CS226 Quiz 2 (Spring 2016)

Mar 17, 2016 $\hspace{1cm}$ **Time: 60 mins**

- *Be brief, complete and stick to what has been asked.*

- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*

- *If you need to make any assumptions, state them clearly.*

- **Please start writing your answer to each sub-question on a fresh page. DO NOT write answers to multiple sub-questions on the same page.**

- **The use of internet enabled devices is strictly prohibited. You will be debarred from taking the examination if you are found accessing the internet during the examination.**

- **Please do not engage in unfair or dishonest practices during the examination. Anybody found indulging in such practices will be referred to the D-ADAC.**

1. *[10 marks]* As a digital logic designer, you have been asked to design a Boolean function $F(x, y, z)$ that satisfies the following properties:

    - The ODC for x (in terms of y and z) is $y' + z'$.
    - The ODC for y (in terms of x and z) is $z.x$.
    - The ODC for z (in terms of x and y) is $y.x$

    Either give the K-map of at least one function $F(x, y, z)$ that satisfies all the above conditions and show how your function satisfies all the three conditions above, or argue why no such function exists.

    Simply saying no such function exists, or simply giving a K-map without showing how it satisfies all three conditions above will fetch 0 marks.

2. Consider a Boolean function of three variables $F(a, b, c) = ((a + b) \oplus c) + c \cdot (a + b)$.

    (a) *[5 marks]* We want to express F as a nested ite expression of the following form:

    $$F(a, b, c) = \mathsf{ite}(a \cdot b, G(a, b, c), \mathsf{ite}(b \cdot c, H(a, b, c), \mathsf{ite}(c \cdot a, M(a, b, c), N(a, b, c))))$$

    Give the simplest possible (i.e. using as few nodes as possible) ROBDDs for $G(a, b, c)$, $H(a, b, c)$, $M(a, b, c)$ and $N(a, b, c)$ using the variable order $a < b < c$ (i.e a above b above c).

    (b) Now suppose we combine four copies of the function F as follows:
    $f(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = F(F(x_1, x_2, x_3), F'(x_3, x_4, x_5), F(x_5, x_6, x_7))$

    i. *[5 marks]* Obtain the ODC of x_4 in terms of $x_1, x_2, x_3, x_5, x_6, x_7$.

    ii. *[5 marks]* A designer proposes replacing $F'(x_3, x_4, x_5)$ in the above expression for $f(x_1, \ldots x_7)$ by simply x_4'. In other words, the function f is proposed to be implemented as
    $F(F(x_1, x_2, x_3), x_4', F(x_5, x_6, x_7))$.
    Show whether the proposed change causes the functionality of the output f to be changed, *using only don't cares, i.e. SDCs and ODCs.* Arguments other than those using SDCs and ODCs will fetch 0 marks.