CS206 Spring 2005 Quiz 2

Time: 60 mins Total marks: 25

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

1. *[3+3+3+6 marks]* In this question, we wish to reason about lists using predicate logic. Note that a list is an ordered sequence *(not a set)* of elements. You are allowed to use the following function and predicate symbols in your formulae. The intent of each of these functions and predicates is as indicated by their names.

   - Unary predicate symbols: $\mathsf{is\_a\_list}(x)$, $\mathsf{is\_empty\_list}(x)$, and $\mathsf{is\_list\_with\_one\_element}(x)$.
   - Binary predicate symbol: $\mathsf{equal}(x, y)$.
   - Unary function symbol: $\mathsf{reverse\_list}(x)$.
   - Binary function symbol: $\mathsf{append\_list\_to\_list}(x, y)$.

   Using *only* the above predicate and function symbols and predicate logic operators, express the following English language statements as predicate logic formulae. Note that your formulae must express the same meaning as the English language sentences even if the universe, S, of a model contains elements that are not lists.

   (a) Every list can be obtained by appending a list to another list.

   (b) Every list x that is obtained by appending list y to list z, can be reversed by appending the reverse of list z to the reverse of list y.

   (c) A list is empty iff it keeps every list unchanged after it is appended to the list.

   (d) There are lists, not all of whose elements are identical.

2. *[5 marks]* Consider the formula: $\phi = \forall x\,((\exists y\,(x = f(y))) \rightarrow (\forall y\,(\exists z\,P(x, y, z))))$. Convert the above formula to Skolem Normal Form (the matrix must be in CNF). You must minimize the arity of all skolem functions, and use function names $sk_1, sk_2, \ldots$ to denote skolem functions.

3. *[5 marks]* Show using natural deduction that
   $\forall x\,\phi_1(x, z, z) \wedge \exists z\,\phi_2(x, x, z) \vdash \exists z_1\,\forall x_1 \phi_1(x_1, z, z) \wedge \phi_2(x, x, z_1)$