Efficient Call Strings Method for Flow and Context Sensitive Interprocedural Data Flow Analysis

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Part 1

About These Slides

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These slides constitute the lecture notes for CS618 Program Analysis course at IIT Bombay and have been made available as teaching material accompanying the book:

- Uday Khedker, Amitabha Sanyal, and Bageshri Karkare. *Data Flow Analysis: Theory and Practice*. CRC Press (Taylor and Francis Group). 2009.

Apart from the above book, some slides are based on the material from the following books

- S. S. Muchnick and N. D. Jones. *Program Flow Analysis*. Prentice Hall Inc. 1981.

Part 2

Value Based Termination of Call String Construction

An Overview

- Value based termination of call string construction (VBTCC)

  No need to construct call strings upto a fixed length
- Only as many call strings are constructed as are required
- Significant reduction in space and time
- Worst case call string length becomes linear in the size of the lattice instead of the original quadratic

All this is achieved by a simple change without compromising on the precision, simplicity, and generality of the classical method

Uday P. Khedker and Bageshri Karkare. *Efficiency, Precision, Simplicity, and Generality in Interprocedural Data Flow Analysis: Resurrecting the Classical Call Strings Method*. International Conference on Compiler Construction (CC 2008), Hungary.

Important Disclaimer

- These slides are aimed at
  - teaching rather than making a short technical presentation,
  - It is assumed that the people going through these slides do not have the benefit of attending the associated lectures
- Hence these slides are verbose with plenty of additional comments (usually not found in other slides)

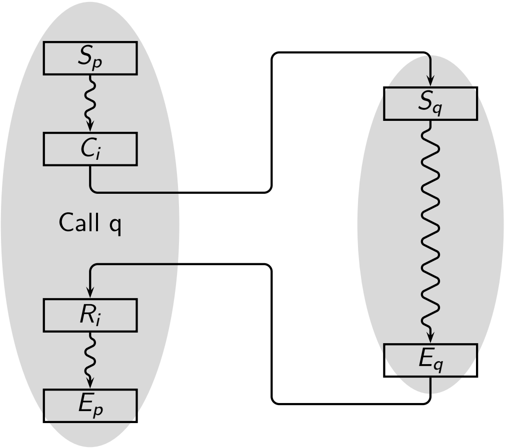
The Limitation of the Classical Call Strings Method

Required length of the call string is:

- K for non-recursive programs
- $K \cdot (|L| + 1)^2$ for recursive programs

M. Sharir and A. Pnueli. *Two Approaches to Interprocedural Data Flow Analysis*. In *Program Flow Analysis: Theory and Applications*. S. S. Muchnick and N. D. Jones (Ed.) Prentice-Hall Inc. 1981.

VBTCC: A Motivating Example



VBTCC: A Motivating Example

We will

- first work out the conventional call strings method on the example program,
- make useful observations about how it works, and
- convert it to VBTCC based call strings method

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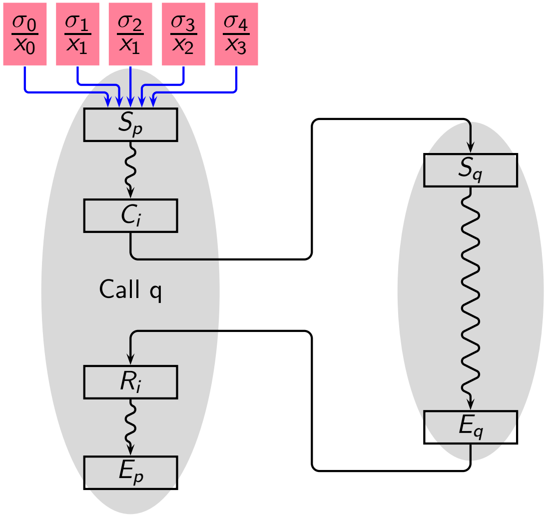

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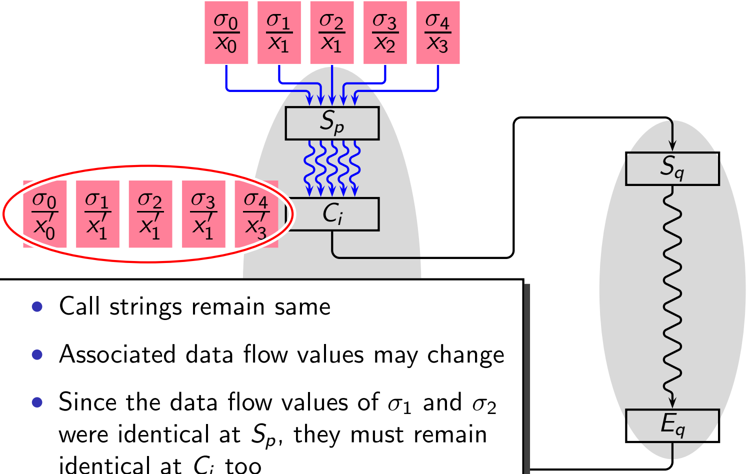

- Call strings remain same
- Associated data flow values may change
- Since the data flow values of σ_1 and σ_2 were identical at S_p, they must remain identical at C_i too
- In this case, the data flow value of σ_3 has become same as that of σ_1 and σ_2

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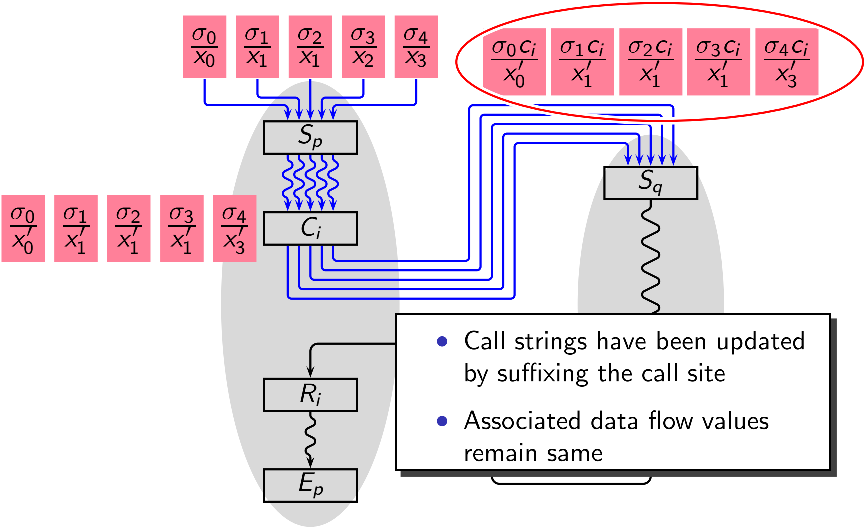

- Call strings have been updated by suffixing the call site
- Associated data flow values remain same

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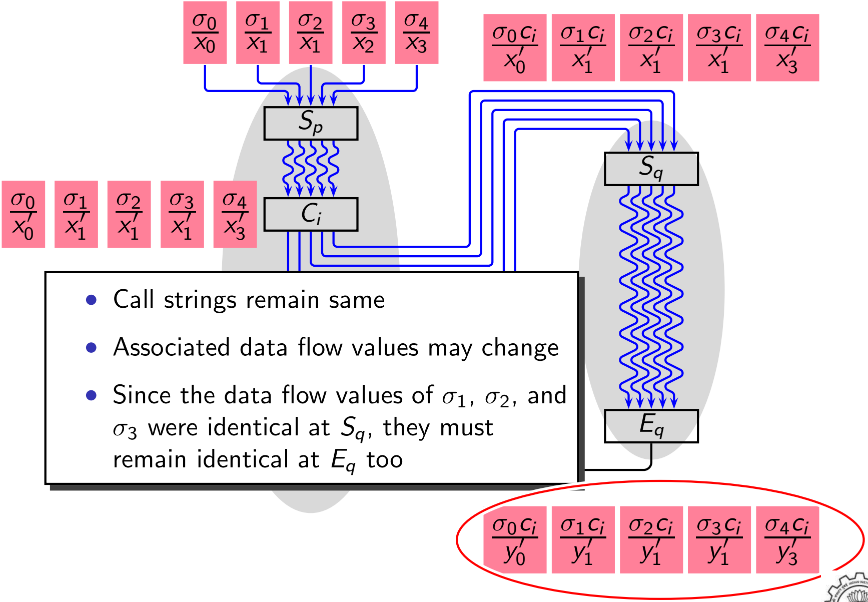
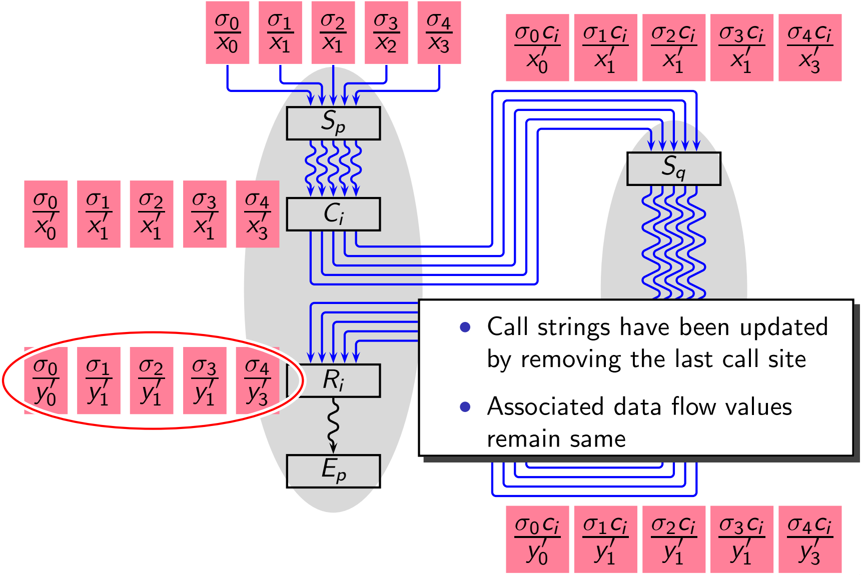


- Call strings remain same
- Associated data flow values may change
- Since the data flow values of σ_1, σ_2, and σ_3 were identical at S_q, they must remain identical at E_q too

VBTCC: A Motivating Example



- Call strings have been updated by removing the last call site
- Associated data flow values remain same

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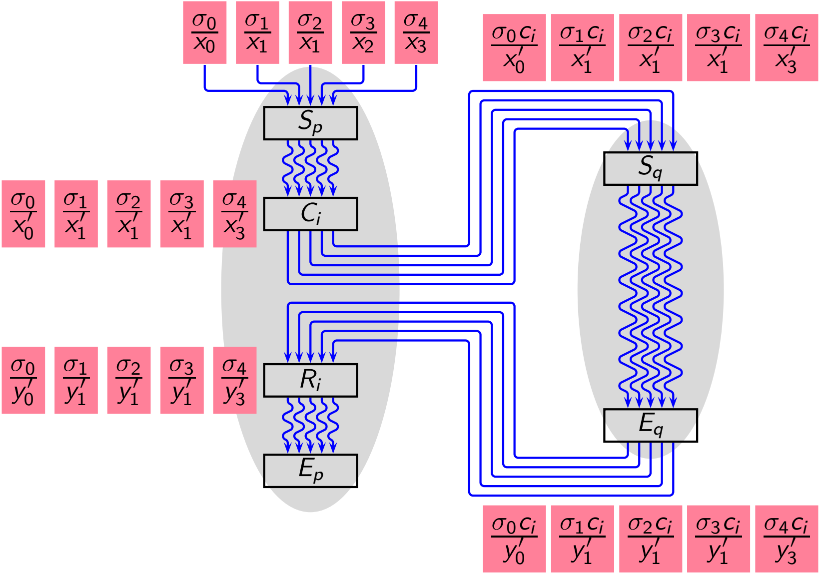

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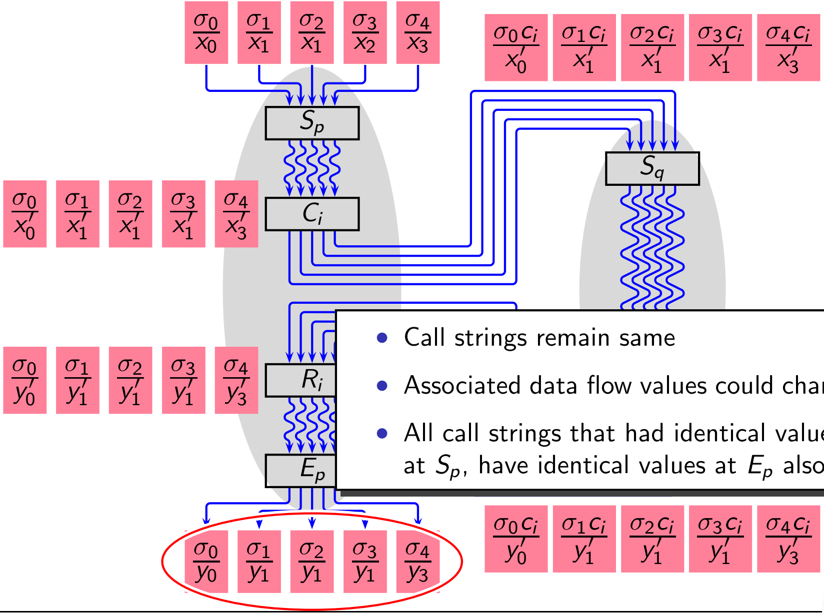
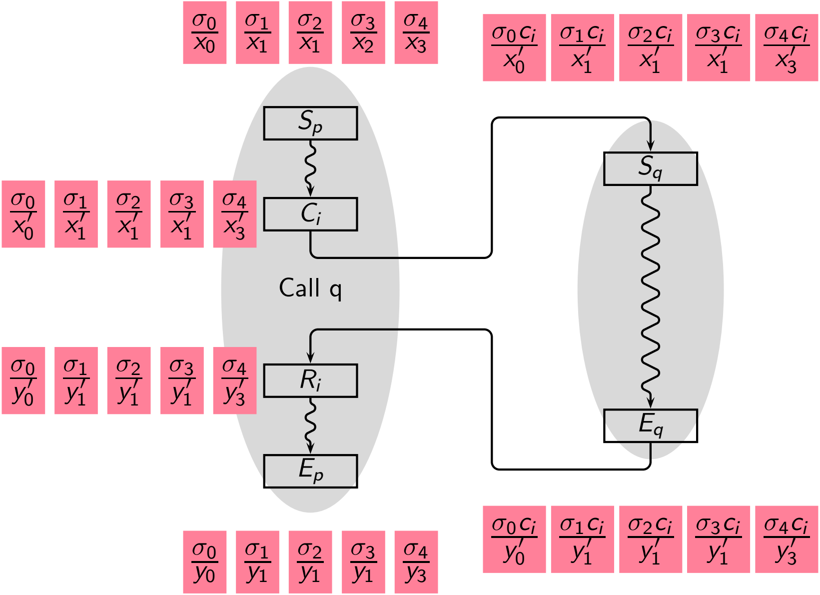


- Call strings remain same
- Associated data flow values could change
- All call strings that had identical values at S_p, have identical values at E_p also

VBTCC: A Motivating Example



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  - the values associated with them will also be transformed in the same manner and will continue to remain equal at E_p.

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- We can reduce the amount of effort by
  - Partitioning the call strings at S_p for each procedure p
  - Replacing all call strings in an equivalence class by its id
  - Regenerating call strings at E_p by replacing equivalence class ids by the call strings in them

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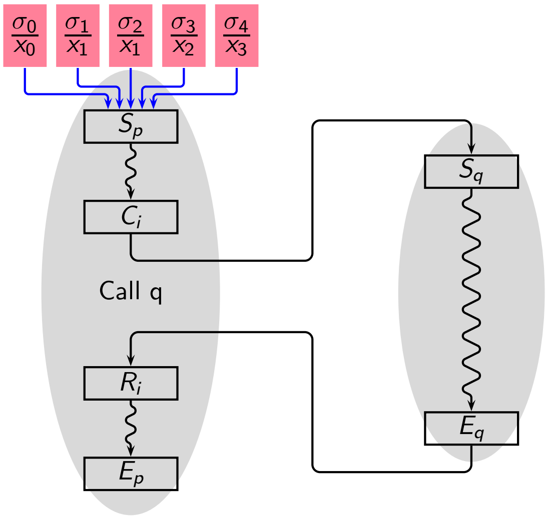
- Can the partitions change?
  - On a subsequent visit to S_p, the partition may be different
  - The data flow values at E_p would also change in a similar manner
  - The data flow value invariant still holds

Understanding VBTCC: Motivating Example Revisited



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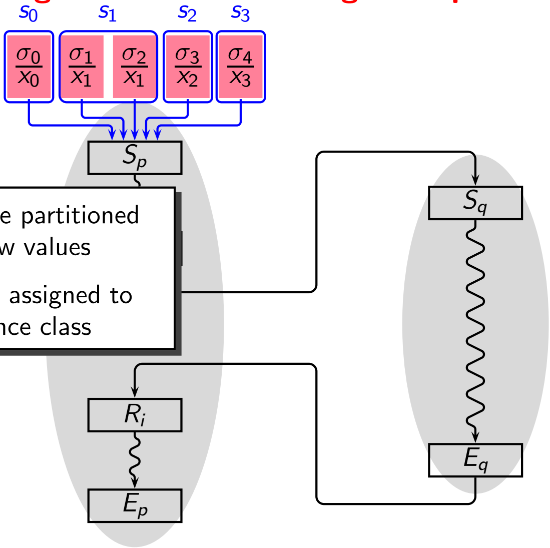
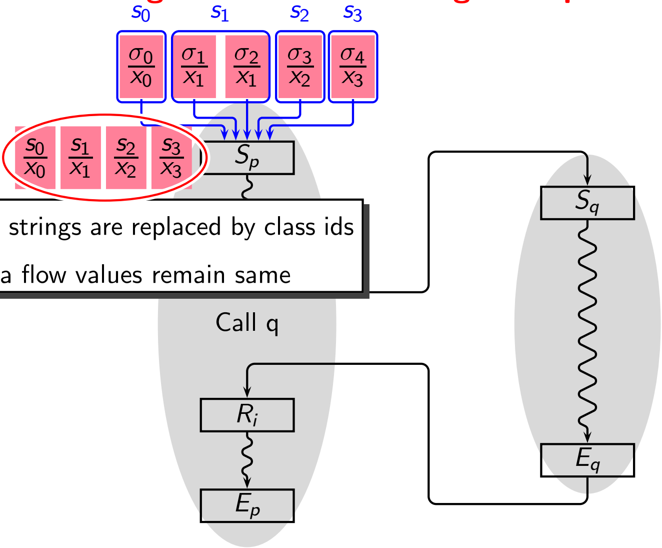


