CS206 Tutorial No. #6

Date: Mar 10, 2006

1. Use Naural Deduction to prove following first order predicate formulae

   (a)

$$\forall(x)[P(x) \rightarrow (x = b)], \qquad P(b),$$
$$\forall(x)[(x = b) \rightarrow P(x)] \;\vdash\; \forall(x)[\forall(y)[P(x) \land P(y) \rightarrow (x = y)]]$$

   (b)

$$\forall(x)[P(x) \lor Q(F(x))],$$
$$\forall(x)[R(F(x)) \rightarrow \neg P(x)], \;\vdash\; \exists(x)[Q(F(x))]$$
$$\forall(x)[R(F(x))]$$

   (c)

$$\forall(x)[\forall(y)[(P(x, y) \land P(y, x)) \rightarrow (x = y)]],$$
$$\forall(x)[P(x, F(x))], \;\vdash\; \exists(x)[u = F(u)]$$
$$\exists(z)[\forall(x)[P(x, z)]]$$

2. Write the Prenex normal form of the formula: $\exists x(P(x) \land \forall y(Q(x, y) \land \forall x \forall w S(x, y, w) \land \exists y R(x, y)))$
   Argue that "it is not the case that only infinite models exist for this formula".