CS615 Quiz 2

Max marks: 30 **Time: 0.5 hours**

- *Be brief, complete and stick to what has been asked.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. Penalty for offenders: FR grade.***

We have studied a bit about intervals as an abstract domain. For a program with n variables (for simplicity, let us consider all variables of type `int`), an element of the interval abstract domain is an n-tuple of intervals, where each interval is of one of the following forms:

- $[l, u]$ where l, u are integers and $l \leq u$
- $(-\infty, u)$ where u is an integer
- $[l, +\infty)$ where l is an integer
- $(-\infty, +\infty)$
- (), denoting the empty interval

Suppose the abstract state at a particular point of program analysis is given by $(I_1, I_2, \ldots I_n)$, where each I_k is an interval of one of the forms mentioned above. Intuitively, I_k gives an overapproximation of the interval in which the value of the k^{th} program variable lies at that point of the program analysis.

Now suppose we were to use a slightly more complex abstract domain where every element of the abstract domain is an n-tuple of *pairs of intervals* (once again, for a program with n integer variables). Let (I_k, J_k) denote the k^{th} pair of intervals in an abstract state at a particular point of program analysis. We wish to interpret the abstract state as specifying that the value of the k^{th} program variable either lies in the interval I_k or J_k. Note that I_k and J_k may represent disjoint intervals on the integer line.

We wish to have the new abstract domain as a lattice.

1. *[10 marks]* Give an effective definition of the $\sqsubseteq$ relation in the new abstract domain. In other words, given abstract elements $a = ((I_1, J_1), \ldots (I_n, J_n))$ and $b = ((I'_1, J'_1), \ldots (I'_n, J'_n))$, describe how you would determine if $a \sqsubseteq b$.

2. *[10 marks]* Give an effective definition of the *lub* and *glb* operators in the new abstract domain. Thus, you must give an algorithm to compute the *lub* and *glb* of a pair of abstract elements.

3. *[10 marks]* Give an effective definition of the ∇ operator in the new abstract domain. Once again, you must give an algorithm to compute the result of applying ∇.