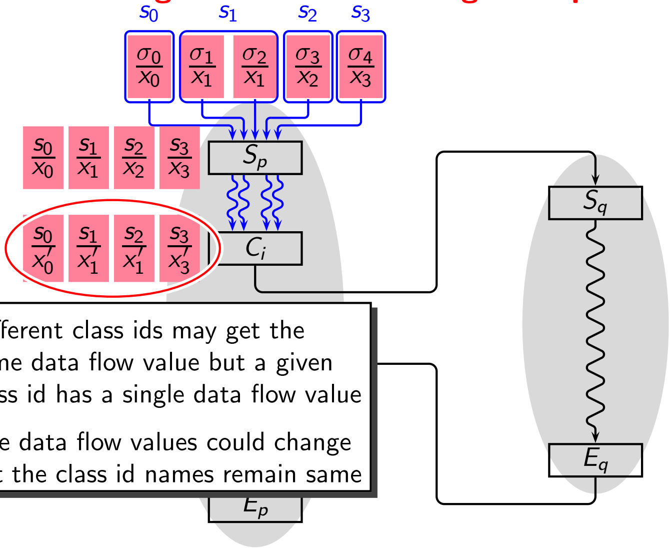
- Call strings are partitioned using data flow values
- A unique id is assigned to each equivalence class

Understanding VBTCC: Motivating Example Revisited



- Call strings are replaced by class ids
- Data flow values remain same

Understanding VBTCC: Motivating Example Revisited



- Different class ids may get the same data flow value but a given class id has a single data flow value
- The data flow values could change but the class id names remain same

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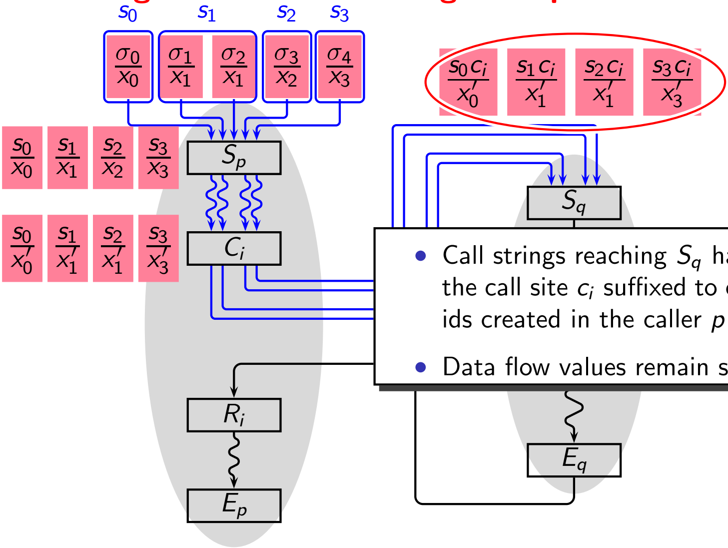

- Call strings reaching S_q have the call site c_i suffixed to class ids created in the caller p
- Data flow values remain same

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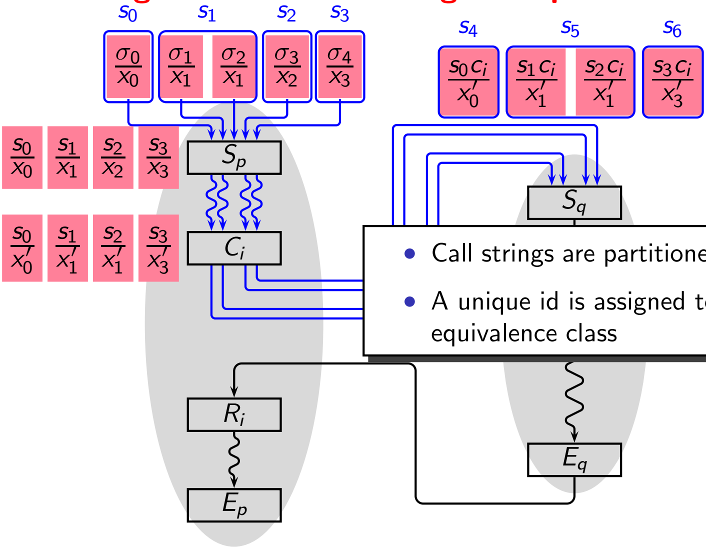

- Call strings are partitioned at S_q
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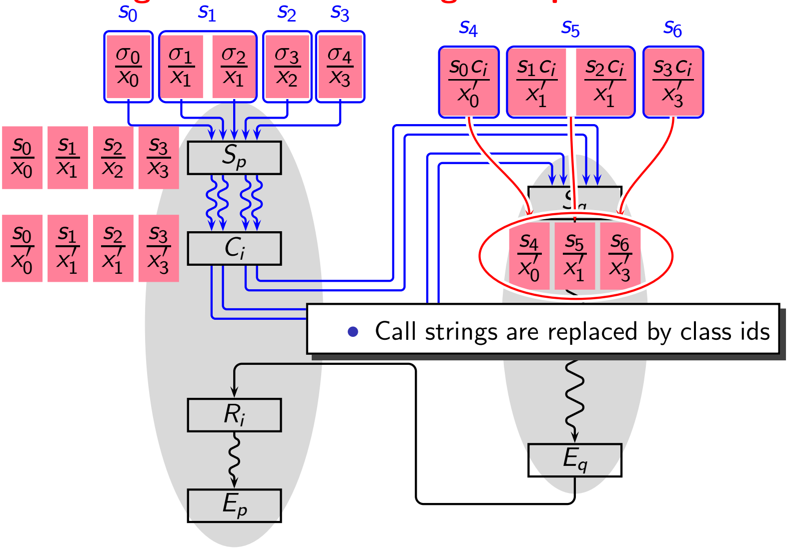

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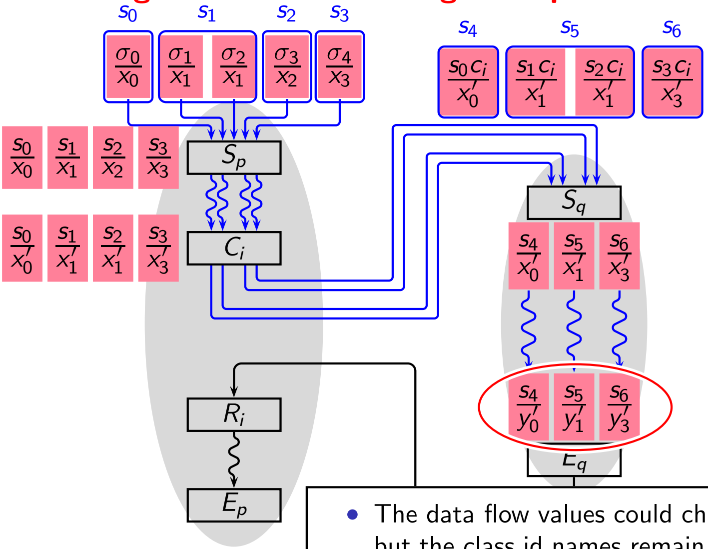

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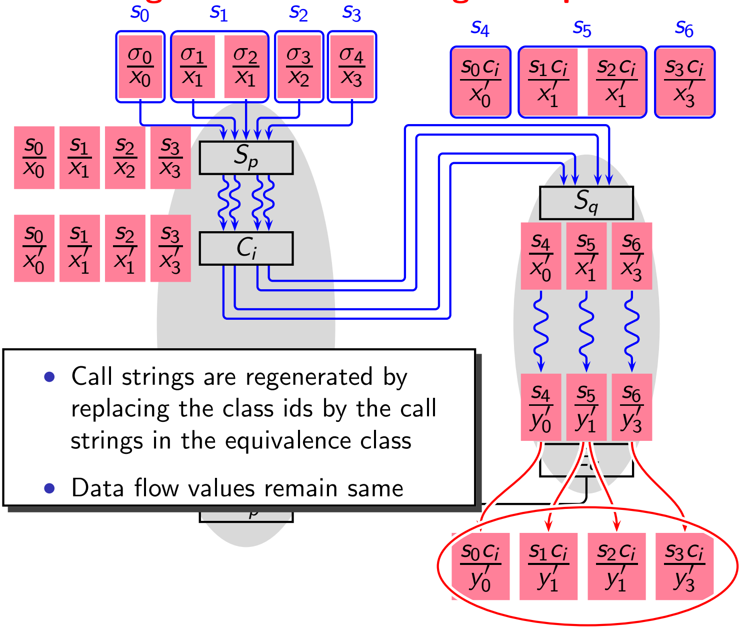

- Call strings are regenerated by replacing the class ids by the call strings in the equivalence class
- Data flow values remain same

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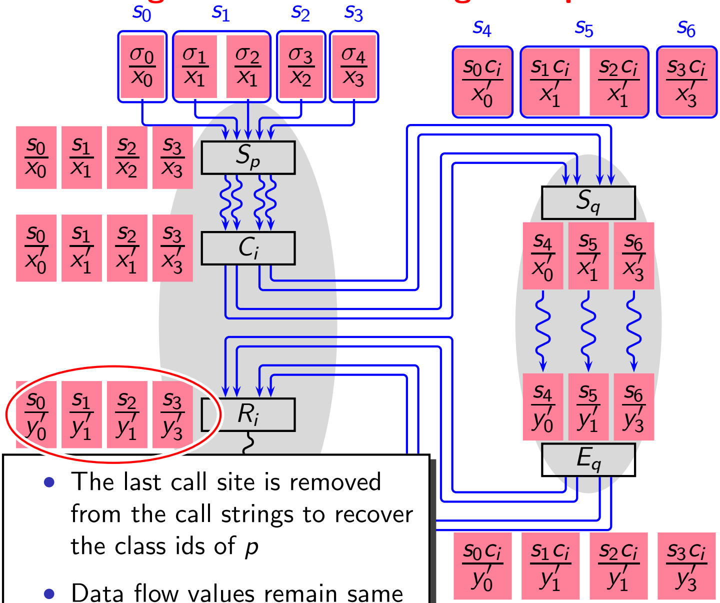

- The last call site is removed from the call strings to recover the class ids of p
- Data flow values remain same

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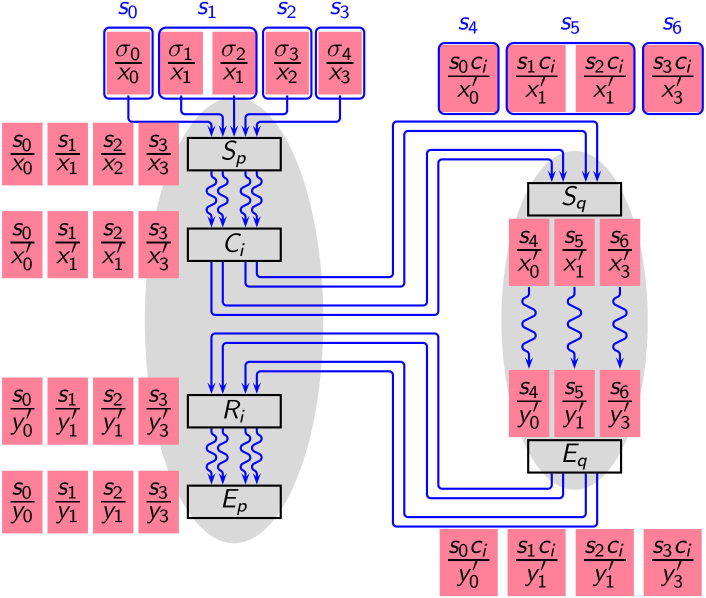

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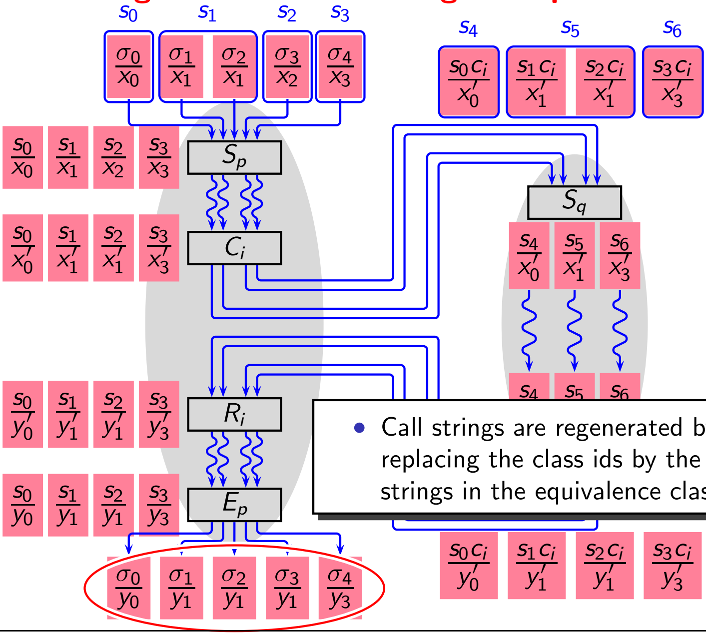

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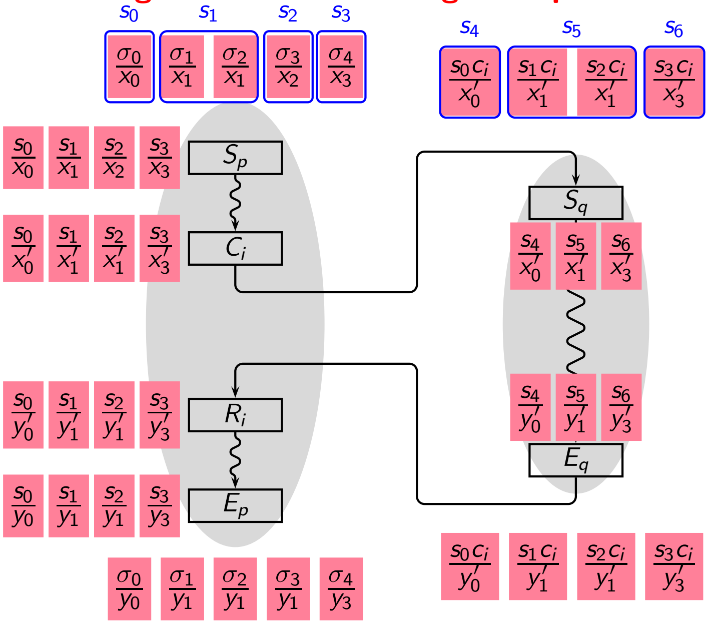

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  - If a new call string is discovered, it will be included in an equivalence class based on its data flow value
- It is possible to adjust equivalence classes at internal nodes too

  However, doing it at each statement may be inefficient

Additional Requirements for VBTCC

- Work list management as described later
- Correctness requirement:
  - Whenever representation is performed at S_p, E_p must be added to the work list
  - In case E_p is to be processed but its predecessors have not been put on the work list (E_p may have been added due to representation), discard E_p from the work list (has the effect of generating $\top$ value).
- Efficiency consideration: Process "inner calls" first

Work List Organization for Forward Analyses

- Maintain a stack of work lists for the procedures being analyzed
  (At most one entry per procedure on the stack)
- Order the nodes in each work list in reverse post order
- Remove the head of work list for the procedure on top (say p)
  - If the selected node is S_p
    - Adjust the call string partition based on the data flow values
    - Replace call strings by class ids
    - Insert E_p in the list for p
  - If the selected node is C_i calling procedure q then
    - Bring q on the top of stack
    - Insert S_q as the head of the list of q
  - If the selected node is E_p
    - Pop p from the stack and add its successor return nodes to appropriate work lists
    - Regenerate the call strings by replacing class ids by the call strings in the class

Work List Organization for Backward Analyses

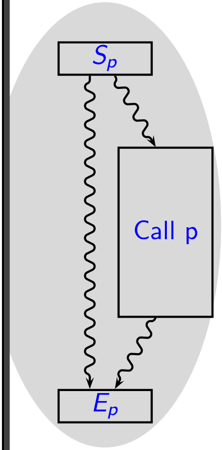
- Swap the roles of S_p and E_p
- Swap the roles of C_i and R_i
- Replace reverse post order by post order

VBTCC for Recursion

- We first make important observations about the role of the length of a call string in recursive contexts in the classical call strings method
- Then we intuitively see how VBTCC serves the same role without actually constructing redundant call strings
- Finally we formally argue that the two methods are equivalent

The Role of Call Strings Length in Recursion (1)

- We consider self recursion for simplicity; the principles are general and are also applicable to indirect recursion



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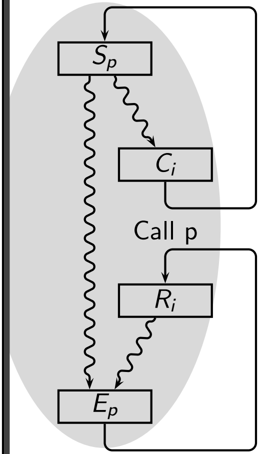

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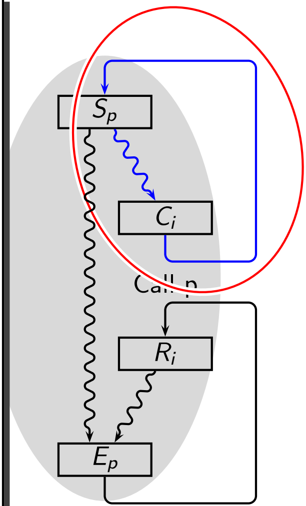

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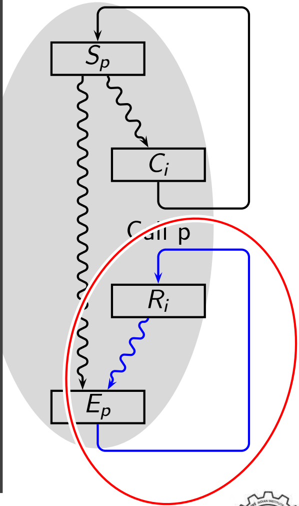

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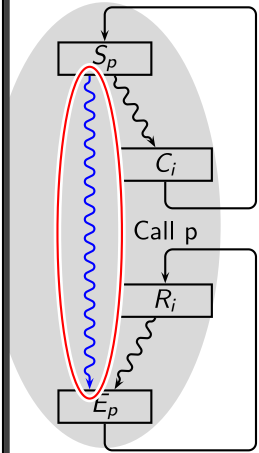

