CS226 End-Semester Examination (Spring 2016)

Max marks: 60 **Time: 180 mins**

Roll No.: ________________ Name: ________________ ________________ ________________

- ***Please write your name and roll number in the space provided at the top.***
- ***Please write your answer to each sub-question only in the space provided in the question paper. Answers written elsewhere will not be graded.***
- ***You must return the question paper along with your answer at the end of the exam.***
- ***The exam is open book and notes brought to the exam hall.***
- *Be brief, complete and stick to what has been asked.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- *If you need to make any assumptions, state them clearly.*
- ***The use of electronic devices is strictly prohibited. You will be debarred from taking the examination if you are found accessing the internet during the examination. All IIT Bombay rules apply in this respect.***
- ***Please do not engage in unfair or dishonest practices during the examination. Anybody found indulging in such practices will be referred to the D-ADAC.***

1. *[10 marks]* Usually, when we draw Karnaugh maps (henceforth called K-maps), we fill in the entries with 1, 0 or − (for don't care). In this question, we want to consider symbolic Karnaugh maps, where we can fill in the entries with symbolic variables or even with other Boolean functions, in addition to using 1, 0 and −.

   Consider the symbolic K-map shown below, where F and G are Boolean variables, and F' and G' represent their respecctive Boolean complements.

$x_1,x_2\rightarrow$ / $x_3,x_4\downarrow$	**00**	**01**	**11**	**10**
00	F	G	F'	1
01	1	G	1	G'
11	F'	F	G	F'
10	G	F'	0	F

   Clearly, the above K-map gives rise to different Boolean functions of x_1, x_2, x_3 and x_4 for different combinations of Boolean values of F and G. In other words, the K-map represents a Boolean function of x_1, x_2, x_3, x_4, F and G. Let us call this function $\varphi(x_1, x_2, x_3, x_4, F, G)$.

   (a) Give a combination of Boolean values of F and G, such that φ with F and G set to these values can be implemented by the circuit shown below (Figure 1). Note that you are not allowed to change the interconnection between the AND and OR gates. You are not allowed to use any additional gates either. However, you are free to label the inputs of the AND gates with $x_1, x_1', x_2, x_2', x_3, x_3', x_4$ or x_4', as you consider appropriate. You must indicate the values of F and G in the space given below, and label the inputs of the AND gates directly in the circuit diagram (Figure 1) shown below.

   Boolean value of F: ________________ , **Boolean value of G:** ________________

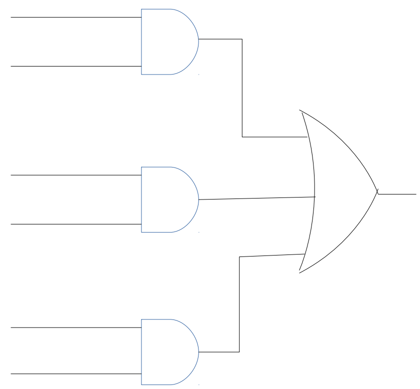

Figure 1: Circuit 1

(b) *[10 marks]* Now suppose F and G are not Boolean variables, but Boolean functions of $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ (or a subset thereof). Therefore, $\varphi(x_1, x_2, x_3, x_4, F, G)$ is now a Boolean function of $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ (or a subset thereof). Give symbolic K-maps for F and G with as many don't cares entries as possible, in the spaces shown below, such that all of the following three conditions hold:

- The ODC for x_5 in the function φ is $x_1.x_2 + x_2'.x_3'$
- The ODC for x_6 in the function φ is $x_3'.x_4 + x_1.x_4'$.
- If the values of x_5 and x_6 are different, then the Boolean functions F and G evaluate to different values (for any combination of values of x_1, x_2, x_3, x_4). *Note that this is an if-then statement, and not an if-and-only-if statement.*

The entries in your K-maps should either be 0, 1, $-$ or functions of $\{x_5, x_6\}$ (or subsets thereof).

Symbolic K-map for F:

$\mathbf{x_1,x_2}\rightarrow$ / $\mathbf{x_3,x_4}\downarrow$	**00**	**01**	**11**	**10**
00				
01				
11				
10				

Symbolic K-map for G:

$\frac{x_1, x_2 \rightarrow}{x_3, x_4 \downarrow}$	**00**	**01**	**11**	**10**
00				
01				
11				
10				

2. Consider the Boolean function $F = (a \oplus b) \cdot (b \oplus c) \cdot (d \oplus e) \cdot (e \oplus f) \cdot (a + f)$, where $\oplus$ denotes EXOR.

   (a) *[10 marks]* Construct an *ROBDD with complement edges* for F using the variable ordering $a < b < c < d < e < f$. You must use only a single 1-leaf (no 0-leaf), and you must *not* have complementing bubbles on solid edges (or 1-labeled edges) in your final answer. If needed, you can use a root edge with a complemening bubble. You must present only your final ROBDD with complement edges in the space below.

