CS615 Autumn 2006 Quiz 7

Time: 30 mins | Total marks: 10

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

Let the concrete domain be the set of all subsets of points in the real (X-Y) plane, with the usual set inclusion ordering. Let the abstract domain be the set of all real 6−tuples (a, b, c, d, e, f). We say that $(a_1, b_1, c_1, d_1, e_1, f_1) \sqsubseteq (a_2, b_2, c_2, d_2, e_2, f_2)$ iff $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$, $d_1 \leq d_2$, $e_1 \leq e_2$ and $f_1 \leq f_2$. We define the abstraction and concretization functions as follows:

- For all $S \subset \Re^2$, $\alpha(S) = (a, b, c, d, e, f)$, where $a = \min_{(x,y)\in S}(x)$, $b = \max_{(x,y)\in S}(x)$, $c = \min_{(x,y)\in S}(y)$, $d = \max_{(x,y)\in S}(y)$, $e = \min_{(x,y)\in S}(x - y)$, $f = \max_{(x,y)\in S}(x - y)$.
- For all $(a, b, c, d, e, f) \in \Re^6$, $\gamma((a, b, c, d, e, f)) = \{(x, y) \mid (a \leq x \leq b) \wedge (c \leq y \leq d) \wedge (e \leq x - y \leq f)\}$.

It can be shown that α and γ as defined above form a Galois connection between the concrete and abstract domains.

Now consider the following program with location labels:

```
L0:  while (y < 100) {
L1:    if (x > 30) {
L2:      x := x - y;
L3:    }
L5:    else {
L6:      x := x + y;
L7:    }
L8:    y := y + 1;
L9:  }
```

Starting with the concrete precondition $\{(0 \leq x \leq 1) \wedge (0 \leq y \leq 1)\}$, you are required to consider execution of the above program in the abstract domain, and determine the following:

1. As strong an abstract invariant as you can at location L3.
2. As strong an abstract invariant as you can at location L7.
3. As strong an abstract invariant as you can at location L0.