CS615 Autumn 2006 Quiz 5

Time: 30 mins Total marks: 10

- ***You must write your answers only in the spaces provided.***
- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

1. In this question, we'll try to reason about abstract interpretation through geometric shapes in two dimensions. Thus, your answers should be geometric shapes rather than formulas and expressions. Of course, you are welcome to justify your *art* (geometric shapes) through formulas and expressions, if you wish to.

   The concrete domain for this question is the powerset of all points on the two-dimensional real plane, i.e., any arbitray (and possibly disconnected) shape. The abstract domain is the set of all two-dimensional *octagons*, i.e., a convex region in the two-dimensional plane that bounded by *at most* eight lines. The slope of each line is either 0, ∞, -1 or $+1$. Note that an octagon has at most eight bounding lines, and could have fewer bounding lines.

   We wish to use the following abstraction and concretization functions. If O is an octagon, and P is an arbitray collection of points in two dimensions, then $\alpha(P)$ is the smallest octagon that contains P, and $\gamma(O)$ is the set of points within the bounding lines of O. It can be shown that these two functions form a *Galois connection.*

   Let us now consider the following program P with real variables `x` and `y`:

```
L1: while (*) {
L2:    assert(x <= 100);
L3:    x := x + 2;
L4:    y := y + 2;}
```

   Suppose the program is started with the precondition $\psi = x^2 + y^2 = 25$.

   In the following questions, you are required to specify an octagon as the answer to each question. Please specify an octagon either by specifying the equations of the (at most) eight lines bounding the octagon, or by specifying the coordinates of its vertices.

   (a) Specify the octagon that represents the abstraction of the precondition.

(b) Specify the octagon that represents the abstraction of the postcondition after executing the loop the first time.

(c) Generalizing from above, if we started the program with $\gamma(\alpha(\psi))$, specify the abstraction of the strongest loop invariant. In other words, what will be the smallest octagon containing the set of states whenever the loop head is reached, if we start with the set of points in $\gamma(\alpha(\psi))$?