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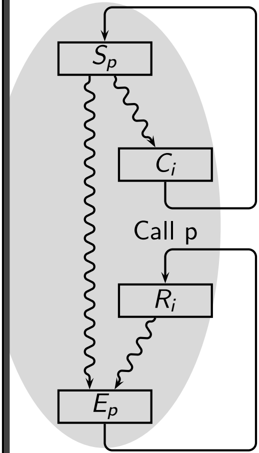
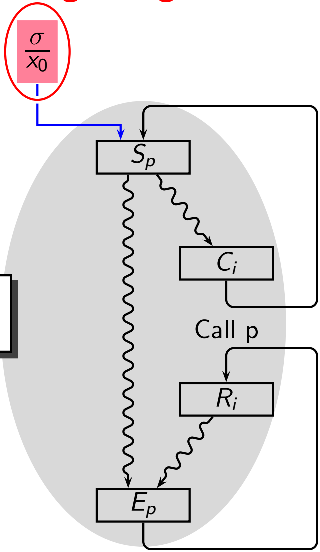
  - The *recursion terminating path* (RTP) refers to the subpath from RCS to RRS
    (We assume that a static analysis can assign arbitrary data flow values to the unreachable parts of the program in the absence of an RTP)

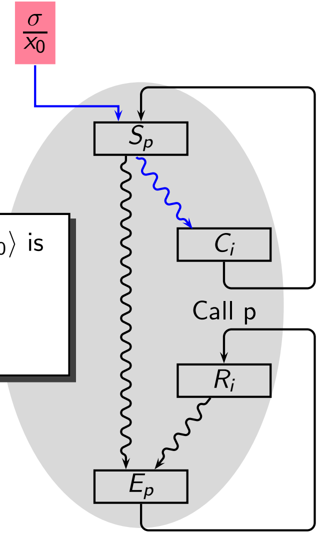


The Role of Call Strings Length in Recursion (2)



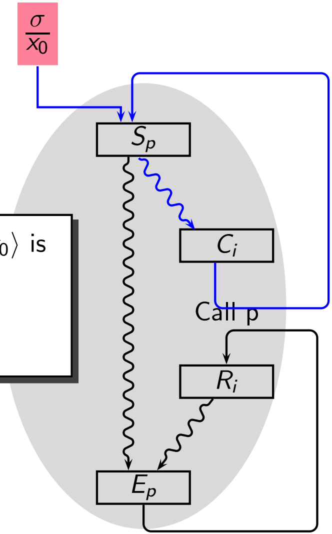
- Data flow value reaching from outside

The Role of Call Strings Length in Recursion (2)



- The data flow value $\langle \sigma, x_0 \rangle$ is propagated over RCS

The Role of Call Strings Length in Recursion (2)



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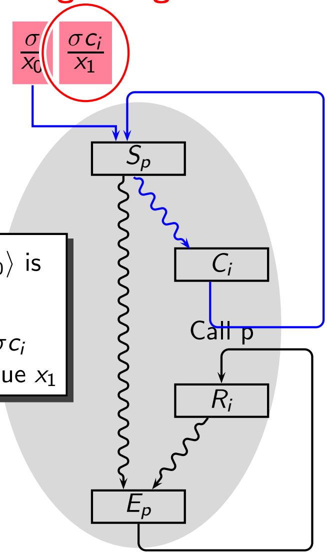
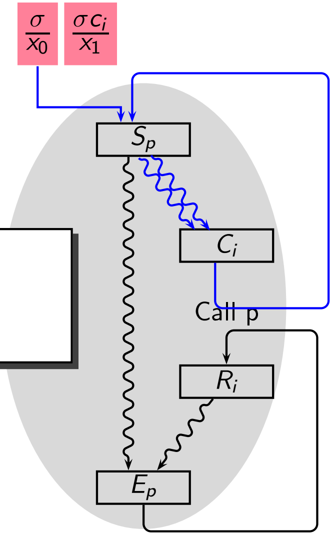


- The data flow value $\langle \sigma, x_0 \rangle$ is propagated over RCS
- We get the new context σc_i and the new data flow value x_1

The Role of Call Strings Length in Recursion (2)

- The new pair $\langle \sigma c_i, x_1 \rangle$ is propagated over RCS



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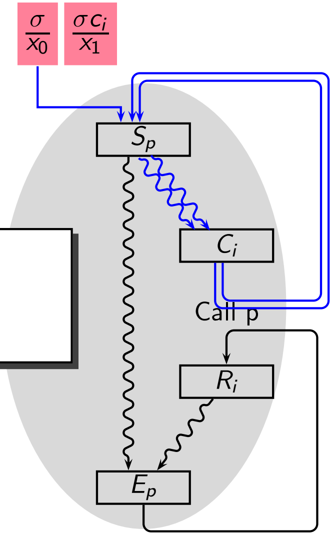

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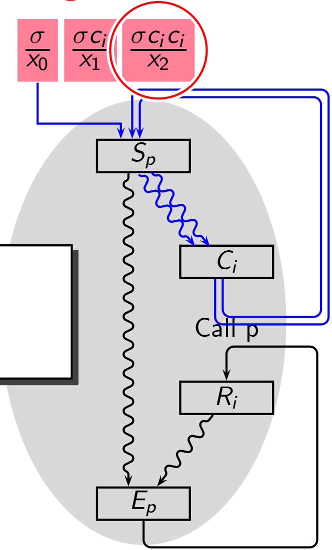
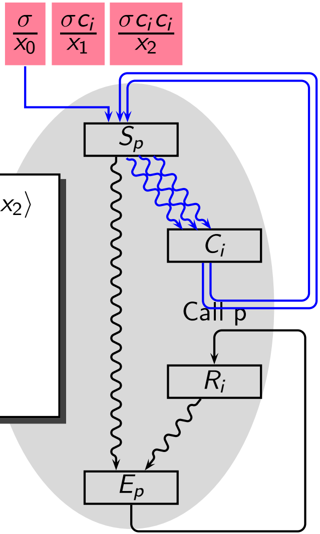


- The new pair $\langle \sigma c_i, x_1 \rangle$ is propagated over RCS
- We get $\langle \sigma c_i c_i, x_2 \rangle$ at S_p

The Role of Call Strings Length in Recursion (2)

- Now the third pair $\langle \sigma c_i c_i, x_2 \rangle$ is propagated over RCS



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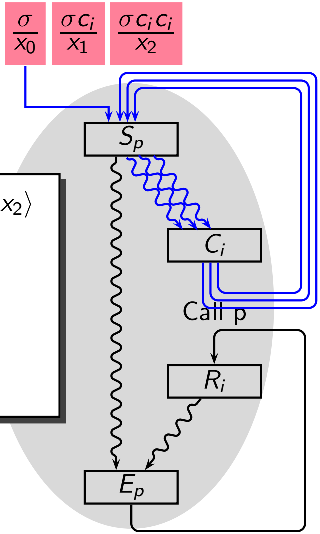

- Now the third pair $\langle \sigma c_i c_i, x_2 \rangle$ is propagated over RCS

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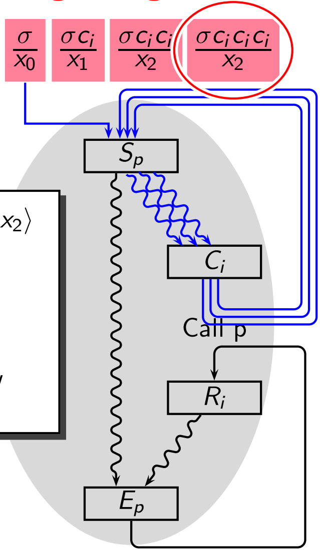
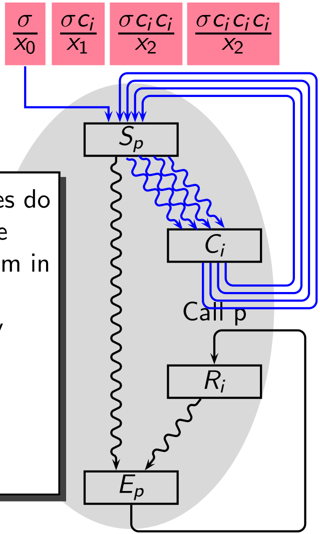


- Now the third pair $\langle \sigma c_i c_i, x_2 \rangle$ is propagated over RCS
- We get the new context $\sigma c_i c_i c_i$ at S_p
- Assume that the data flow ceases to change

The Role of Call Strings Length in Recursion (2)



- Even if the data flow values do not change any further, we still need to propagate them in RCS in order to build call strings that are sufficiently large

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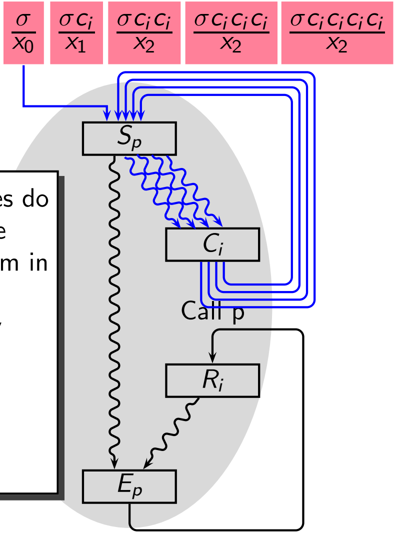

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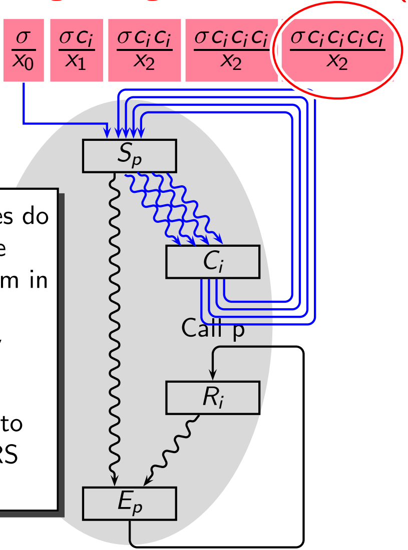

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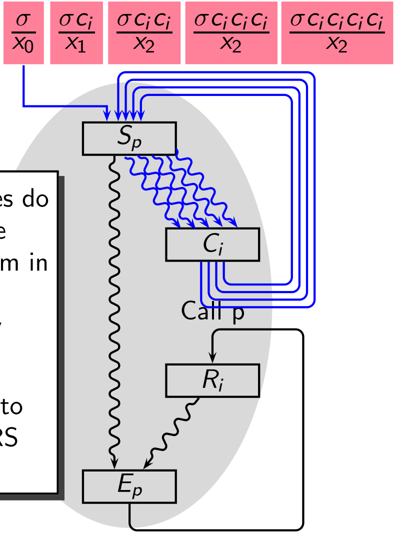

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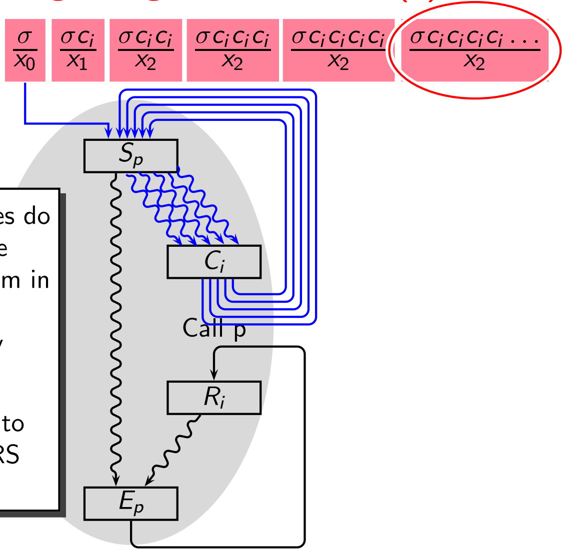

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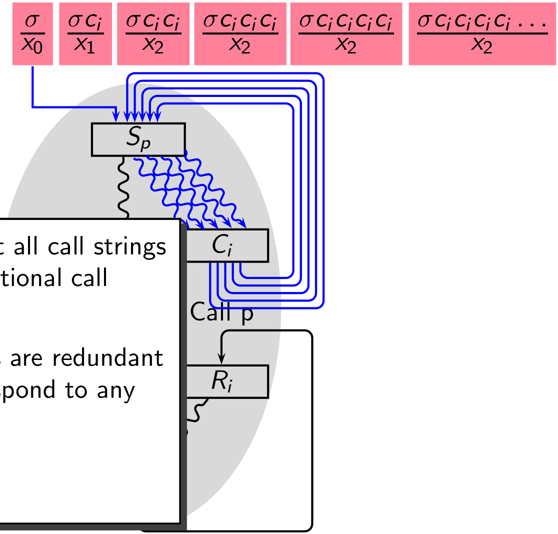

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- Many of these call strings are redundant in that they do not correspond to any new data flow value

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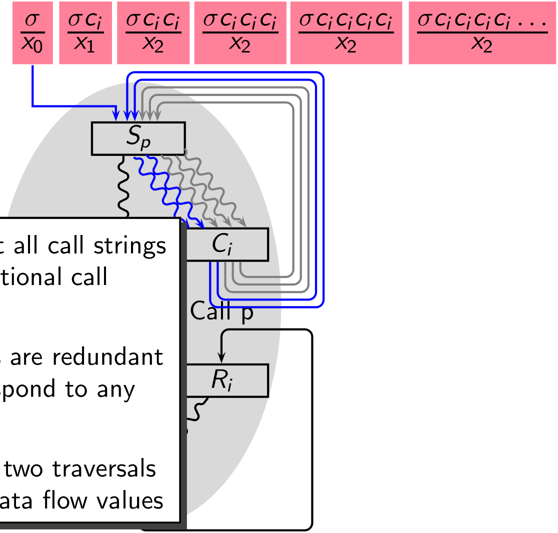
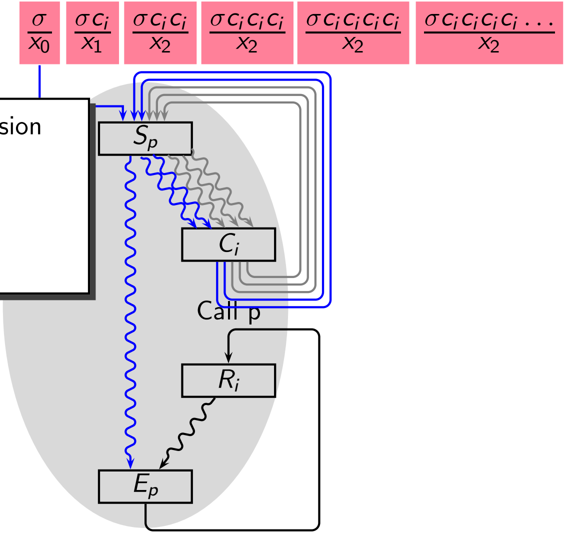


- Assume that we construct all call strings as required by the conventional call strings method
- Many of these call strings are redundant in that they do not correspond to any new data flow value
- In our case, only the first two traversals over RCS compute new data flow values

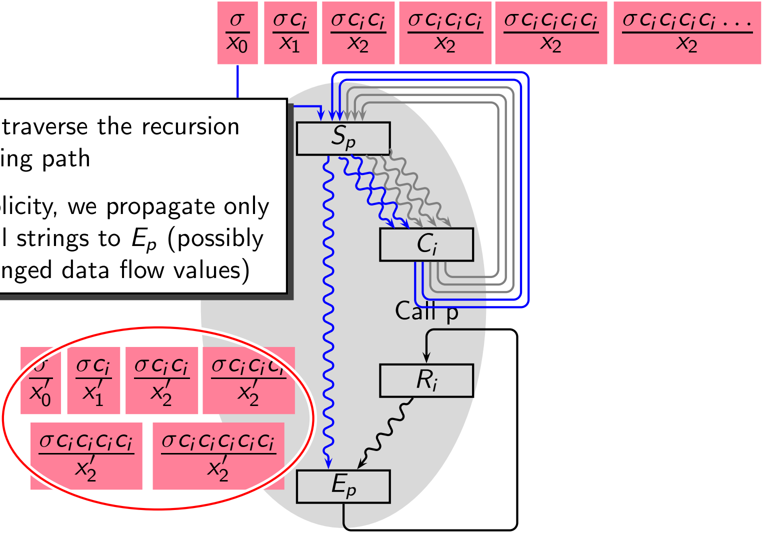
The Role of Call Strings Length in Recursion (2)



- Now we traverse the recursion terminating path

The Role of Call Strings Length in Recursion (2)

- Now we traverse the recursion terminating path
- For simplicity, we propagate only some call strings to E_p (possibly with changed data flow values)



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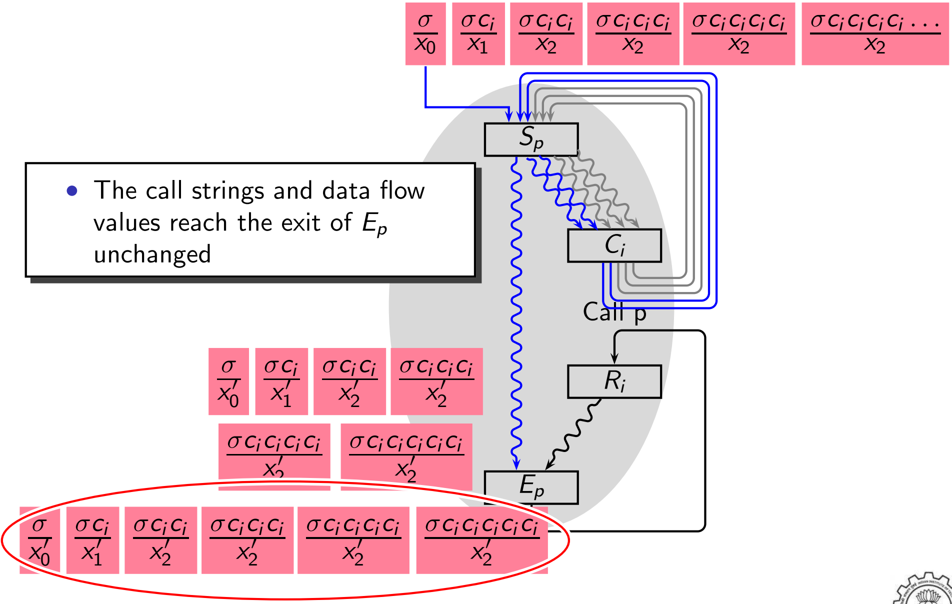
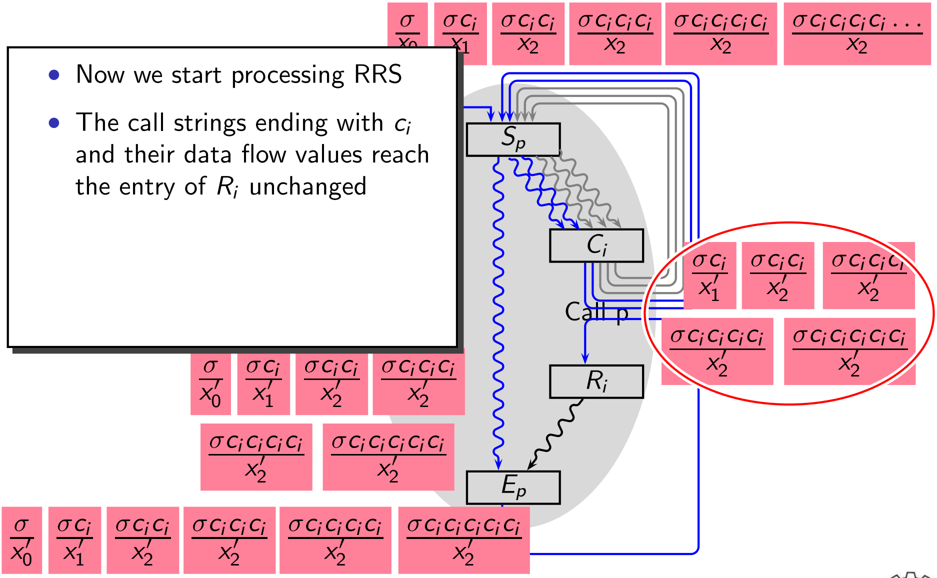


