CS615 Autumn 2005 Quiz 1

Time: 90 mins | Total marks: 15

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***<u>Do not copy solutions from others or indulge in unfair means.</u>***

1. Consider the following program with location labels `Li`:

```
L1:
     while ((x <= y + 2) AND (x <= 100))
L2:  // begin while
       if (p >= 0) then
L3:
           if (q >= 0) then
              x := x + 1;
           else
              y := y + 1;
           // end if
       else
L4:
           if (q >= 0) then
              x := y - 1;
           else
              y := x - 1;
           // end if
       // end if
L5:
       p := -(p + 1);
       q := -q;
L6:  // end while
```

   You are told that all variables in the above program are of type `integer` and the precondition for this program is $(x = y) \wedge (x \geq 0) \wedge (y \geq 0) \wedge (p \geq 0) \wedge (q \geq 0)$.

   Using this precondition, we wish to compute the strongest loop invariant of the form $(\phi_1(p,q) \vee \phi_2(p,q)) \wedge (\psi_1(x,y) \vee \psi_2(x,y) \vee \psi_3(x,y) \vee \psi_4(x,y))$ at `L1`, where $\phi_i(p,q)$ denote finite sets of linear inequalities involving only the variables p and q, and $\psi_j(x,y)$ denote finite sets of linear inequalities involving only the variables x and y.

   You may use the location labels given in the program to identify sets of linear inequalities between the appropriate variables at these locations. Note also that since you are allowed to use disjunctions, you can represent a non-convex region as the union of two or more convex regions. You must use exploit this freedom to obtain as strong a loop invariant as you can.

   Answers without justification will fetch no marks. Also, your reasoning must involve only applications of Hoare Logic axioms and rules, and manipulations of linear inequalities across assignment statements, while statements and if-then-else statements as discussed in class.