

CS206 Spring 2006 Quiz 1

Time: 60 mins

Total marks: 30

- *The exam is open book and notes.*
- *Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.*
- *If you need to make any assumptions, state them clearly.*
- *Do not copy solutions from others or indulge in unfair means.*

1. [10 marks] Use natural deduction to prove the following sequent, popularly known in logic as the *Resolution Rule*: $\phi \vee \psi, \neg\phi \vee \psi \vdash \psi$.

To score marks, your proof must have no more than 15 steps, and **must not** use *Law of the Excluded Middle (LEM)*.

2. [10 marks] Consider the following set of atomic propositions, representing English language sentences:

- (a) Proposition p_0 : Shyam is a liar.
- (b) Proposition p_1 : The universe started with a soft meow.

In addition, consider the set of atomic propositions $\{q_0, q_1\}$, where q_i represents the sentence “Shyam said that the sentence corresponding to proposition p_i is true”.

For purposes of this question, we will assume a simple binary world, where a person either always speaks the truth or always lies. It then follows that $p_0 \wedge q_i \rightarrow \neg p_i$ and $\neg p_0 \wedge q_i \rightarrow p_i$ for all $i \in \{0, 1\}$. These four formulae can therefore be used as premises for purposes of this question.

Show using natural deduction that “If Shyam said that the sentence corresponding to p_0 is true, then the universe started with a soft meow”.

To score marks, you may use LEM **at most once** in your proof, and your proof must have no more than 20 steps.

3. [5+5 marks] Consider the formula $\phi = \neg((x \wedge \neg(y \wedge z)) \vee (x \wedge y \wedge z))$, where x, y, z are atomic propositions.

- (a) Convert ϕ to negation-normal form (NNF).
- (b) Convert ϕ to conjunctive normal form (CNF) with the *minimum number of clauses*. You may use commonly used propositional logic identities (e.g., distributive laws, $\top \vee \psi = \top$, $\psi \wedge \neg\psi = \perp$, etc.) to minimize the number of clauses. To score marks, you **must mention** the identities you are using.