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## CS206 End-semester Re-examination

Max marks: 60

Time: 3 hours

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- *Be brief, complete and stick to what has been asked.*
  - *If needed, you may cite results/proofs covered in class without reproducing them.*
  - *If you need to make any assumptions, state them clearly.*
  - **Do not copy solutions from others. Penalty for offenders: FR grade.**
1. [10 + 10 marks] Let  $\phi(x, y, z)$  be a propositional logic formula, and let  $\psi(x, y, z) = \phi(x, y, \phi(x, y, z))$ .
    - (a) Show using natural deduction that  $z \rightarrow \phi(x, y, z) \vdash z \rightarrow \psi(x, y, z)$ .
    - (b) Give an equisatisfiable formula for  $\psi(x, y, z)$  in which  $\phi$  is not fed as an argument to itself. You must give complete justification why you think your formula is equisatisfiable with  $\psi(x, y, z)$ .
  2. [10 + 10 marks] Use natural deduction to prove the following sequents:
    - (a)  $\forall x (P(x) \rightarrow (x = b)), \forall x (x = b) \rightarrow P(x) \vdash P(b) \wedge \forall x \forall y (P(x) \wedge P(y) \rightarrow (x = y))$ .
    - (b)  $\forall x (P(x) \vee Q(f(x))), \forall x (R(f(x)) \rightarrow \neg P(x)), \exists x R(f(x)) \vdash \exists x Q(f(x))$ .
  3. [5+5+5+5 marks] Let  $\{a, b, c\}$  be a set of atomic propositions used to construct formulae in CTL and LTL.
    - (a) For each of the following properties, either express it in CTL, or give justification why it cannot be expressed in CTL.
      - i. Whenever  $a$  is true in two consecutive states,  $a$  cannot become true again in the future until a state is reached where either  $b$  or  $c$  is true.
      - ii. Between every two states where  $b$  is true, there is a state where  $c$  is true and  $b$  is false.
    - (b) Express the following property in LTL.
      - i. Either both  $a$  and  $b$  become true infinitely many times, or both become true only finitely many times.
      - ii.  $c$  or  $b$  can become true only after  $a$  has become true in at least two states.