

- *The exam is open book and notes.*
- *Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.*
- *Unnecessarily lengthy solutions will be penalized.*
- *If you need to make any assumptions, state them clearly.*
- *Do not copy solutions from others or indulge in unfair means.*

1. [10 + 10 marks] Consider the following program  $P$  in the language studied in class.

```

L1:    t := x;
L2:    while (t != y)
L3:        t1 := *t;
L4:        if (t1 = 0) then
L5:            *t := y;
L6:        else
L7:            *t := t1;
L8:        t := *t;
L9:    // end of while loop

```

Let  $\text{list1}(u, v)$  and  $\text{list2}(u, v)$  be recursive predicates defined using separation logic as follows:

$$\text{list1}(u, v) = (u \mapsto v) \vee \exists w. ((u \mapsto w) \star \text{list1}(w, v))$$

$$\text{list2}(u, v) = ((u \mapsto 0) \star (v \mapsto 0)) \vee \exists p, q. ((u \mapsto p) \star (v \mapsto q) \star \text{list2}(p, q))$$

Prove the following Hoare triples.

- $\{\text{list1}(x, p) \star \text{list1}(y, p) \star \text{list1}(p, 0)\} P \{\text{list1}(x, p) \star \text{list1}(p, p)\}$
- $\{\text{list2}(x, y)\} P \{\text{list1}(x, 0)\}.$

In the above proofs, you can make use of the fact that  $\text{list1}(u, v) \star \text{list1}(v, w) \vdash \text{list1}(u, w)$  for all  $u, v, w$ .

2. [10 marks] The following program  $Q$  is written in the language studied in class.

```

L1:  let f(x) =
L2:      if (x >= 0) then
L3:          ret := -x;
L4:      else
L5:          ret := 1 - f(x+1);
L6:  in
L7:      i := 0;
L8:      while (v != 0)
L9:          v := f(v);
L10:         i := i + 1;
L11:  // end of program

```

Indicate whether  $\{\text{True}\} Q \{0 \leq i \leq 5\}$  is a valid Hoare triple. If yes, you must give a proof. Otherwise, you must provide a counterexample, i.e. a state that satisfies the pre-condition, but violates the post-