- The call strings and data flow values reach the exit of E_p unchanged

The Role of Call Strings Length in Recursion (2)

- Now we start processing RRS
- The call strings ending with c_i and their data flow values reach the entry of R_i unchanged



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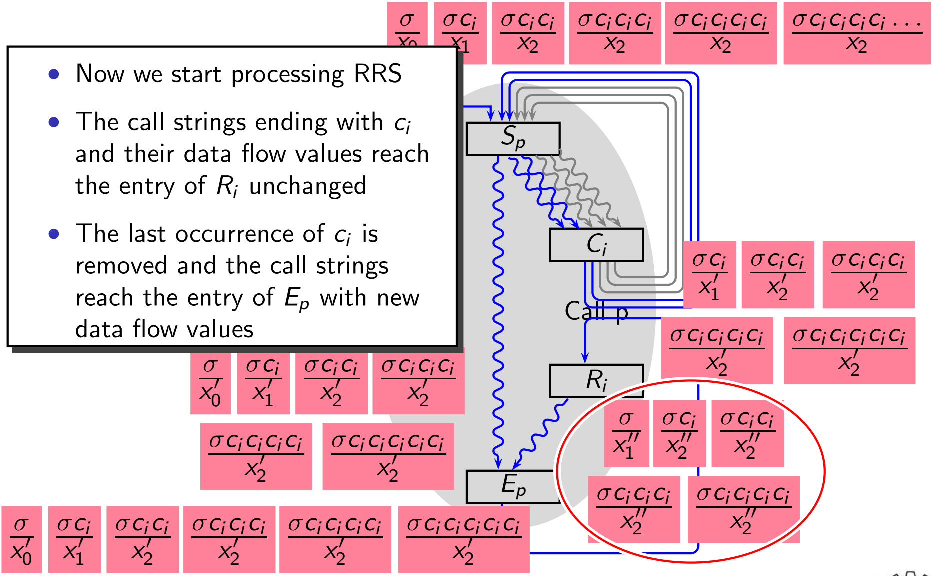
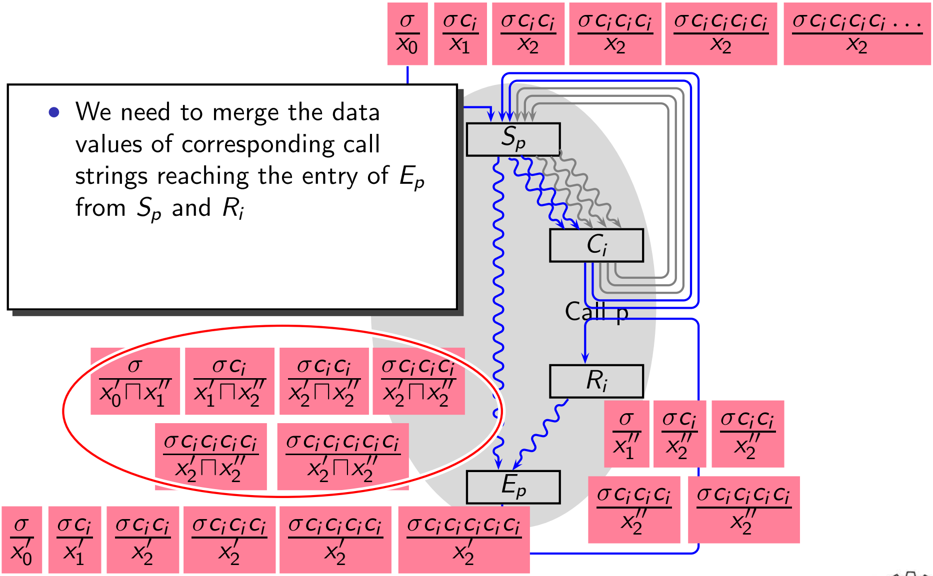


The Role of Call Strings Length in Recursion (2)

- We need to merge the data values of corresponding call strings reaching the entry of E_p from S_p and R_i



The Role of Call Strings Length in Recursion (2)

- We need to merge the data values of corresponding call strings reaching the entry of E_p from S_p and R_i
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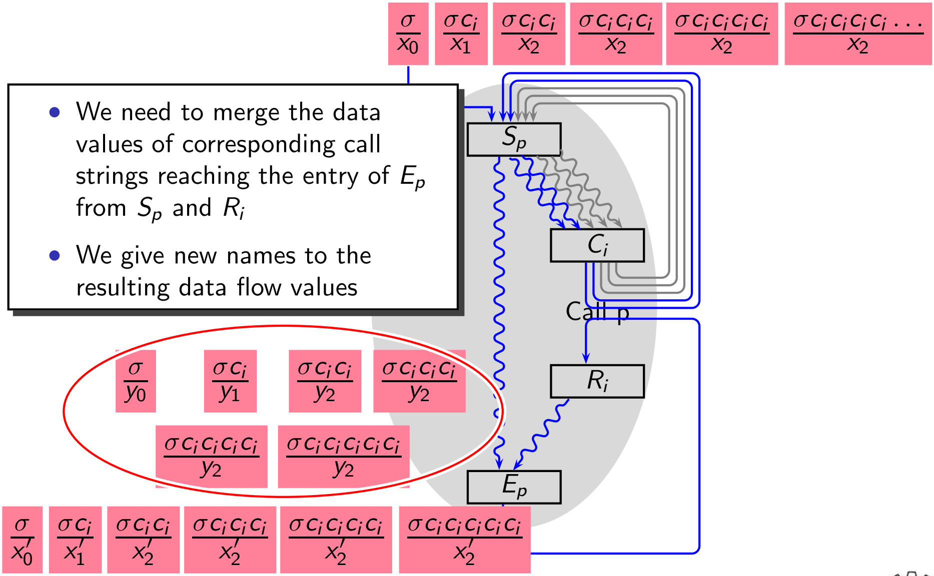
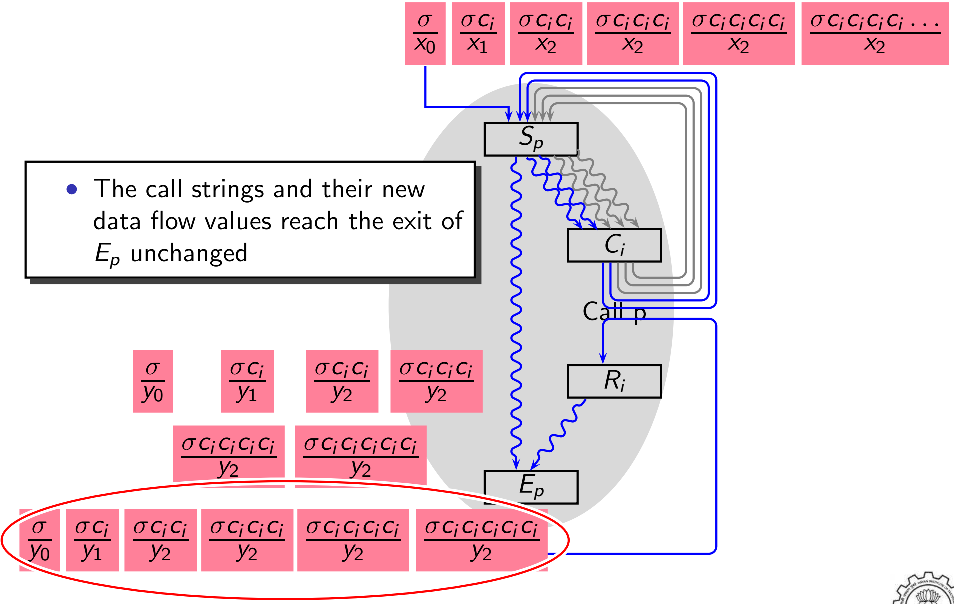


The Role of Call Strings Length in Recursion (2)

- The call strings and their new data flow values reach the exit of E_p unchanged



The Role of Call Strings Length in Recursion (2)

- We process RRS once again
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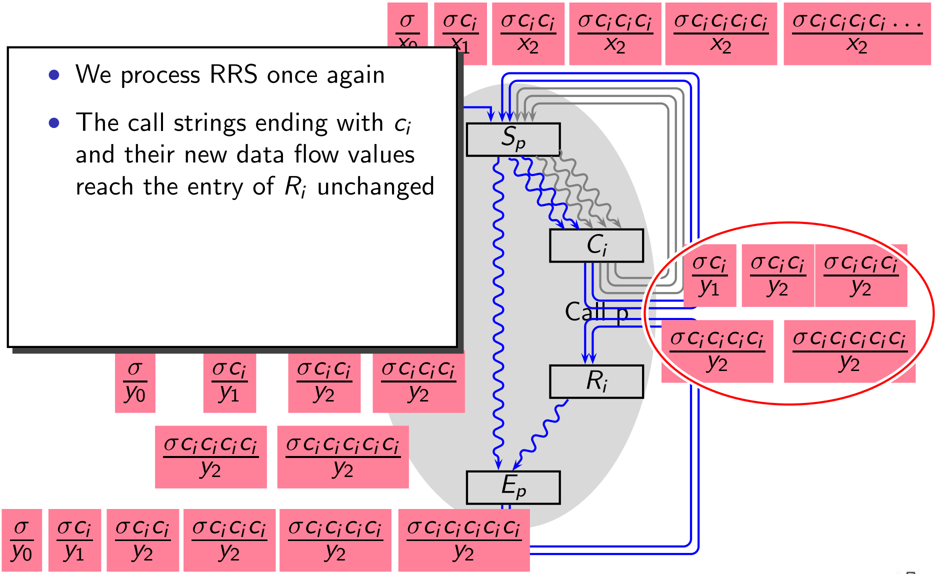

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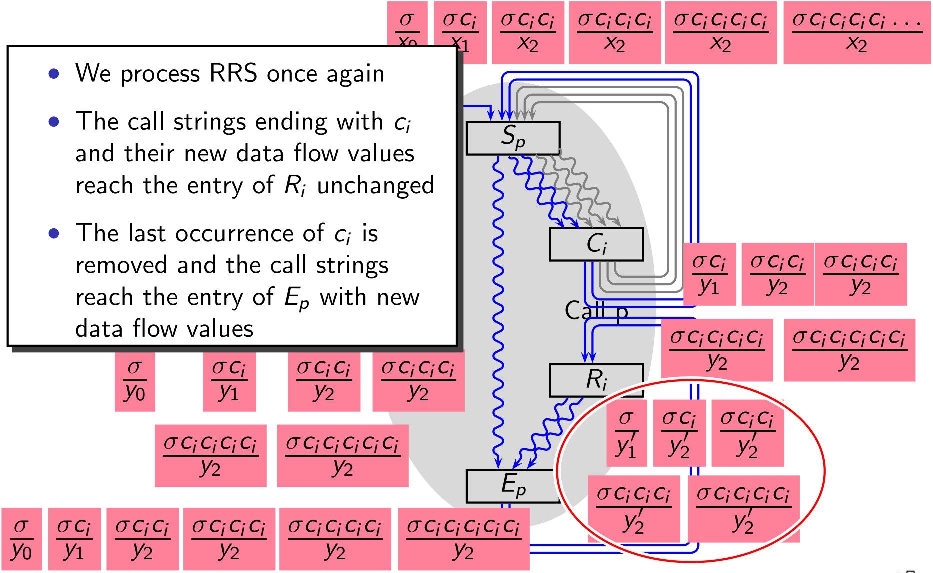
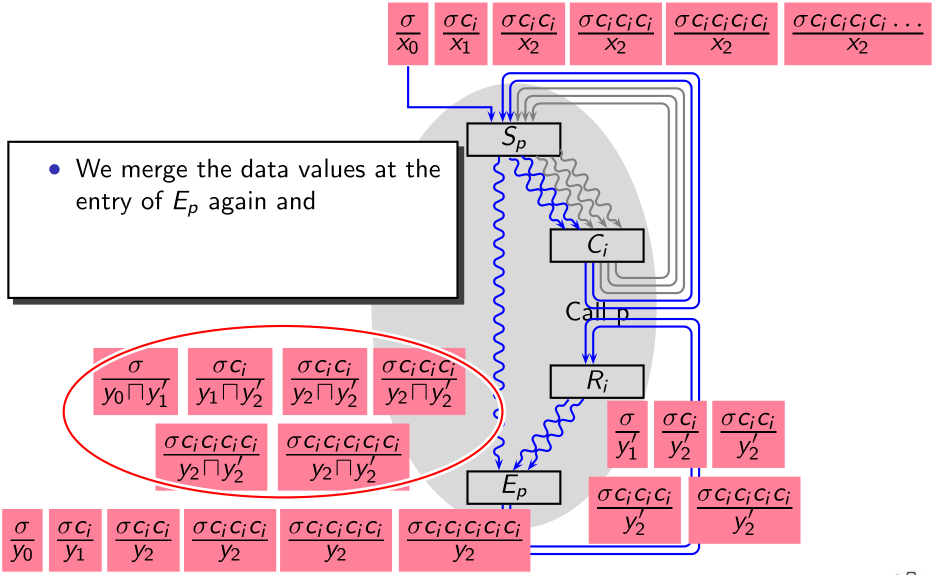


The Role of Call Strings Length in Recursion (2)

- We merge the data values at the entry of E_p again and



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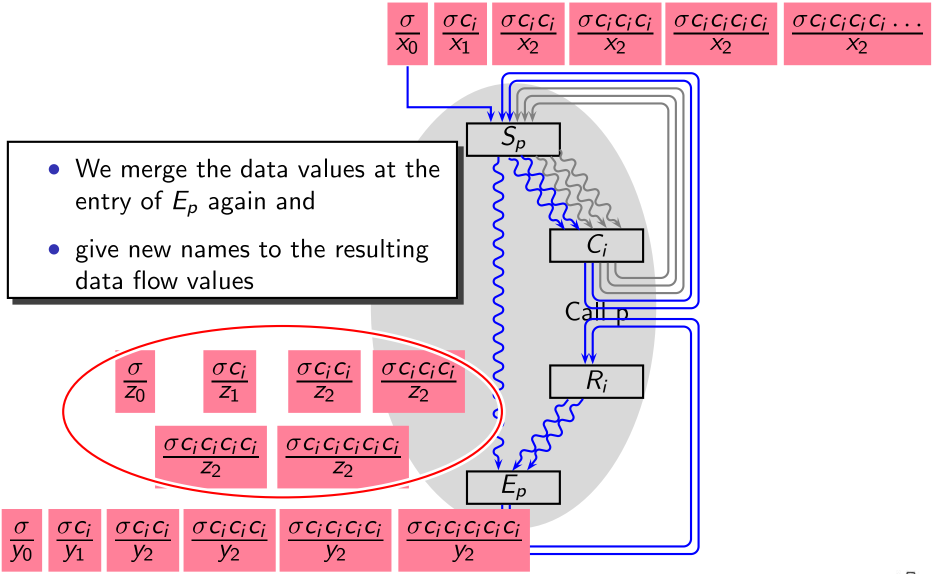

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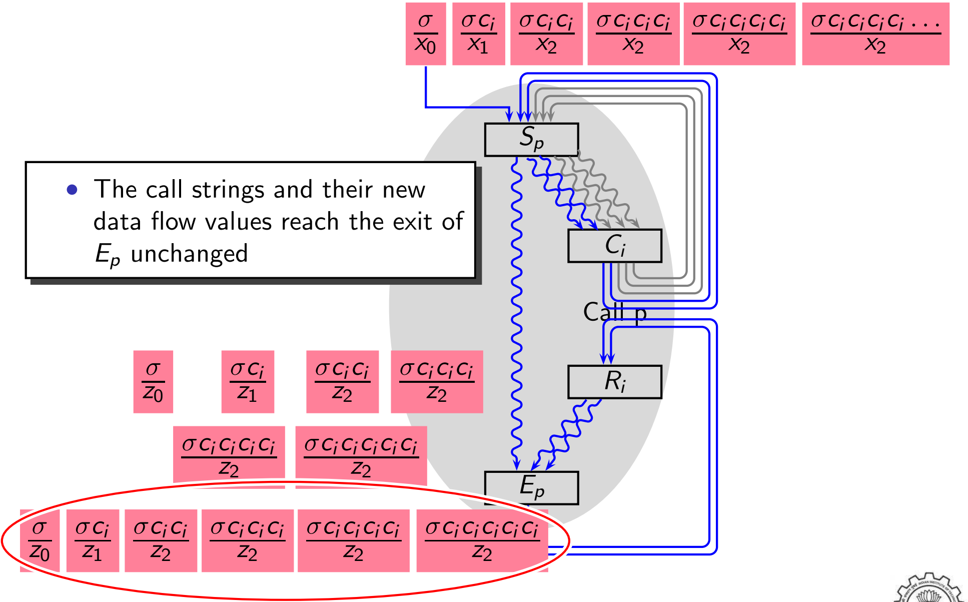

