CS615 Autumn 2006 Quiz 2

Time: 30 mins Total marks: 10

- ***You must write your answers only in the spaces provided.***
- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***If you need to make any assumptions, state them clearly.***
- ***<u>Do not copy solutions from others or indulge in unfair means.</u>***

1. In a program with n variables, let D_i be the domain of the i^{th} variable. Let the powerset of $D_1 \times \ldots D_n$ be denoted $\mathcal{P}(D_1 \times \ldots D_n)$.

   Now consider the two complete lattices $(\mathcal{P}(D_1 \times \ldots D_n), \subseteq, \emptyset, D_1 \times \ldots D_n)$ and $(A, \sqsubseteq, \bot^\#, \top^\#)$, where A is a set, $\sqsubseteq$ is a partial order on A and $\bot^\#$ and $\top^\#$ are the bottom and top elements of the second lattice. Let $\alpha : \mathcal{P}(D_1 \times \ldots D_n) \rightarrow A$ and $\gamma : A \rightarrow \mathcal{P}(D_1 \times \ldots D_n)$ be two *monotone functions*.

   Show that

   (a) If $S \subseteq \gamma(\alpha(S))$ for all $S \in \mathcal{P}(D_1 \times \ldots D_n)$ and if $\alpha(\gamma(\hat{S}) \sqsubseteq \hat{S}$ for all $\hat{S} \in A$, then for all $S \in \mathcal{P}(D_1 \times \ldots D_n)$ and $\hat{S} \in A$, $\alpha(S) \sqsubseteq \hat{S}$ iff $S \subseteq \gamma(\hat{S})$.

   (b) If for all $S \in \mathcal{P}(D_1 \times \ldots D_n)$ and $\hat{S} \in A$, $\alpha(S) \sqsubseteq \hat{S}$ iff $S \subseteq \gamma(\hat{S})$, then $S \subseteq \gamma(\alpha(S))$ for all $S \in \mathcal{P}(D_1 \times \ldots D_n)$ and $\alpha(\gamma(\hat{S}) \sqsubseteq \hat{S}$ for all $\hat{S} \in A$.