilibrium allocation, i.e., $\mathbb{X}(S') = \{X'\}$.
- $u_1(X') = 11.41$, $u_2(X') = 5.29$.
- Recall: $w_1(S) = 11.42$, hence $u_1(X') > w_1(S) - \delta$.

Characterization of NESPs - Necessary Conditions

Theorem

If there is a Nash equilibrium S then it is conflict-free, i.e., $\exists X \in \mathbb{X}(S)$ such that $u_i(X) = w_i(S), \forall i \in \mathcal{B}$, i.e., S is conflict-free.

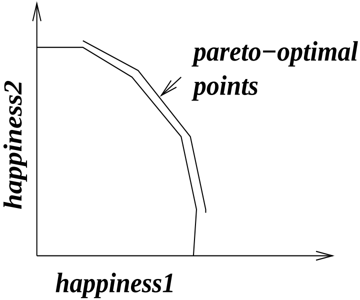
Symmetric NESP

Definition

A strategy profile $S = (\mathbf{s_1}, \ldots, \mathbf{s_m})$ is said to be a *symmetric* strategy profile if $\mathbf{s_1} = \cdots = \mathbf{s_m}$. "Unanimity" on the relative importance of goods.

Lemma

The payoff w.r.t. a symmetric NESP is Pareto optimal.



Complete Characterization of Symmetric NESPs

Proposition

A symmetric strategy profile S is a NESP iff it is conflict-free.

Corollary

A symmetric NESP can be constructed, whose payoff is the same as the Fisher payoff. The truthful strategy is not NE.

Example - Asymmetric NESP (not Pareto Optimal)

$$\text{Input:} \quad U = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 2 & 3 \end{bmatrix}, \quad \mathbf{m} = \langle 50, 100, 50 \rangle.$$

- $\mathbf{s_1} = \langle 2, 0 \rangle, \mathbf{s_2} = \langle 2, 3 \rangle, \mathbf{s_3} = \langle 0, 3 \rangle$, and $\mathbf{s} = \langle 2, 3 \rangle$.
- $S_1 = (\mathbf{s_1}, \mathbf{s_2}, \mathbf{s_3})$ and $S_2 = (\mathbf{s}, \mathbf{s}, \mathbf{s})$ are NESPs.
- $\mathcal{P}(S_1) = (1.25, 6.75, 1.25)$, $\mathcal{P}(S_2) = (1.25, 7.5, 1.25)$.
- $\mathcal{P}(S_1) \leq \mathcal{P}(S_2)$.

The Two-Buyer Markets

- Arise in numerous scenarios: two firms (buyers) in a duopoly with a large number of suppliers (goods).
- Results:
  - All NESPs are symmetric and they are a union of at most $2n$ convex sets.
  - The set of NESP payoffs constitute a PLC curve and all these payoffs are Pareto optimal.
  - A buyer gets the maximum NESP payoff when she imitates the other buyer.
  - There may exist NESPs, whose social welfare is larger than that of the Fisher payoff.
  - Behavior of prices - incentives.

Complete Characterization of NESPs

Lemma

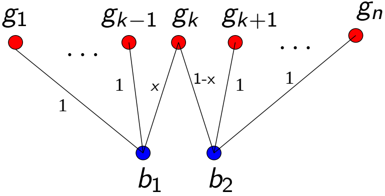
All NESPs for a two-buyer market game are symmetric.

- Assumption: $\frac{u_{1j}}{u_{2j}} \geq \frac{u_{1(j+1)}}{u_{2(j+1)}}$, for $j = 1, \ldots, n-1$.
- For a NESP $S = (\mathbf{s}, \mathbf{s})$, where $\mathbf{s} = (s_1, \ldots, s_n)$.
  - $G(S)$ is a complete bipartite graph.
  - $(p_1, \ldots, p_n) = \mathbf{p}(S)$.
  - $m_1 + m_2 = \sum_{j=1}^{n} s_j = 1 \Rightarrow p_j = s_j, \forall j \in \mathcal{G}$.
  - In a conflict-free allocation $X \in \mathbb{X}(S)$, if $x_{1i} > 0$ and $x_{2j} > 0$, then clearly $\frac{u_{1i}}{p_i} \geq \frac{u_{1j}}{p_j}$ and $\frac{u_{2i}}{p_i} \leq \frac{u_{2j}}{p_j}$.

Nice Allocation

Definition

An allocation $X = [x_{ij}]$ is said to be a nice allocation, if it satisfies the property: $x_{1i} > 0$ and $x_{2j} > 0 \;\Rightarrow\; i \leq j$.



$x \in [0, 1]$

Lemma

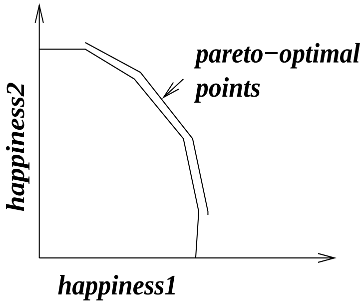
Every NESP has a unique conflict-free nice allocation.

The Happiness Curve

- $\mathbb{F} = \{(\mathcal{P}_1(S), \mathcal{P}_2(S)) \mid S \in S^{NE}\}$ is the set of all possible NESP payoff tuples.

