CS615 Autumn 2005 Quiz 3

Time: 60 mins | Total marks: 40

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

1. *[10+15+5 marks]* We wish to analyze the recursive program below to determine if location `L` is reachable. All variables are assumed to be of type `integer`.

```
    x := A(m)
    if ((x > 0) AND (m < 100)) then
L:   skip;
    else
      skip;

    A(n)
     if (n >= 100) then
       return(100);
     else
       if (n <= 50) then
         return(0);
       else
         return(A(A(n-10)));
```

The program is converted to the following (partially described) form for ease of analysis:

```
    ap_n := m;
    A(ap_n);
    x:= ret_A;

    if (*) then
      assert(...);
L:    ...
    else
      ...
```

(see next column)

```
A(fp_n)
  if (*) then
    ...
    return;
  else
    ...
    if (*) then
      ...
      ret_A := 0;
      return;
```

(see next column)

```
    else
      ...
      ap_n := fp_n - 10;
      A(ap_n);
      ...
      return;
```

(a) Complete the partially described program above, such that every trace of the original program corresponds to a trace of the transformed program that does not end in an assertion failure, and vice versa. You must not use any additional variables other than those shown above in the partially described program.

(b) Using Boolean variables corresponding to *only the following predicates*, give a Boolean abstraction of the program (i.e., a Boolean program) that preserves behaviour as the original program as far as reachability of `L` is concerned.

Bool var	Predicate	Bool var	Predicate	Bool var	Predicate
b_0	$m \geq 100$	b_1	$m \leq 50$	b_2	$ap\_n \geq 100$
b_3	$ap\_n \leq 50$	b_4	$fp\_n \geq 100$	b_5	$fp\_n \leq 50$
b_6	$ret\_A = 0$	b_7	$ret\_A \leq 50$	b_8	$ret\_A \geq 100$
b_9	$x > 0$				

Note: Your Boolean program must follow Ball and Rajamani's construction, except for the possible use of suitable underapproximations to the $\mathcal{F}$ functions. You must choose your underapproximations, if any, such that the behavior with respect to reachability of `L` *is preserved.*

(c) Does your Boolean program always terminate? Give brief justification for your answer.

2. *[10 marks]* Show using first-order logic that every path π that satisfies $\mathbf{FGFG}p$ also satisfies $\mathbf{FG}p$, where p is an atomic proposition.