CS615 Spring 2010 Alternative Quiz1

Time: 30 mins | Total marks: 15

- ***The quiz is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

Consider the program P given below:

```
L0:  t1 := x;
L1:  t2 := y;
L2:  i := 0;

L3:  while ((t1 != 0) AND (t2 != 0))
L4:    t1 := *t1;
L5:    t2 := *t2;
L6:    i := i+1;
L7:  // end of while loop
```

Let $\mathsf{mylist}(x)$ and $\mathsf{mylistlen}(x, n)$ be recursive predicates defined as follows:

$$\mathsf{mylist}(x) \equiv (x = 0) \vee \exists z. ((x \mapsto z) \star \mathsf{mylist}(z))$$

$$\mathsf{mylistlen}(x, n) \equiv ((n = 0) \wedge (x = 0)) \vee ((n > 0) \wedge \exists z. ((x \mapsto z) \star \mathsf{mylistlen}(z, n-1)))$$

Informally, $\mathsf{mylist}(x)$ says that x is a NULL-terminated list (including the degenerate case when x is itself 0), and $\mathsf{mylistlen}(x, n)$ says that x is a NULL-terminated list of n elements (for $n \geq 0$).
We want to prove $\{\varphi\}$ P $\{\psi\}$, where $\varphi \equiv (\mathsf{mylistlen}(x, m) \star \mathsf{mylistlen}(y, n))$ and $\psi \equiv (i = \min(m, n))$. Note that m and n are auxiliary variables that do not appear in the program. Instead of proving $\{\varphi\}$ P $\{\psi\}$ (call this Hoare triple HT1) directly, we will first prove $\{\varphi \wedge (m \leq n)\}$ P $\{(i = m) \wedge (m \leq n)\}$ (call it HT2). By symmetry of the formulas and of the program P, this would also give us a proof of $\{\varphi \wedge (n \leq m)\}$ P $\{(i = n) \wedge (n \leq m)\}$ (call it HT3). HT1 can then be obtained from HT2 and HT3 by disjoining the corresponding pre-conditions and post-conditions.
So our goal now is to prove HT2: $\{\mathsf{mylistlen}(x, m) \star \mathsf{mylistlen}(y, n) \wedge (m \leq n)\}$ P $\{(i = m) \wedge (m \leq n)\}$. The real meat of such a proof is obtaining the loop invariant at L3.
Show that $\xi \equiv \mathsf{mylistlen}(t1, m - i) \star \mathsf{mylistlen}(t2, n - i) \wedge (m \leq n)$ suffices as a loop invariant to prove HT2. In other words, you must show the following three facts:

1. *[3 marks]* $\{\varphi \wedge (m \leq n)\}$ `L0: ... L1: ... L2: ...` $\{\xi\}$
2. *[8 marks]* $\{\xi \wedge (t1 \neq 0) \wedge (t2 \neq 0)\}$ `L4: ... L5: ... L6: ...` $\{\xi\}$
3. *[4 marks]* $\xi \wedge ((t1 = 0) \vee (t2 = 0)) \Rightarrow \{(i = m) \wedge (m \leq n)\}$.