CS615 Autumn 2006 Quiz 6

Time: 30 mins | Total marks: 10

- ***You must write your answers only in the spaces provided.***
- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

In this quiz, we will investigate some properties of threshold widening studied in class.
Let $(L, \sqsubseteq, \bot, \top)$ be a complete lattice with $\sqcup$ as the least upper bound operator and $\sqcap$ as the greatest lower bound operator. Let $l_1 \sqsubseteq l_2 \sqsubseteq \ldots \sqsubseteq l_k = \top$ be a chain of k elements (or thresholds) in the lattice, with the last element being the top element. For any two elements $x, y \in L$, we define (as done in class) $x \nabla y = l_i$, where

- $1 \leq i \leq k$
- For all j such that $1 \leq j < i$, either $x \not\sqsubseteq l_j$ or $y \not\sqsubseteq l_j$. In other words, i is the least index such that $x \sqsubseteq l_i$ and $y \sqsubseteq l_i$.

We have seen in class that the above definition of ∇ satisfies the properties of a widening operator.

In the following subquestions, x, y and z are arbitrary elements in L.

(a) Prove in the space below that $x \nabla (x \sqcup y) = x \nabla y$.

(b) Prove in the space below that $x \nabla (x \sqcap y) = x \nabla x$

(c) You are told that $x \nabla (y \nabla z) = w \nabla w$. Express w in terms of x, y, z and the $\sqcap$ and $\sqcup$ operators, with justification.

END OF PAPER