CS615 Theory Quiz 3 (Autumn 2017)

Max marks: 15 **Time: 60 mins**

- *Be brief, complete and stick to what has been asked.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. Penalty for offenders: FR grade.***

1. *[15 marks]* Consider the predicate $BT(x, n)$ that defines a special kind of binary tree of height n and with root r, as follows:

$$BT(x,n) \quad \equiv \quad (x \neq NULL) \wedge \begin{pmatrix} ((x \mapsto [l : NULL, r : NULL]) \wedge (n = 1)) \quad \vee \\ \exists u, v, j, k \;\; ((x \mapsto [l : u, r : v]) \star BT(u, j) \star BT(v, k) \wedge (n = \max(j, k) + 1)) \end{pmatrix}$$

In the above formulation, we have used l and r to denote the "left child" and "right child" fields of a node in a binary tree.

Now consider the function `myFunc1` given below:

```
myFunc(BTNodePtr x)  {
  lchild = x->l;
  rchild = x->r;
  if (lchild == NULL) {
    addChildren(x);
  }
  else {
    myFunc(lchild);
    myFunc(rchild);
  }
}
```

You are given the following specification of the function `addChildren(BTNodePtr x)`:

$$\{BT(x, 1)\} \quad \texttt{a}ddChildren(x) \quad \{BT(x, 2)\}$$

Use the Frame Rule and separation-logic based Hoare-style reasoning to show that

$$\{BT(x, n)\} \quad \texttt{m}yFunc(x) \quad \{BT(x, n + 1)\}$$