CS615 Quiz 1 (Spring 2019)

Max marks: 20 **Time: 60 mins**

- *Be brief, complete and stick to what has been asked.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. Penalty for offenders: FR grade.***

1. *[5 + 5 marks]* We've studied a bit about the theory of abstract interpretation. Consider two abstract domains $\mathcal{A}_i = (A_i, \sqsubseteq_i, \sqcup_i, \sqcap_i, \top_i, \bot_i, \nabla_i)$ for $i \in 1, 2$. Let the corresponding abstraction and concretization functions be α_i and γ_i respectively. Assume that (α_i, γ_i) form Galois connections for $i \in \{1, 2\}$. In addition, you are told the following facts:

   - $\forall a_1 \in A_1,\ \alpha_1(\gamma_1(a_1)) = a_1$. In other words, (α_1, γ_1) forms a Galois insertion.
   - $\forall a_2 \in A_2\, \exists a_1 \in A_1\ \gamma_2(a_2) = \gamma_1(a_1)$. In other words, the shape of concrete state sets that A_2 can represent is subsumed by that which A_1 can represent.

   Show that $(\alpha_2 \circ \gamma_1 : A_1 \to A_2,\ \alpha_1 \circ \gamma_2 : A_2 \to A_1)$ forms a Galois connection between $(A_1, \sqsubseteq_1)$ and $(A_2, \sqsubseteq_2)$.

2. *[10 marks]* We've learnt about threshold widening as a way to improve the precision of the widening operator using a *finite* set of thresholds. A student proposes the following improvement to threshold widening using a set of thresholds that is not restricted to be finite.

   Let $\mathcal{A} = (A, \sqsubseteq, \sqcup, \sqcap, \top, \bot, \nabla)$ be the abstract domain with ∇ being the native widen operator for $\mathcal{A}$. Let $T = (t_0, t_1, t_2 \ldots \top)$ be an increasing chain of elements in $(A, \sqsubseteq)$, i.e. $\bot = t_0 \sqsubset t_1 \sqsubset t_2 \sqsubset \cdots$. The student proposes the following adaptation of ∇, denoted ∇_T:

   Given a sequence $a_0 \sqsubseteq a_1 \sqsubseteq a_2 \sqsubseteq \cdots$, the threshold widened limit (i.e. using ∇_T) is defined as follows.

   - Define $z_0 = a_0$ and $z_i = (z_{i-1} \nabla a_i) \sqcap t_{f(i)}$, where $i \geq 0$ and $f(i)$ is defined by the following relation: $t_{f(i)-1} \sqsubseteq (z_{i-1} \sqcap a_i)$ and $(z_{i-1} \sqcup a_i) \sqsubseteq t_{f(i)}$.
   - The threshold widened limit of the sequence $a_0 \sqsubseteq a_1 \sqsubseteq a_2 \sqsubseteq \cdots$ is given by the native widened limit (i.e. using ∇) of the sequence $z_0 \sqsubseteq z_1 \sqsubseteq \cdots$ obtained above.

   Does ∇_T form a valid widening operator for $\mathcal{A}$? Give reasons for your answer.