CS206 Tutorial No. #7

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1. Express the following mathematical sentences using first order predicates. Only natural numbers are considered.

   (a) x divides y.

   (b) x is a prime.

   (c) $x \equiv y \pmod{n}$.

   (d) Twin Prime Conjecture
   There are an infinite number of pairs of primes that differ by the number 2.

   (e) There are infinitely many primes.

   (f) If $a \equiv b \pmod{p}$, then
   $(a + c) \equiv (b + c) \pmod{p}$

2. Let

$$\begin{aligned}
\phi_1 &= \exists(x)[P(x, G(x))] \\
\phi_2 &= \forall(y)[P(y, F(y))] \\
\phi_3 &= \forall(u)[\forall(v)[\forall(w)[P(u, v)) \wedge P(v, w) \rightarrow P(u, w)]]] \\
\phi_4 &= \exists(z)[P(z, F(G(z))]
\end{aligned}$$

   $\phi = ((\phi_1 \wedge \phi_2 \wedge \phi_3) \rightarrow \phi_4)$
   Also, ϕ is valid iff ψ is unsatisfiable.
   Use Herbrand Theorem, to prove that ψ is unsatisfiable.

3. Skolemize the formula $\forall x \forall y(x < y \rightarrow \exists z(x < z \wedge z < y))$

4. Skolemize the formula $\exists x \forall y \exists z(x \rightarrow y \wedge z) \wedge \exists x \forall y \exists z \neg(x \rightarrow y \wedge z)$

5. Write a formula using the binary predicate R, and equality predicate and whose only model is an infinite linear chain with one start node.