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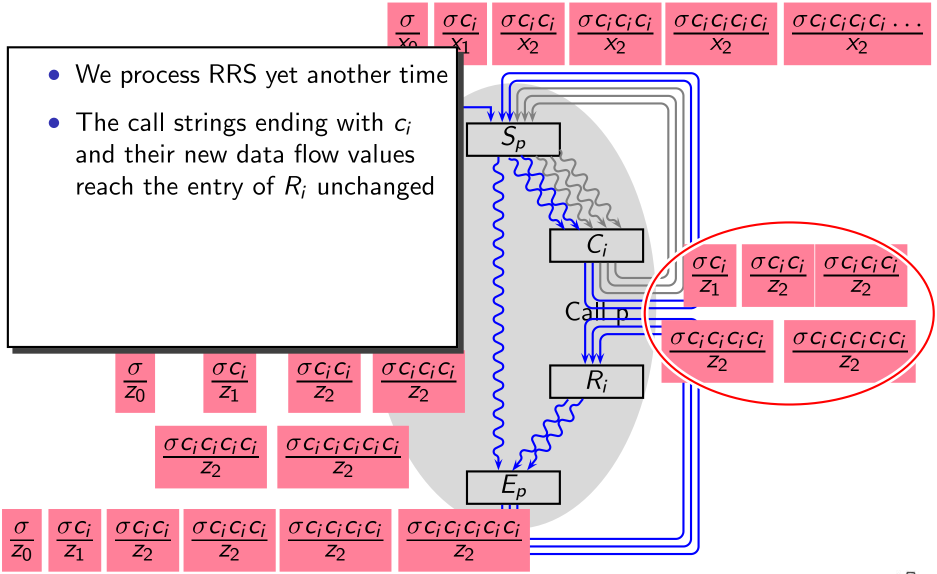

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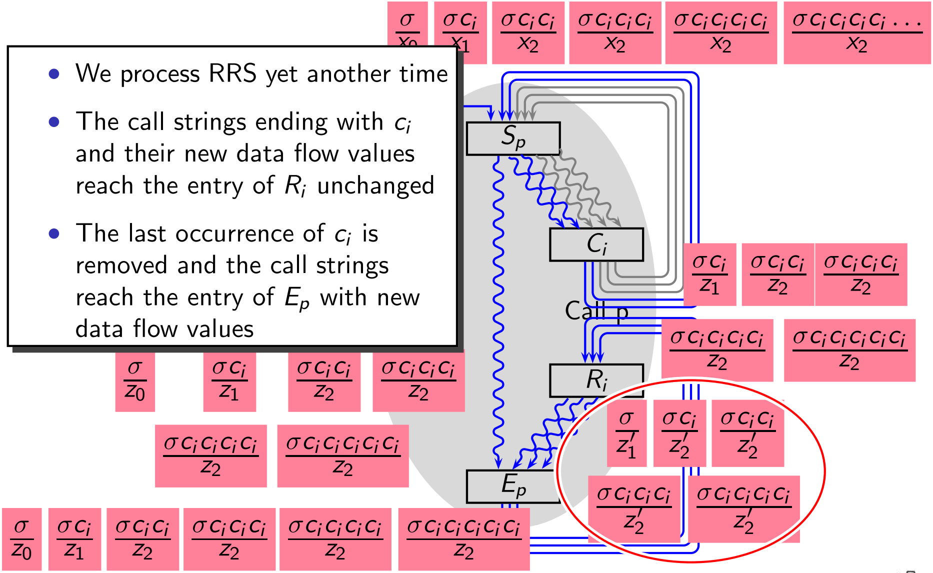
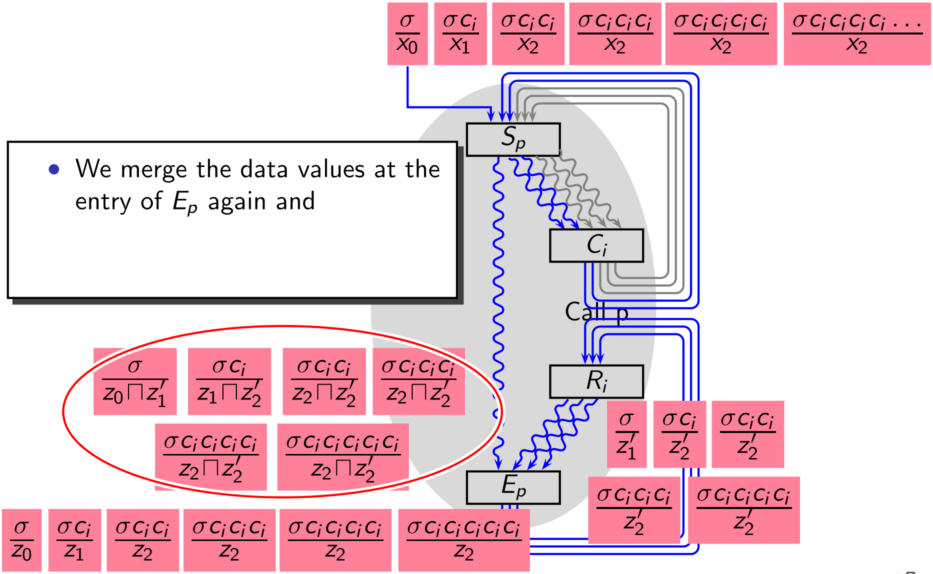


The Role of Call Strings Length in Recursion (2)

- We merge the data values at the entry of E_p again and



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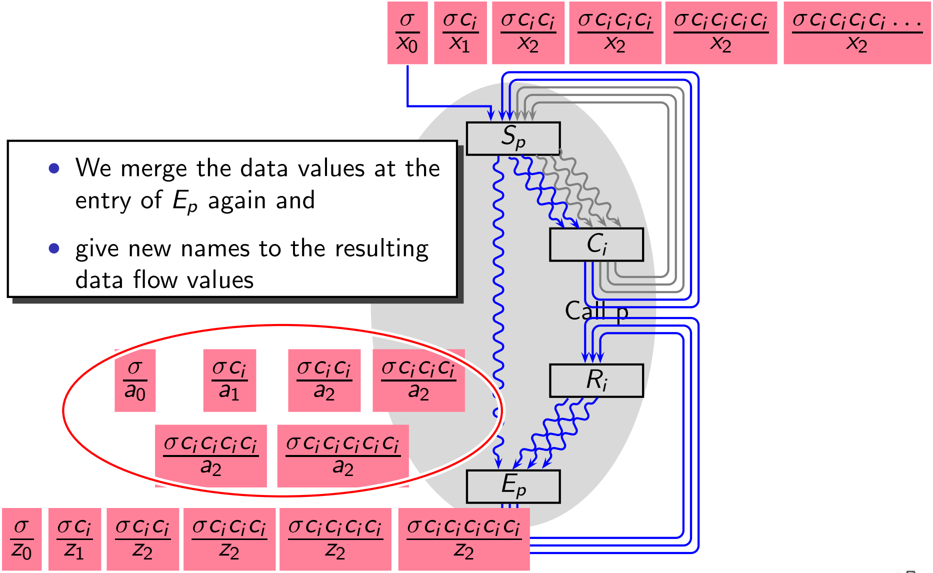
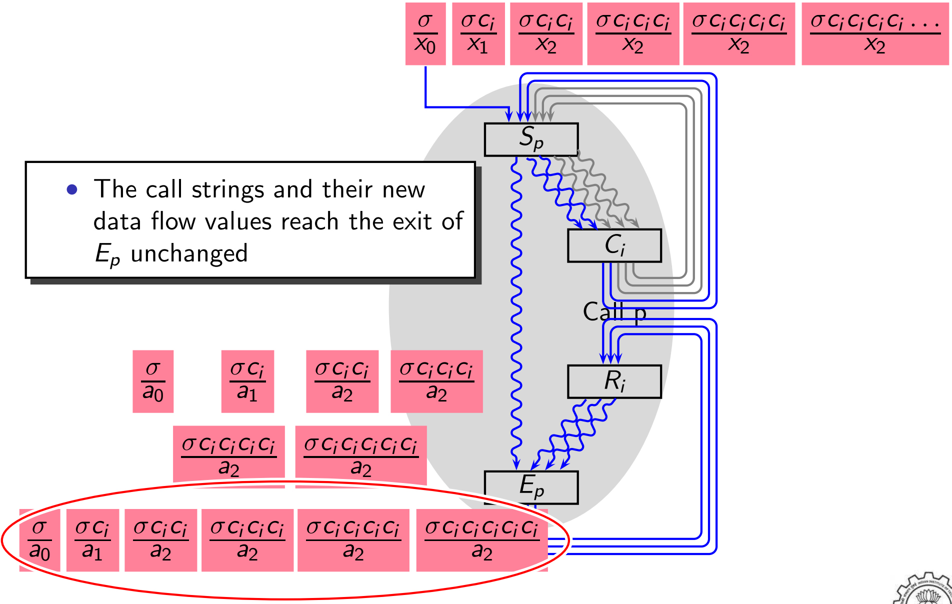


The Role of Call Strings Length in Recursion (2)

- The call strings and their new data flow values reach the exit of E_p unchanged



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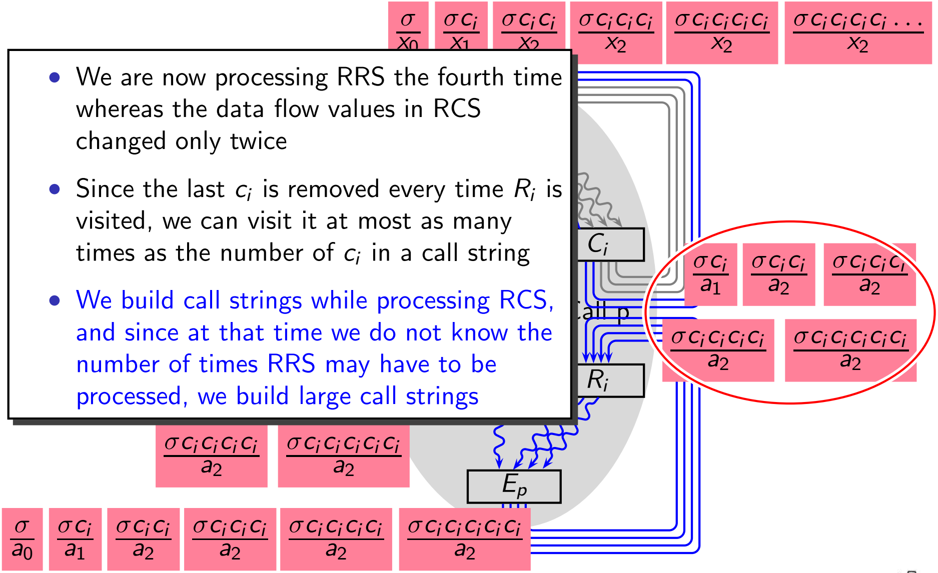

- We are now processing RRS the fourth time whereas the data flow values in RCS changed only twice
- Since the last c_i is removed every time R_i is visited, we can visit it at most as many times as the number of c_i in a call string
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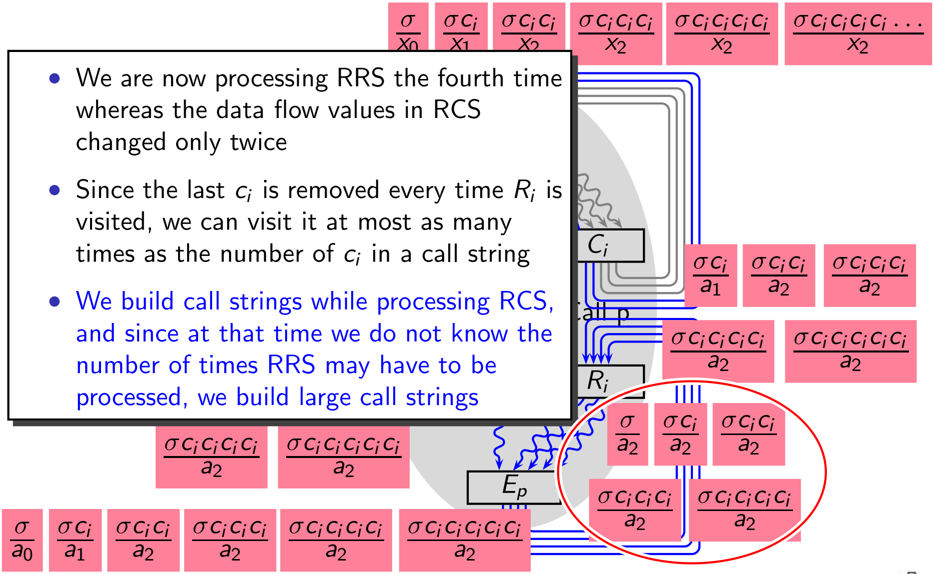

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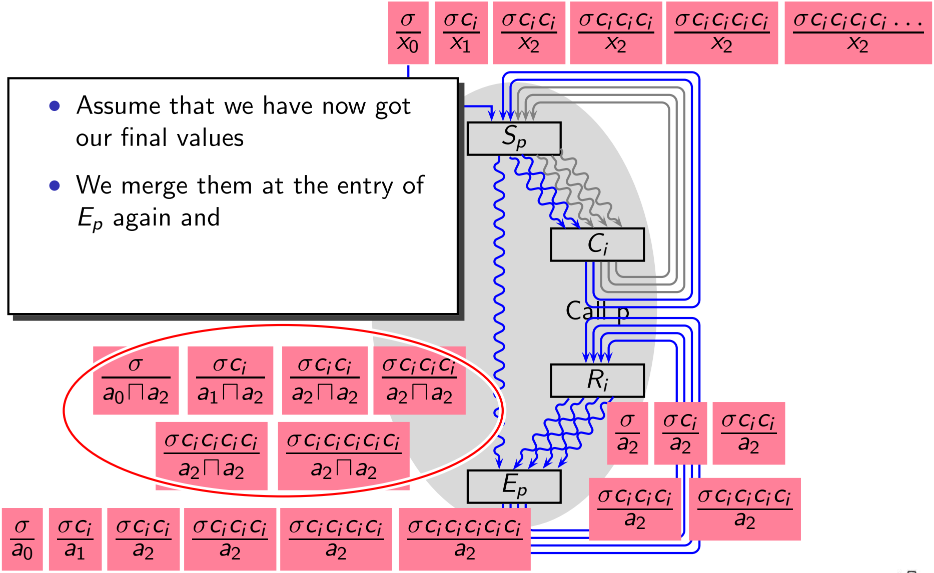

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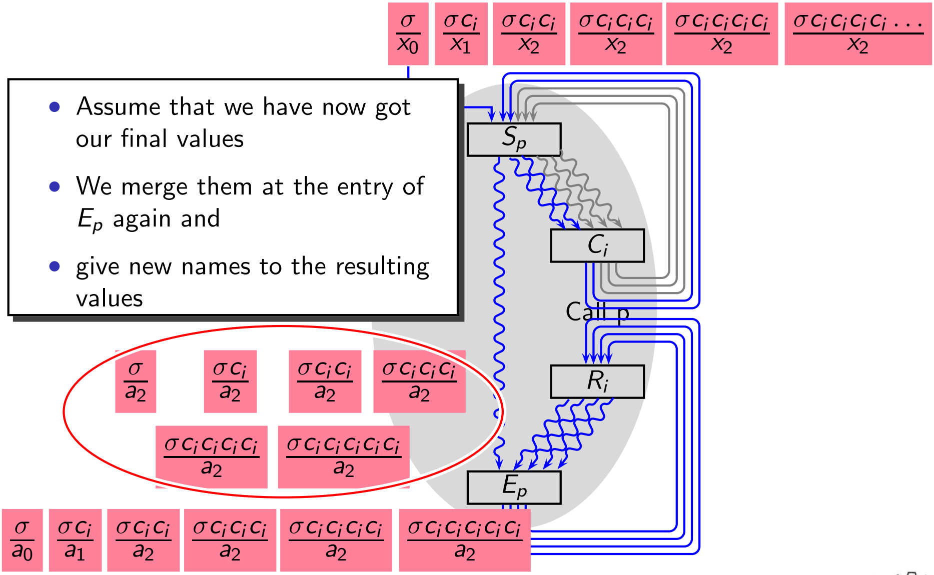

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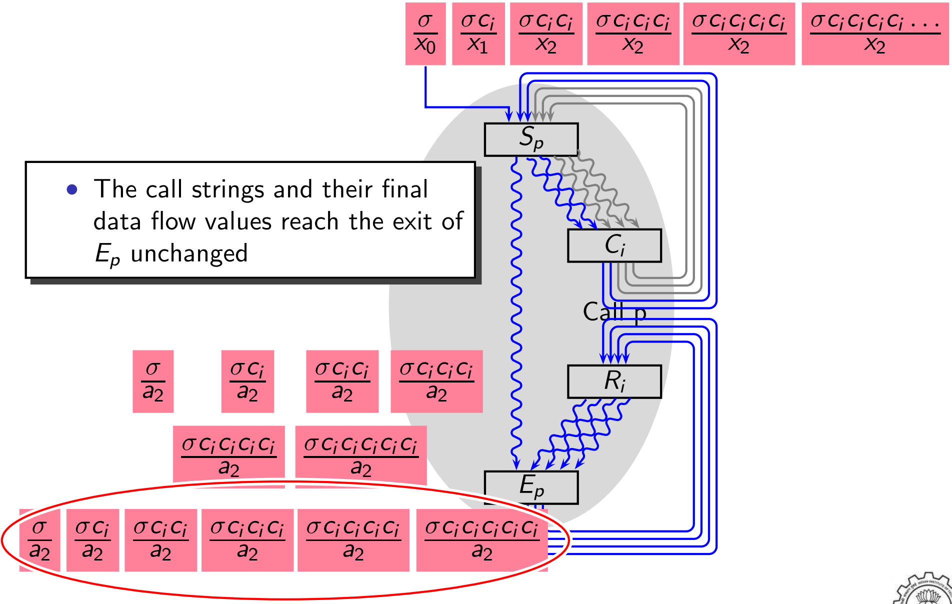
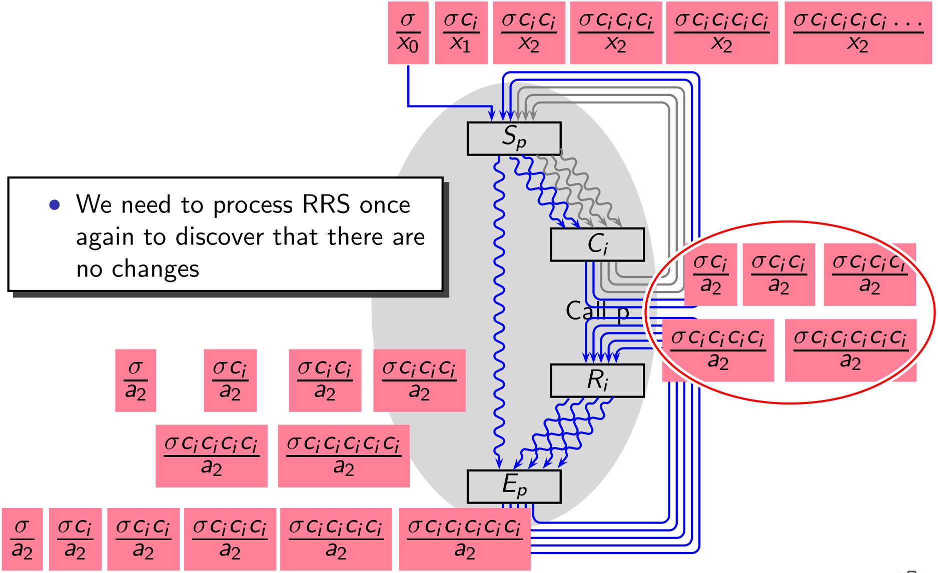
- The call strings and their final data flow values reach the exit of E_p unchanged