(b) *[10 marks]* You are required to implement the function F by interconnecting 2-*to*-1 *multiplexors and a* 2-*to*-4 *decoder*. You are given three working 2-to-1 multiplexors, four defective 2-to-1 multiplexors and a single defective 2-to-4 decoder, for this purpose. You are not allowed to use any other gate (not even an inverter) in your implementation. Assume that you have access to $a, a', b, b', c, c', d, d', e, e', f$ and f', so you can feed any of the literals directly to an input of the multiplexors or the decoder.

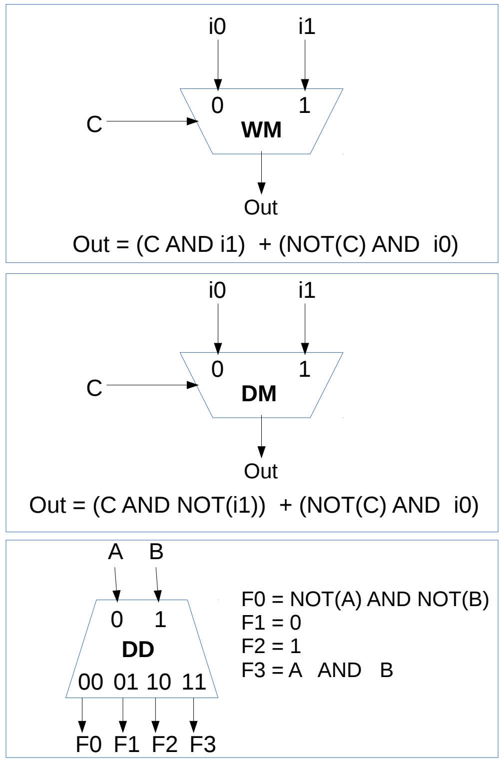

Figure 2: Circuit 1

Figure 2 shows a working 2-to-1 multiplexor, a defective 2-to-1 multiplexor and a defective 2-to-4 decoder, along with the functions they implement. Indicate in the space below how you would connect the (working and/or defective) multiplexors and decoder to implement F correctly. You must label each working multiplexor by "WM", and each defective multiplexor by "DM". You must also label the defective decoder by "DD".

3. *[10 marks]* Consider the circuit shown in Figure 3. The delays of the various gates are as indicated within the gates. Assume that all wires have zero delays.

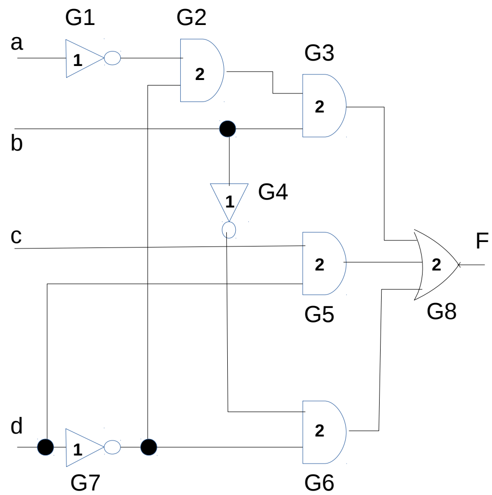

Figure 3: Circuit with delays

(a) We want a single transition on input a to propagate to the output F. Clearly, the delay involved is 7 time units. Assume that a transitions at time $t = 0$; so we want this transition to reach F at time $t = 7$. Note that F may have other transitions before and after $t = 7$. However, we require that the transition on F at time $t = 7$ be the one that propagated from a.

For purposes of this question, we will assume that c stays constant at 1 during the entire time interval from $t = 0$ to $t = 7$. However, we want each of b and d to transition *at least once* during the interval from $t = 0$ to $t = 7$, such that if *any* of the transitions on b or d did not happen, the transition on a would not be able to propagate to F in 7 time units. Note that this means that each of the transitions on b and d is essential for the transition on a to propagate to F in 7 time units.

Indicate the waveforms on a, b and d in the space given below for the above to happen.

(b) *[10 marks]* Consider a circuit consisting of n identical (except delays) stages, as shown in Figure 4. Note that this means that while the structure of each stage of the circuit is the same, the delays of gates in the different stages could indeed be different.

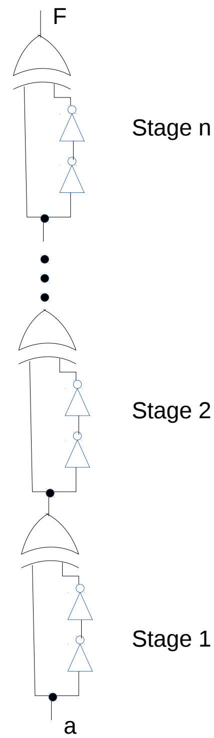

Figure 4: n-stage circuit

Suppose the delay of each EXOR gate is 0, and the delay of each inverter in the i^{th} stage of the circuit is d_i time units.

Indicate what value of d_i would ensure that a single nsition at the input a of the circuit results in 2^n transitions at the output F.

You must clearly indicate your reasoning in around 10 lines in the space provided below.

d_i (as a function of i) is: ________________