

- *The quiz is open book and notes.*
- *Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.*
- *Unnecessarily lengthy solutions will be penalized.*
- *If you need to make any assumptions, state them clearly.*
- *Do not copy solutions from others or indulge in unfair means.*

Consider the following program P along with symbolic assertions:

```

L0:  n := 0;

      {phi1}

L1:  while (n = 0)
L2:      n := f(n);
      {phi2}
L3:      *x := *y;
      {phi3}
L4:      *y := x;
      {phi4}
L5:      x := *x;
      {phi5}

L6: {phi6} // end of program

```

Let $\text{list}(y, x)$ be a recursive predicate defined as follows:

$$\text{list}(y, x) = (y \mapsto x) \vee \exists z. (y \mapsto z) \star \text{list}(z, x)$$

Suppose further that the definition of function $\mathbf{f}(n)$ is not known, but it is known that $\mathbf{f}(n)$ does not access the heap and therefore never causes a memory error.

Show that $\{\text{list}(y, x) \star \exists w. x \mapsto w\} \vdash P \{ \exists v. ((y \mapsto v) \star (v \mapsto x)) \star \mathbf{true} \}$ is a valid Hoare triple. Note that $(s, h) \models \mathbf{true}$ for any stack s and heap h .

To be specific,

- Give assertions $\mathbf{phi1}$, $\mathbf{phi2}$, $\mathbf{phi3}$, $\mathbf{phi4}$, $\mathbf{phi5}$ and $\mathbf{phi6}$ such that these assertions hold at the locations indicated in the program. Note that $\mathbf{phi1}$ must be a loop invariant, and $\mathbf{phi6}$ should be implied by $\mathbf{phi1} \wedge (n \neq 0)$.
- Ensure that $\{\text{list}(y, x) \star \exists w. x \mapsto w\}$ logically entails $\mathbf{phi1}$, and $\mathbf{phi6}$ logically entails $\{\exists v. ((y \mapsto v) \star (v \mapsto x)) \star \mathbf{true}\}$.

Hint: Rewrite the precondition as the disjunction of two separation logic formulae, obtained from the recursive definition of list and from the fact that \star distributes over \vee . Then consider what happens if you start from each of these preconditions separately.