The Role of Call Strings Length in Recursion (2)

- We need to process RRS once again to discover that there are no changes



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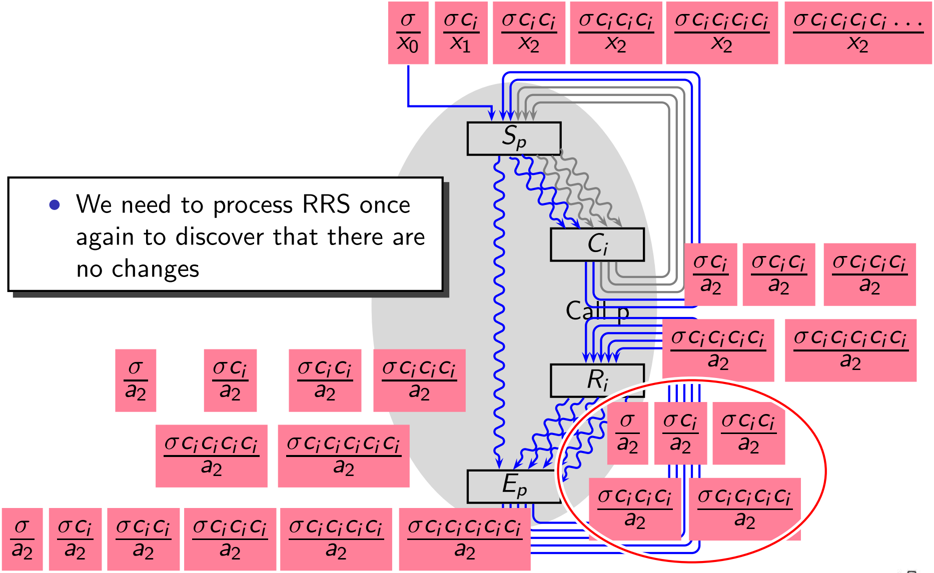

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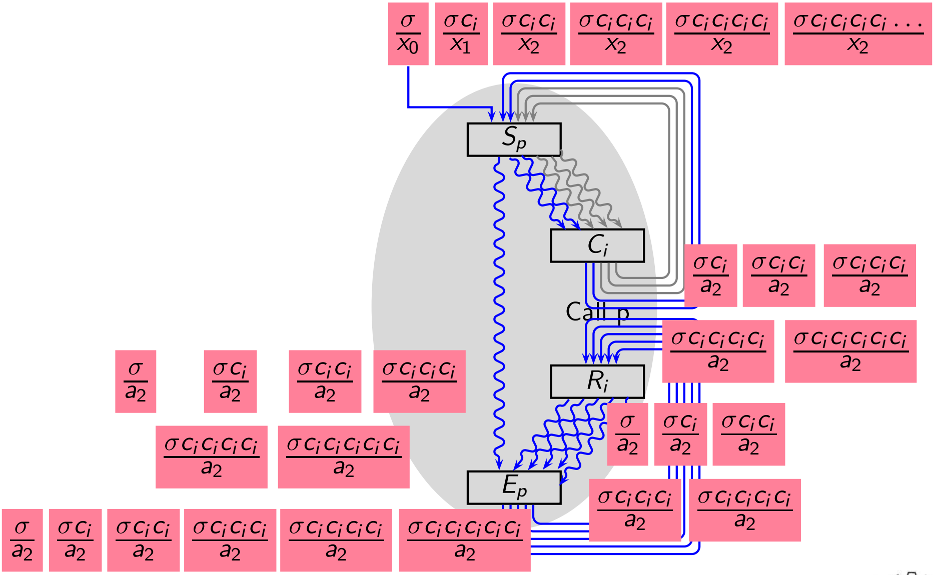

The Role of Call Strings Length in Recursion: Summary

- Context sensitivity in recursion requires matching the number of traversals over RCS and RRS
- For a forward analysis the call strings are constructed while traversing RCS and are consumed while traversing RRS
- At the time of traversing RCS, we do not know how many times do we need to traverse the corresponding RRS
- In order the allow an adequate number of traversals over RRS, we construct large call strings in anticipation while traversing RCS

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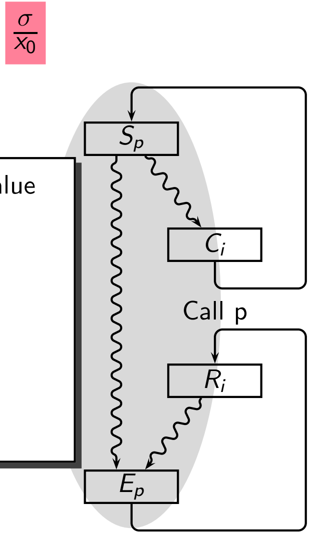
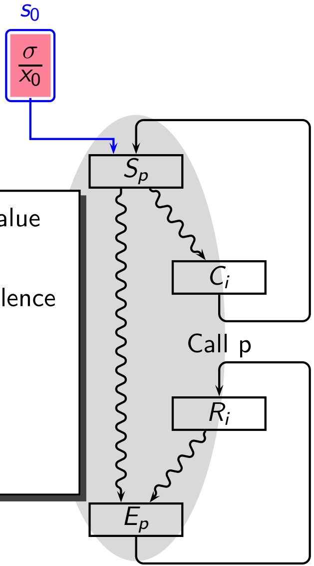
The main reason behind building long call strings is to allow an adequate number of traversals over RCS

VBTCC in Recursion: Motivating Example Revisited



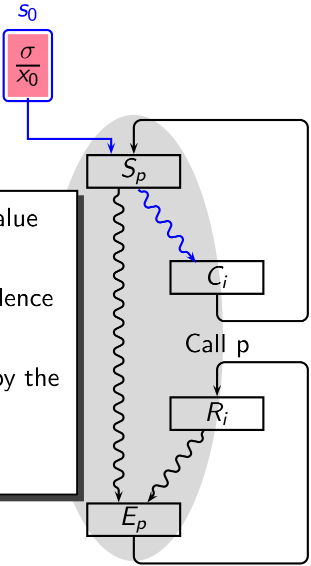
- We have a single data flow value reaching S_p from the outside

VBTCC in Recursion: Motivating Example Revisited



- We have a single data flow value reaching S_p from the outside
- Thus we have a single equivalence class in the partition

VBTCC in Recursion: Motivating Example Revisited



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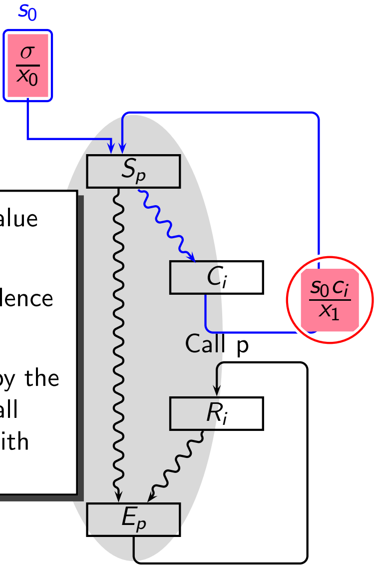
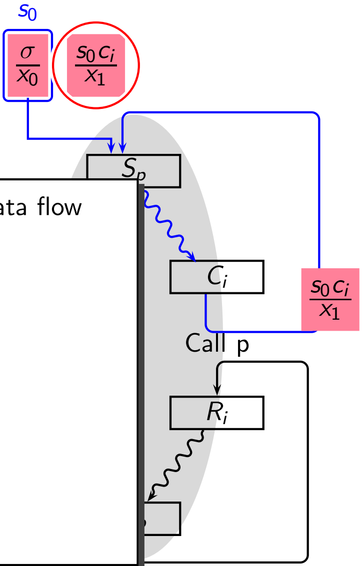
- We replace the call string σ by the class id s_0 and get the new call string s_0c_i at the exit of C_i with new data flow value x_1

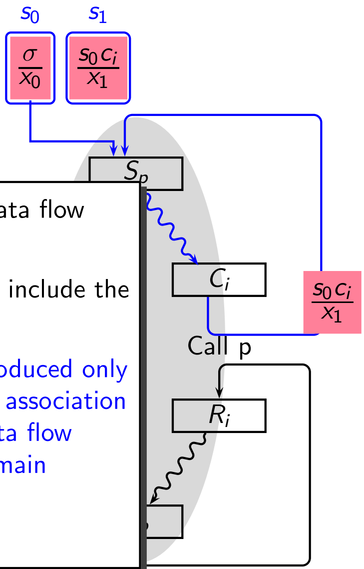


VBTCC in Recursion: Motivating Example Revisited



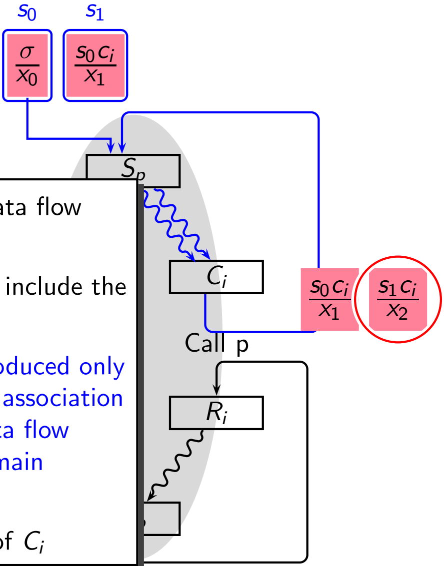
- The new call string and its data flow value reaches S_p

VBTCC in Recursion: Motivating Example Revisited



- The new call string and its data flow value reaches S_p
- We adjust the partitioning to include the new call string
- New partition names are introduced only for new data flow values; the association between a partition and a data flow value in a procedure must remain invariant during analysis

VBTCC in Recursion: Motivating Example Revisited



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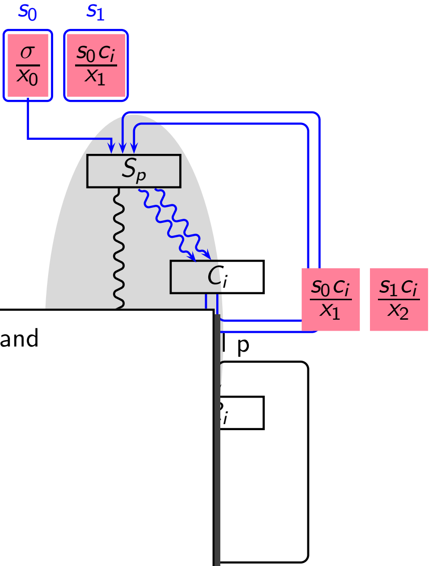
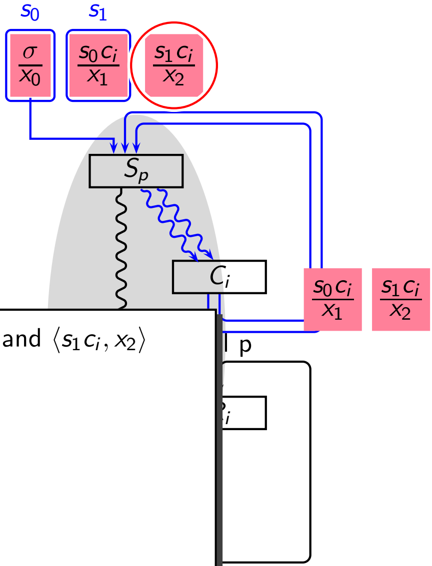
- New partition names are introduced only for new data flow values; the association between a partition and a data flow value in a procedure must remain invariant during analysis
- We get $\langle s_1 c_i, x_2 \rangle$ at the exit of C_i

VBTCC in Recursion: Motivating Example Revisited



- We continue to process RCS and

VBTCC in Recursion: Motivating Example Revisited



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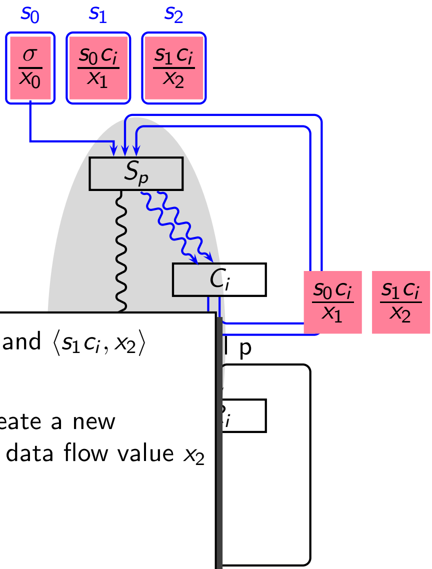
VBTCC in Recursion: Motivating Example Revisited



- We continue to process RCS and $\langle s_1 c_i, x_2\rangle$ reaches S_p
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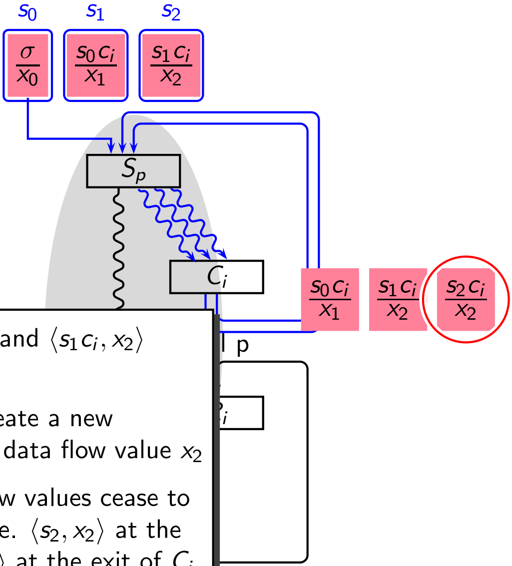
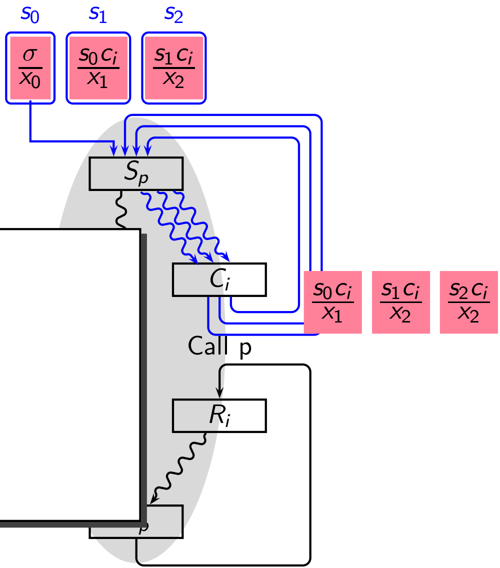
VBTCC in Recursion: Motivating Example Revisited



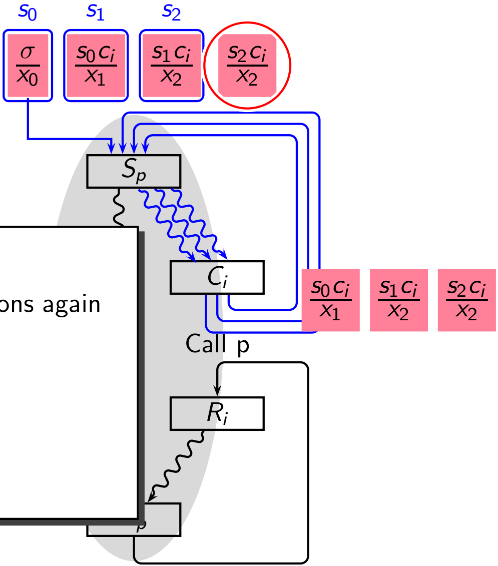
- We continue to process RCS and $\langle s_1 c_i, x_2 \rangle$ reaches S_p
- We adjust the partition to create a new equivalence class for the new data flow value x_2
- For our example, the data flow values cease to change after computing x_2, i.e. $\langle s_2, x_2 \rangle$ at the exit of S_p reaches as $\langle s_2 c_i, x_2 \rangle$ at the exit of C_i

VBTCC in Recursion: Motivating Example Revisited



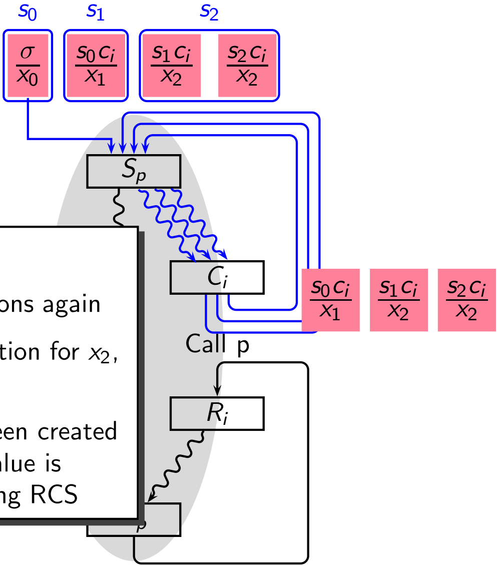
- $\langle s_2 c_i, x_2 \rangle$ reaches S_p

VBTCC in Recursion: Motivating Example Revisited



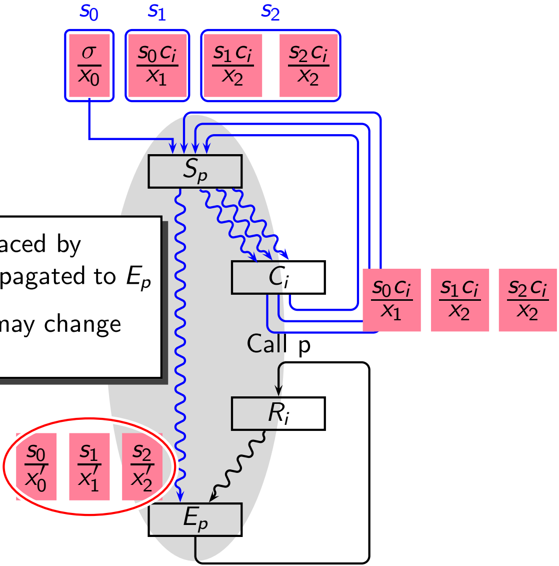
- $\langle s_2 c_i, x_2 \rangle$ reaches S_p
- We need to adjust the partitions again