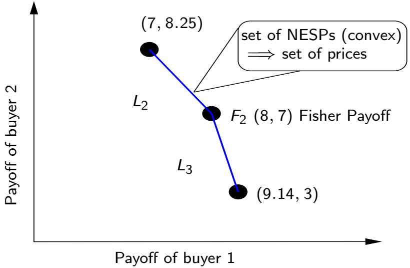
Proposition

$\mathbb{F}$ *is a piecewise linear concave curve.*



Example - Happiness Curve $\mathbb{F}$

Input: $U = \begin{bmatrix} 6 & 2 & 2 \\ 0.5 & 2.5 & 7 \end{bmatrix}, \quad \mathbf{m} = \langle 7, 3 \rangle.$



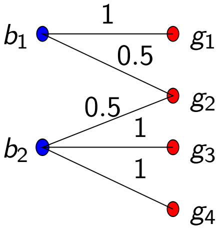
- L_i corresponds to the sharing of good i.
- Social welfare from the Fisher payoff $(8, 7)$ is lower than the payoff $(7, 8.25)$ from the NESP $S = (\mathbf{s}, \mathbf{s})$, where $\mathbf{s} = \langle 6, 2, 2 \rangle$.

Example - Incentives

Input: $U = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \quad \mathbf{m} = \langle 10, 10 \rangle.$

- $\mathbf{s_1} = \langle \frac{20}{3}, \frac{20}{3}, \frac{10}{3}, \frac{10}{3} \rangle$, $\mathbf{s_2} = \langle \frac{20}{3}, \frac{20}{3}, \frac{9}{3}, \frac{11}{3} \rangle$.
- $S_1 = (\mathbf{s_1}, \mathbf{s_1})$, $S_2 = (\mathbf{s_2}, \mathbf{s_2})$.

$G(S_1) = G(S_2)$



$\mathbf{p}(S_1)$	$\mathbf{p}(S_2)$
$\frac{20}{3}$	$\frac{20}{3}$
$\frac{20}{3}$	$\frac{20}{3}$
$\frac{10}{3}$	$\frac{9}{3}$
$\frac{10}{3}$	$\frac{11}{3}$

Can Regulators Help?

Question: Is there a correlated equilibrium π such that the payoff w.r.t. π is liked by both the players?

Proposition

The correlated equilibrium cannot give better payoffs than every NE payoffs to all the buyers.

Srirang

Srirang is a cowherd from Gokul. He has a single cow. By god's grace:

- The cow gives 50 litres of milk everyday.
- The expense of maintaining this cow is Rs. 250 per day.

Srirang wishes to sell this milk. Every evening, Srirang gets bids from various parties. Each bid is of the form:

- Name of the bidder.
- The price at which he/she will purchase milk.
- The volume that he/she requires.

Looking at the bids, Srirang decides on a price for the next day, say X. This price is offered to all customers. The customers who can afford the price collect the milk and pay Rs. X/litre.

Srirang

name	volume	price
roshni	5	20
radha	20	10
prema	15	8
neha	10	6
rukmi	10	5
gauri	10	3

He fixes a price of Rs.5. Gauri goes away. There is an overall demand of 60. The others distribute the supply of 50 liters somehow. Sriang earns Rs. 250.

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He gets a bit greedy and fixes the price to Rs. 8.

Declared Price	8
Demand	40
Supply	40
Earnings	320

Srirang and Siddhartha

- Two identical cows, each giving 25 liters, same *gopis*.
- Each *gopi* has an option to bid either at Srirang or at Siddhartha.

name	volume	price
roshni (5)	1	20
radha (20)	1	10
prema	15	8
neha	10	6
rukmi	10	5
gauri	10	3

- Game with *gopis* as the players and cowherd choice as the strategy.
- Standard NE is randomization.
- An asymmetric solution: Price at Sid is 10 and Price at Srirang is 6.
- rukmi and gauri go away.

Market Segmentation! Different prices for identical goods.

- Sid will differentiate and *add packaging*.

The placement game

Facts! 70% placements over at IIT Bombay.

	M.Techs	Non M.Techs	Remarks
Total jobs	350	563	60% more
Total salary (crores)	31.4	66.7	
Average salary (lakhs)	8.97	11.84	32% more
Engg. and Tech* jobs	84	82	
E&T salary (crores)	5.97	6.37	
E&T average (lakhs)	7.1	7.7	8% more
E&T salary fraction	19%	9.5%	

* E& T is as marked by HR/company/placement officer. Other categories are Finance, Analytics, FMCG, R&D, IT, Consultancy, Education, Services, NA, Others.

The Stiglitz (1975) signalling game.

- Students with capabilities $\theta_1 < \theta_2$, known to students.
- Education system as a labelling agent paid for by society.
- Companies recruit based on label. Salary equals average firm productivity.
- Generally θ_2 wages rise *at the cost of* θ_1.

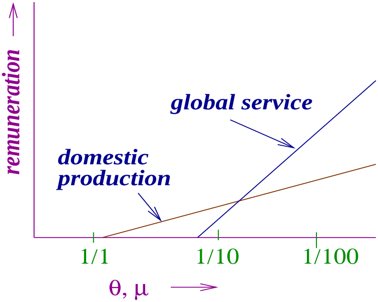
The Question : Under what conditions would/should society pay for labelling?

- The productivity of θ_2-firm is non-linear and overall society wealth increases.
- Mechanisms exist to redeistribute wealth created so that *everyone is better off*.
- **In India–both absent! Instead we have merit and transfer.**

Even more complicating...

θ–societally relevant productive skills
μ– globally relevant service skills

$\mu \downarrow \theta \rightarrow$	1/1	1/10	1/100
1/1	□	□	
1/10	□	□	□
1/100		□	□



The case against *excessive merit*

- Huge labelling costs, borne by public. Subsidy to μ-discovery!
- Need to orient curricula to domestic production.

Conclusions

- Markets–Need to understand basic notions of *efficiency* and *equilibrium*
- *Unpack* consequences for society.
- Indian scenario poses many interesting situation.
  - FDI and agricultural supply chains.
  - Engineering placements.
  - NREGA–Rs. 30,000 crores. PDS.

Thanks!

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