CS615 Autumn 2005 Take-home Quiz

Due by: Sat, Nov 12, 12 noon   Total marks: 70

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Do not copy solutions from others or indulge in unfair means.***
- ***Penalty for suspected copying is an FR grade.***
- ***Your solutions must be turned in to the instructor's office in CFDVS or to the TA by 12noon, Sat, Nov 12.***

1. Consider the following program with location labels `Li`:

```
      /* precondition: (0 <= x <= A) AND (0 <= y <= B) */

L1:   while ((C * x * x) + (D * y * y) <= E)
         t1 := (F * x) + (G * y);
         y  := (H * x) + (I * y);
         x  := t1;

      if ((C * x * x) + (D * y * y) <= J) then
L2:      x := 0;
      else
         x := 1;
```

   In the above program, `x` and `y` are variables of type `real`, and `A` through `J` are symbolic rational constants.

   You are required to check http:://www.cse.iitb.ac.in/∼supratik/courses/cs615/takehomeconst.txt to obtain values to be used for the symbolic constants before answering the following questions. A separate set of values has been assigned to each student. Depending on the values assigned to you, answers to the following questions may differ.

   You are required to use abstract interpretation to determine if location `L2` can be reached in the above program starting from the given precondition.

   (a) *[30 marks]* Define a suitable abstract interpretation for the analysis of the above program. You must show that the elements in the abstract domain form a complete lattice, and must give abstraction (α) and concretization (γ) functions.

   Your choice of abstract interpretation must be such that you obtain a precise answer to question (c) below. Use of an abstract interpretation that gives an imprecise answer to question (c) will fetch no marks for any of the three questions. You are therefore advised to think sufficiently before choosing a suitable abstract interpretation

   (b) *[20 marks]* Compute the loop invariant of the while loop in the abstract domain defined above. You must show that the transformations effected by the loop body in the concrete domain can be abstracted by a monotonic function in the abstract domain. You must then compute the least fixed point of this function.

   Note that the loop invariant must be satisfied everytime location `L1` is reached. In particular, the invariant must be satisfied at the end of the final iteration of the loop (if it terminates) when `((C * x * x) + (D * y * y) > E)`. Therefore, `((C * x * x) + (D * y * y) <= E)` cannot be a loop invariant if the loop terminates.

   (c) *[20 marks]* Use the concretization function on the loop invariant computed above to determine whether program location `L2` is reachable. Your choice of abstract interpretation **must** lead to a precise answer to this question.