VBTCC in Recursion: Motivating Example Revisited



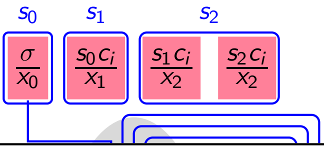
- $\langle s_2 c_i, x_2 \rangle$ reaches S_p
- We need to adjust the partitions again
- Since we already have a partition for x_2, $s_2 c_i$ gets included in it
- Since no new partition has been created (because no new data flow value is discovered), we stop processing RCS

VBTCC in Recursion: Motivating Example Revisited



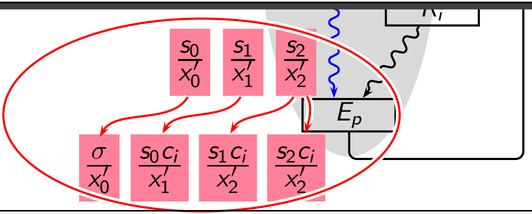
- The call strings are replaced by partition ids and are propagated to E_p
- Their data flow values may change along the way

VBTCC in Recursion: Motivating Example Revisited



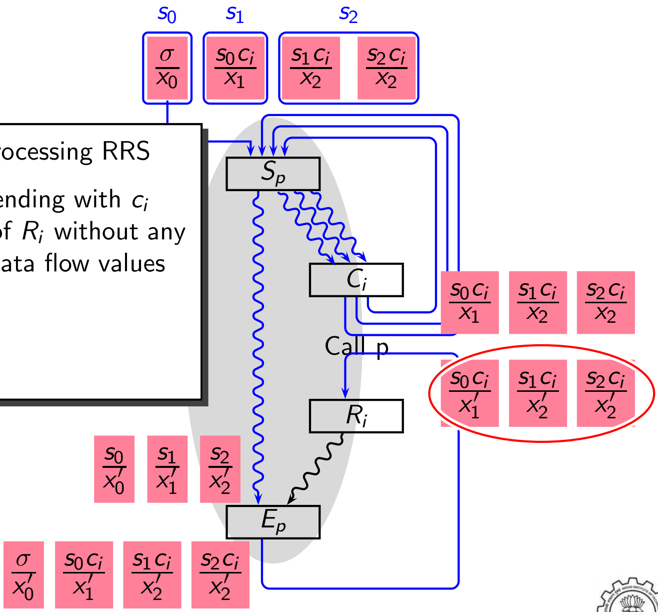
- At E_p, the class ids are replaced by call strings contained in equivalence classes regenerating the call strings with their new values
- Observe the effect of de-partitioning: The data flow value of s_2 has been copied to $s_1 c_i$ *as well as* $s_2 c_i$

  This has the effect of pushing the data flow values into "deeper" recursion without constructing the call strings because all these call strings would have identical data flow values



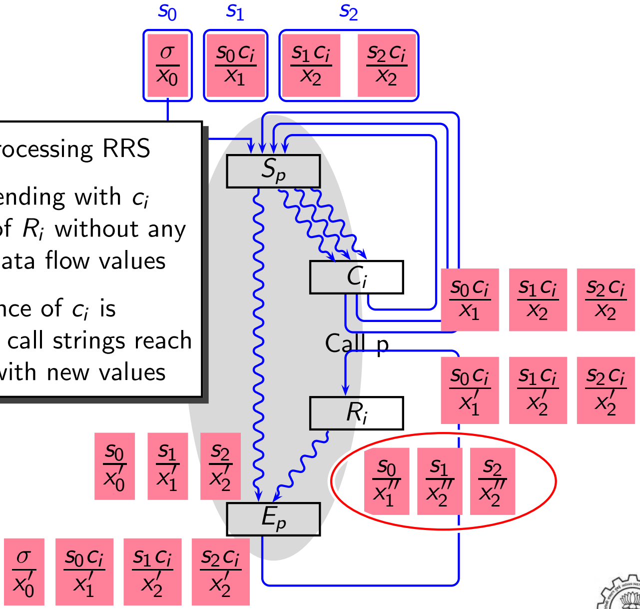
VBTCC in Recursion: Motivating Example Revisited

- We now begin processing RRS
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values



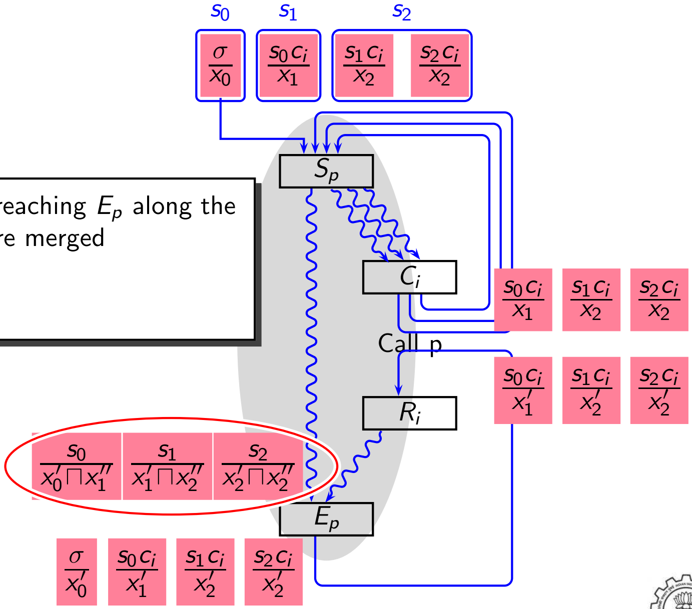
VBTCC in Recursion: Motivating Example Revisited

- We now begin processing RRS
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values
- The last occurrence of c_i is removed and the call strings reach the entry of E_p with new values



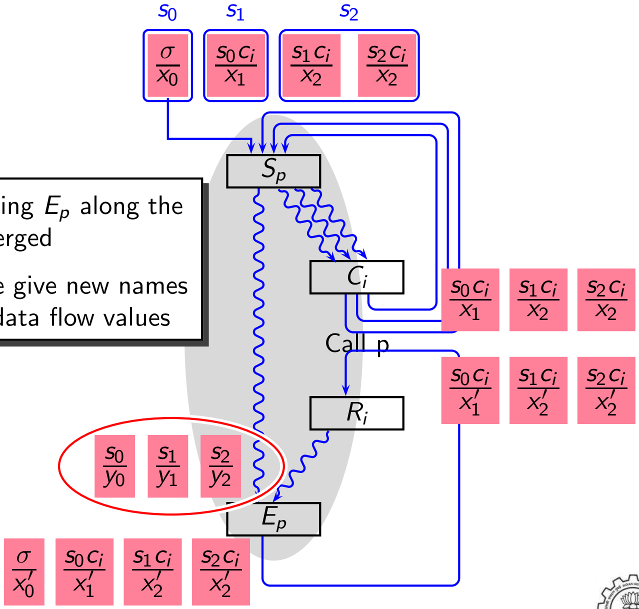
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged



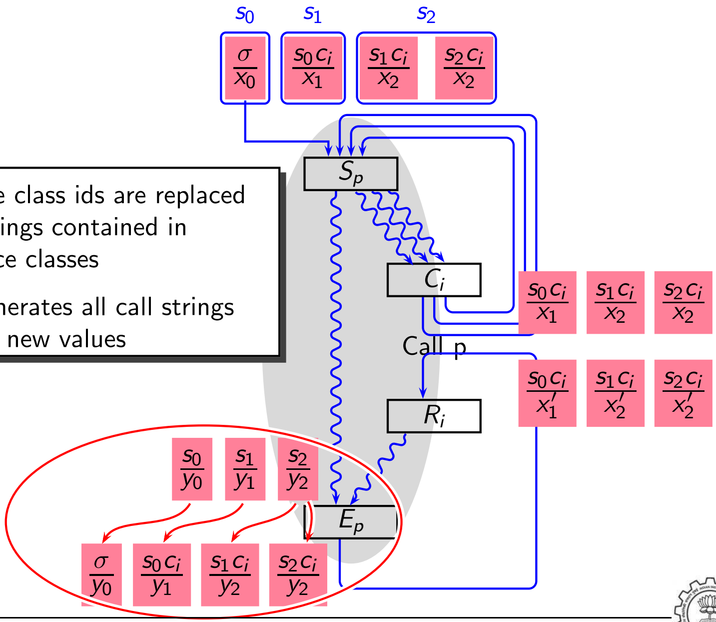
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged
- For simplicity, we give new names to the resulting data flow values



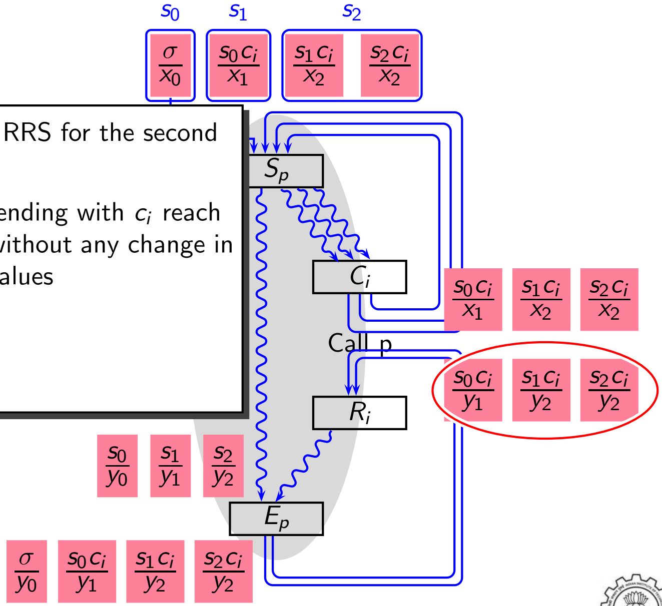
VBTCC in Recursion: Motivating Example Revisited

- At E_p, the class ids are replaced by call strings contained in equivalence classes
- This regenerates all call strings with their new values



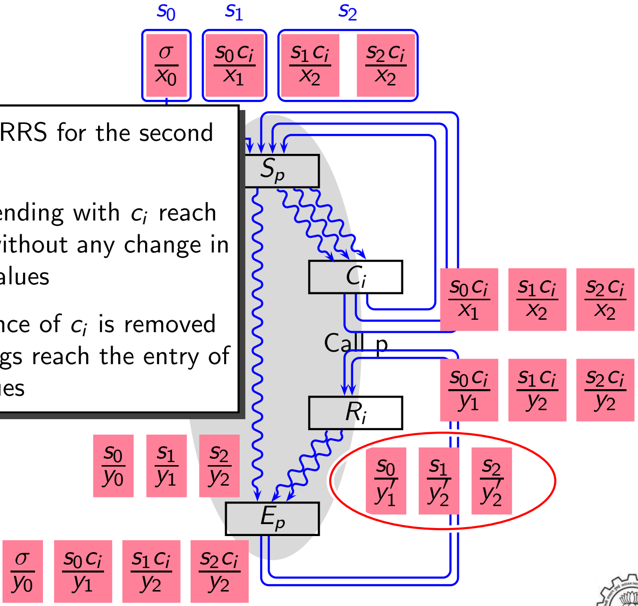
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the second time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values



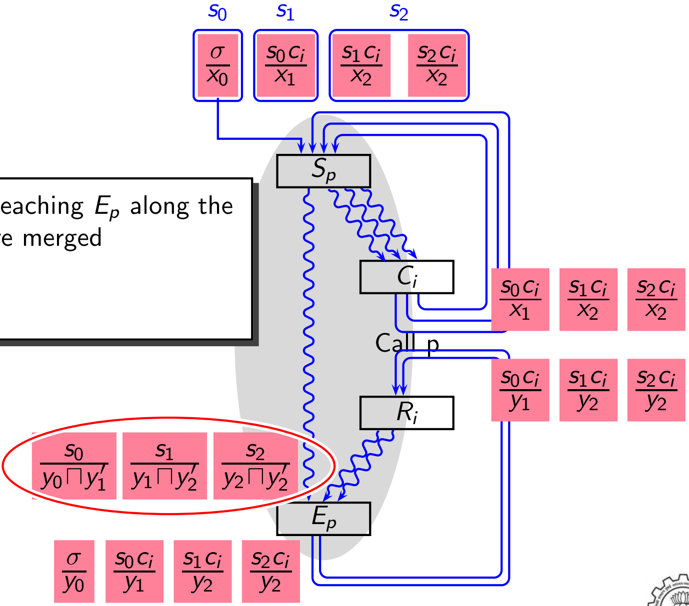
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the second time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values
- The last occurrence of c_i is removed and the call strings reach the entry of E_p with new values



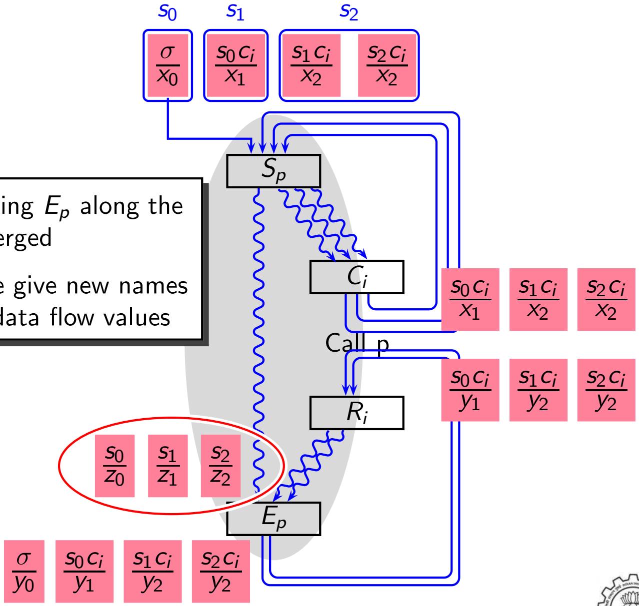
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged



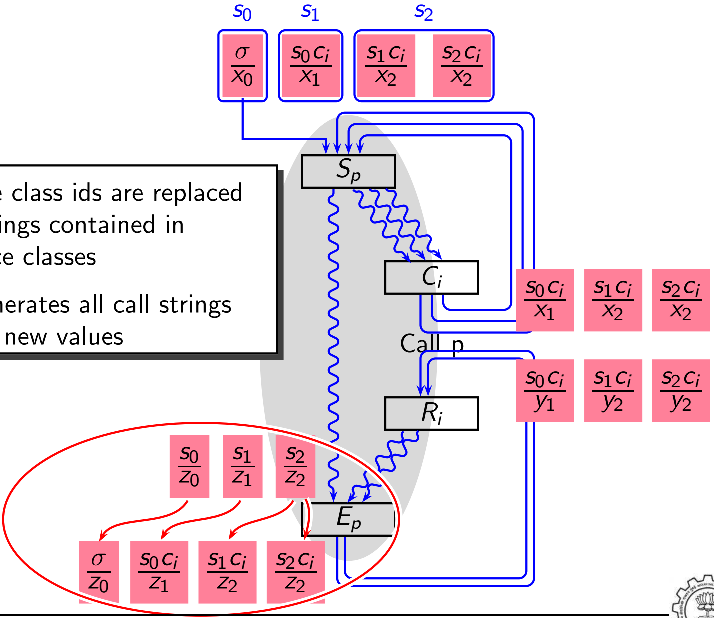
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged
- For simplicity, we give new names to the resulting data flow values



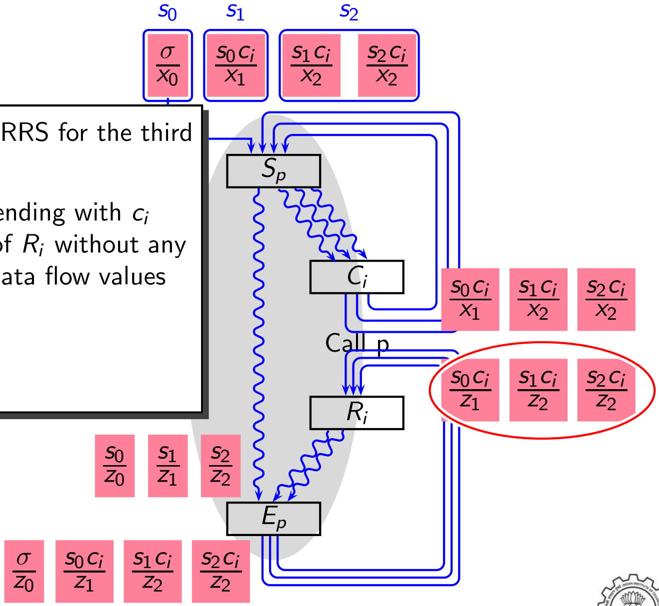
VBTCC in Recursion: Motivating Example Revisited

- At E_p, the class ids are replaced by call strings contained in equivalence classes
- This regenerates all call strings with their new values



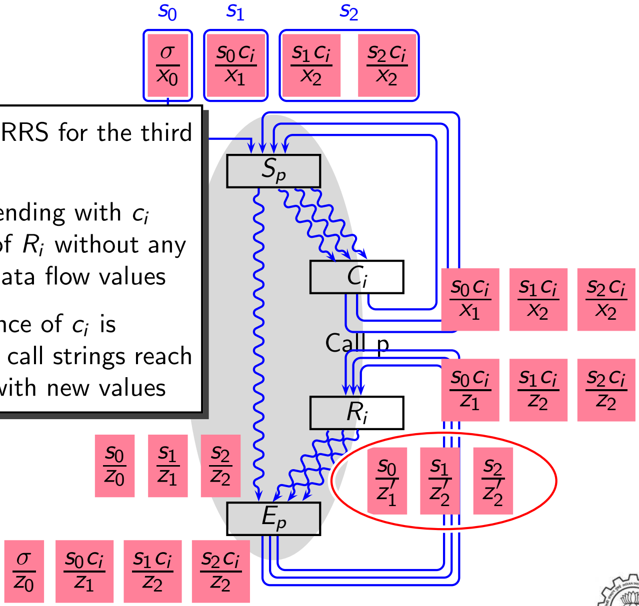
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the third time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values



