CS615 Autumn 2004 End Semester Examination

Time: 3 hours | Total marks: 75

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***
- ***<u>Do not copy solutions from others or indulge in unfair means.</u>***

1. Consider the following program, `C`, where all variables are of type `integer`:

```
# Program C

 # Subprogram C0 (next 3 lines)
   t := 2*m - 1;
   n := t;
   c := 0;

 # Subprogram C1 (while loop)
   while (n > 0)

     # Subprogram C2 (if then else)
       if (odd(t))  then
         t := t - 1;
         c := c + 1;
       else
         n := n - 1;
         t := n;
       # endif
   # endwhile
```

Here `even` and `odd` denote the obvious predicates on natural numbers.

(a) *[10 marks]* Give the relational semantics of subprogram `C2` above. Note that the set of state variables in the above program is `t, c, n` and `m`.

(b) *[15 marks]* Let $\underline{C_2}$ denote the relational semantics of subprogram `C2`. Give the result of the composition $((n > 0)\rceil\, \underline{C_2}) \circ ((n > 0)\rceil\, \underline{C_2})$.

(c) *[15 marks]* You are told that a loop invariant of the `while` loop in the above program is:
$L = (n-1 \leq t \leq n) \wedge ((\mathtt{even}(t) \wedge (2*c+t = 2*m)) \vee (\mathtt{odd}(t) \wedge (t = n) \wedge (2*c+t = 2*m-1)))$.
Using the axioms of Hoare Logic and simple algebra, show the following:
$\{L \wedge \mathtt{odd}(t)\}$ `C2` $\{L \wedge (t = n-1) \wedge \mathtt{even}(t)\}$.

2. (a) *[10 marks]* Express the following property as an LTL formula. In your formula, you may assume that op and oq are atomic propositions that evaluate to True in a state if and only if events p and q, respectively, occur in that state. You may assume that two events never occur in the same state.

   Desired property: *Occurrences of events p and q always alternate.*

   (b) *[10 marks]* Is it possible to express the above property in CTL? If so, give the CTL formula. Else, give an intuitive explanation why the above property cannot be expressed as a CTL formula.

3. Consider the following symbolic transition relation, where x_0, x_1 denote present state variables and x'_0 and x'_1 denote next state variables.

   $T(x_0, x_1, x'_0, x'_1) = (\neg x_0 \wedge x'_1) \vee (x_0 \wedge \neg x'_i) \vee (x_0 \wedge \neg x'_0)$.

   (a) *[7.5 marks]* Draw the Kripke structure corresponding to the above formula. You must label each state with values of x_0 and x_1 and draw directed edges to denote state transitions.

   (b) *[7.5 marks]* Let $F(Z(x_0, x_1)) = Z(x_0, x_1) \vee \exists x'_0 \exists x'_1 (T(x_0, x_1, x'_0, x'_1) \wedge Z(x'_0, x'_1))$.

   We wish to compute the greatest fixed point (gfp) of F. Show each step of the gfp computation and give the gfp as a Boolean expression in x_0 and x_1.