Max marks: 20+20(bonus)

1. Consider the following program P in a C-like language, in which all variables are of type int.

```
L1: while (a <= 1000) {
L2:    a = b + i;
L3:    b = a + 1;
L4:    i = i + 1;
L5: }
L6: if (b > 2000) {
error:    exit(-1);
L7: }
```

We wish to check if starting from the pre-condition  $\{(a \le 1000) \land (b = 0) \land (i = 0)\}$  the program P can reach the **error** location. We will try to do this using predicate abstraction, i.e. using the technique of Boolean programs.

- (a) [10 marks] Suppose we start off with the predicates  $(a \le 1000), (b \ge 0), (i = 0)$ , and  $(b \le 2000)$ . Let the boolean variables  $p_1, p_2, p_3$ , and  $p_4$  denote the above predicates respectively. Construct a Boolean program  $BP_1$  from P using the above predicates. You must clearly indicate how each statement of the Boolean program is derived.
- (b) [5 marks] Show that  $BP_1$  constructed above can indeed reach the error location, when started with the pre-condition  $\{p_1 \wedge p_2 \wedge p_3\}$ . Give the shortest trace  $\tau$  (i.e., sequence of program locations) in  $BP_1$  that illustrates how the error location is reached in  $BP_1$ .
- (c) [5 marks] Show that the shortest trace  $\tau$  obtained above does not correspond to an execution of the original program  $P_1$ . In other words, show that the counterexample provided by the Boolean program is *spurious*.
- (d) [10 marks] Using the idea of Craig interpolants, derive a location-specific set of predicates from the above shortest trace  $\tau$ , that would eliminate the spurious counterexample obtained above. In other words, you are required to identify one or more predicate(s) corresponding to each location of the program, such that if we construct a new Boolean program  $BP_2$  using the new set of predicates, the trace  $\tau$  would not correspond to an execution of  $BP_2$ .
- (e) [10 marks] Construct  $BP_2$ , i.e. a Boolean program from P using the location-specific predicates identified in the previous sub-question.