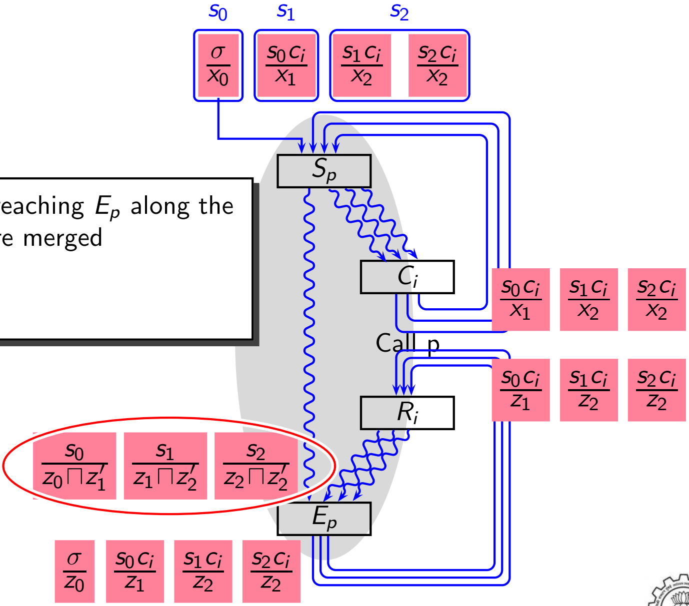
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the third time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values
- The last occurrence of c_i is removed and the call strings reach the entry of E_p with new values



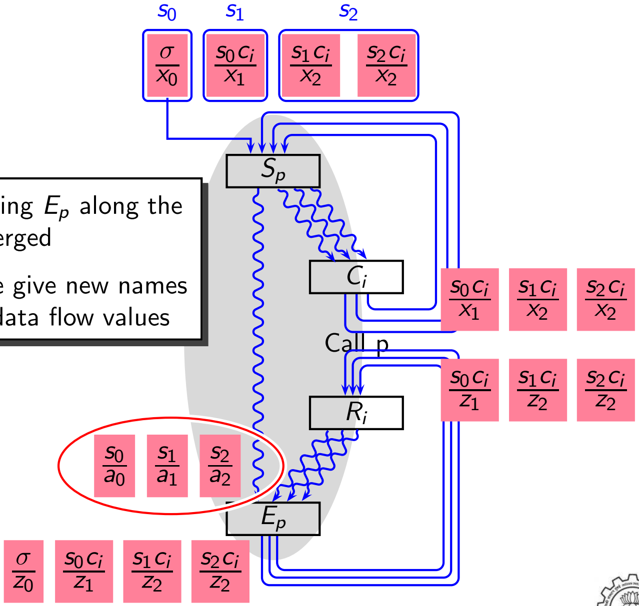
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged



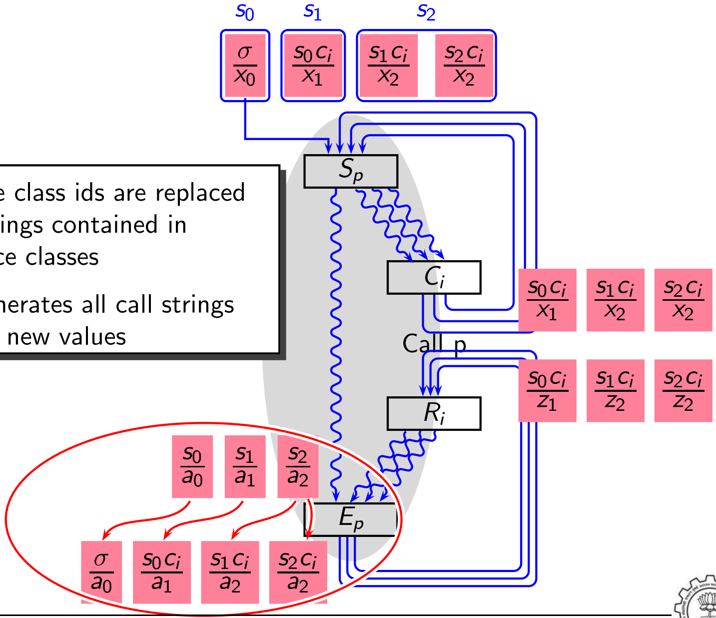
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged
- For simplicity, we give new names to the resulting data flow values



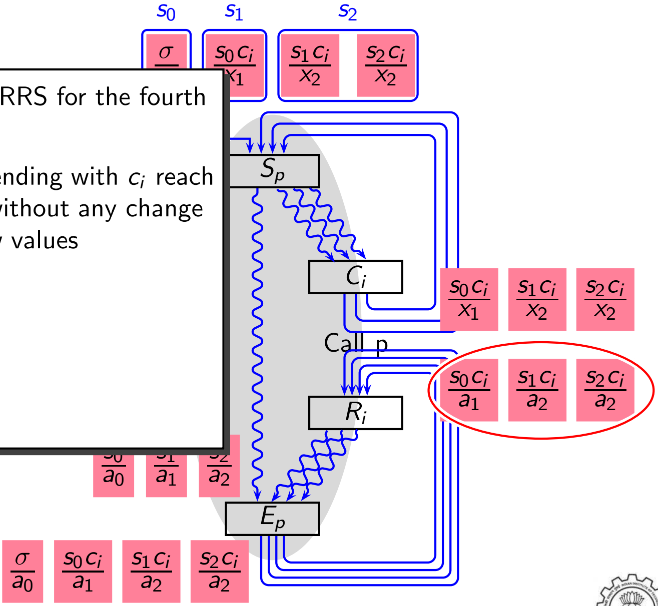
VBTCC in Recursion: Motivating Example Revisited

- At E_p, the class ids are replaced by call strings contained in equivalence classes
- This regenerates all call strings with their new values



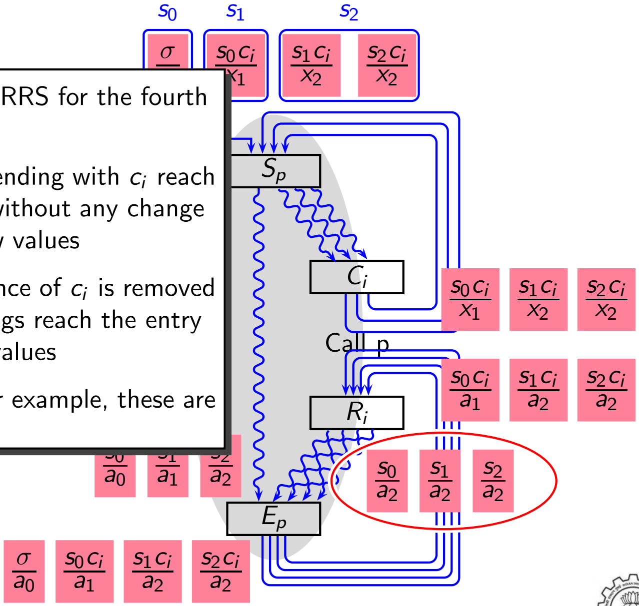
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the fourth time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values



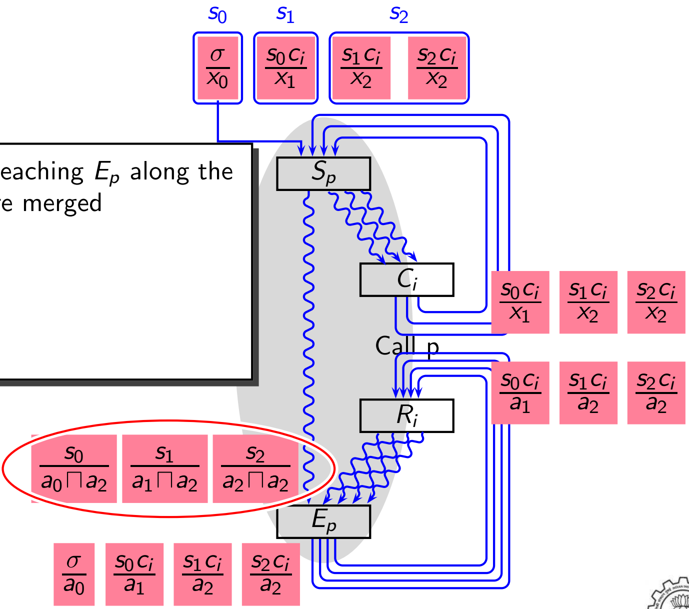
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the fourth time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values
- The last occurrence of c_i is removed and the call strings reach the entry of E_p with new values
- Note that for our example, these are the final values



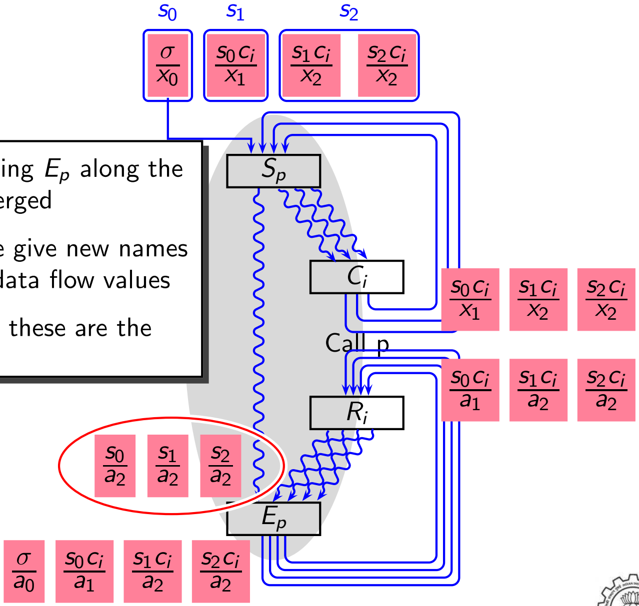
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged



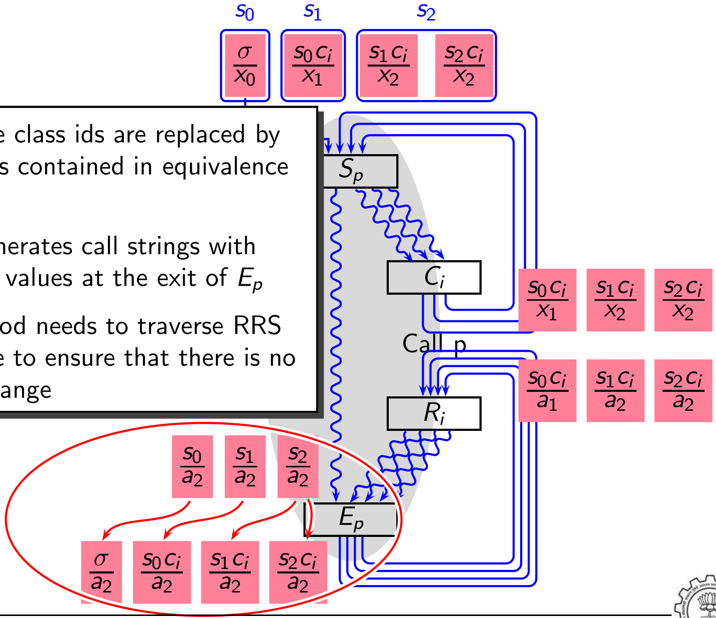
VBTCC in Recursion: Motivating Example Revisited

- The values reaching E_p along the two paths are merged
- For simplicity, we give new names to the resulting data flow values
- For our example, these are the final values



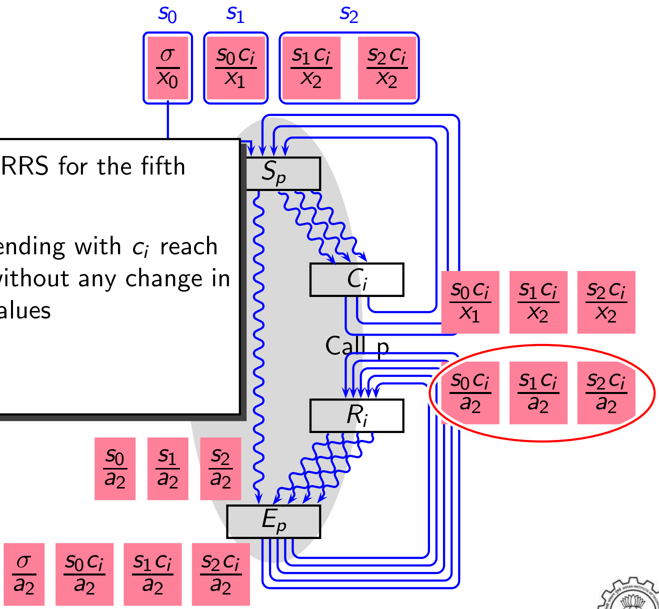
VBTCC in Recursion: Motivating Example Revisited

- At E_p, the class ids are replaced by call strings contained in equivalence classes
- This regenerates call strings with their final values at the exit of E_p
- The method needs to traverse RRS once more to ensure that there is no further change



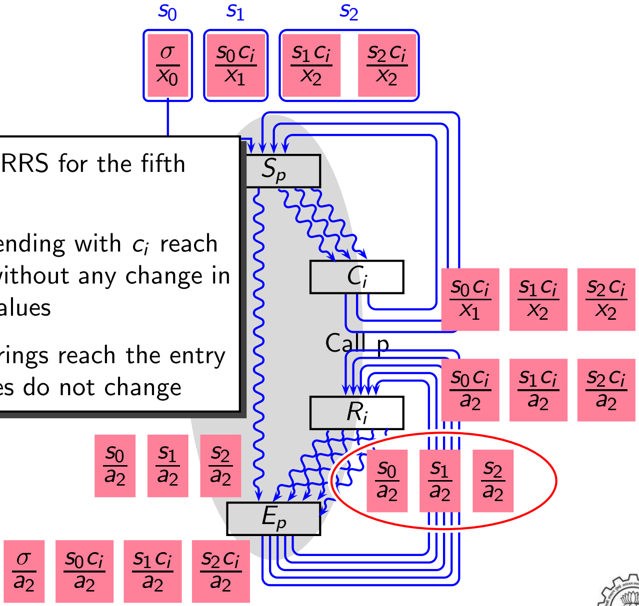
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the fifth time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values



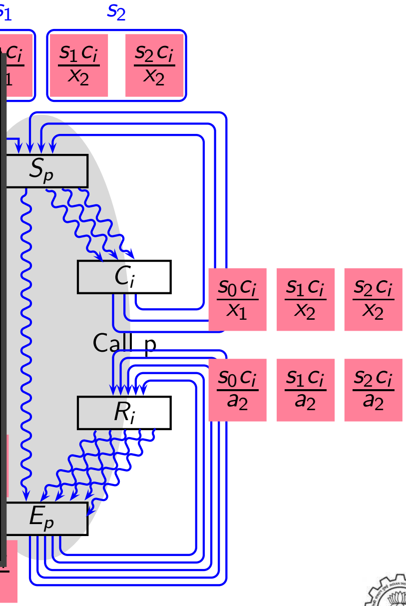
VBTCC in Recursion: Motivating Example Revisited

- We now process RRS for the fifth time
- The call strings ending with c_i reach the entry of R_i without any change in their data flow values
- When the call strings reach the entry of E_p, their values do not change

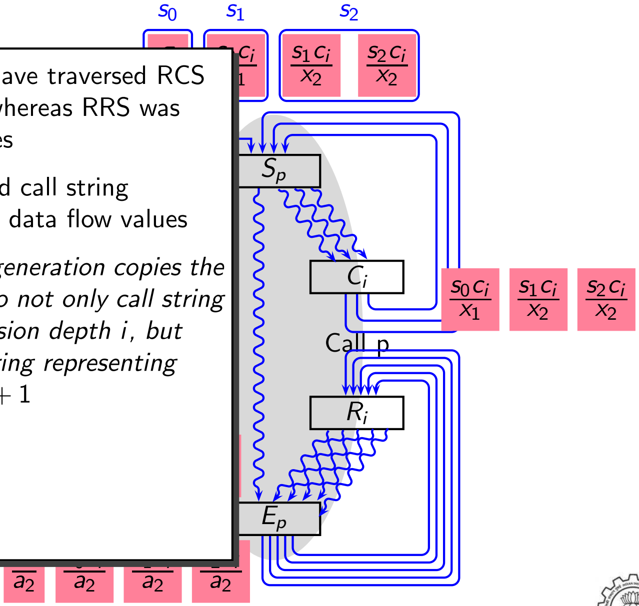


VBTCC in Recursion: Motivating Example Revisited

- Observe that we have traversed RCS only three times whereas RRS was traversed five times
- We had terminated call string construction using data flow values



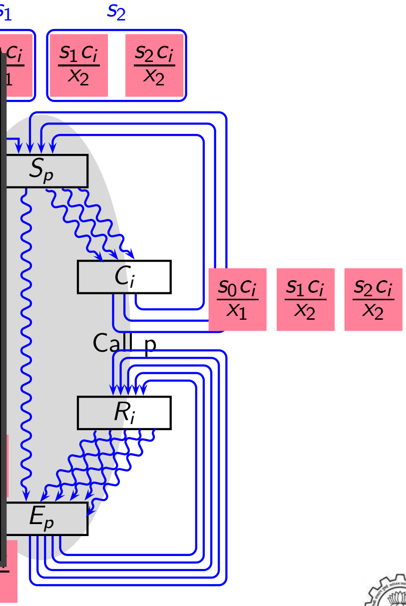
VBTCC in Recursion: Motivating Example Revisited



- Observe that we have traversed RCS only three times whereas RRS was traversed five times
- We had terminated call string construction using data flow values
- *The process of regeneration copies the data flow values to not only call string representing recursion depth i, but also to the call string representing recursion depth $i + 1$*

VBTCC in Recursion: Motivating Example Revisited

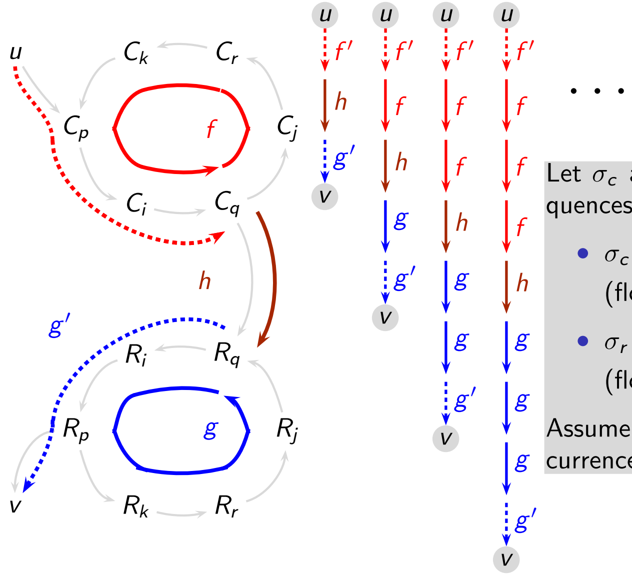
- Observe that we have traversed RCS only three times whereas RRS was traversed five times
- We had terminated call string construction using data flow values
- *The process of regeneration copies the data flow values to not only call string representing recursion depth i, but also to the call string representing recursion depth $i+1$*
- Since this happens repeatedly, we end up propagating values to call strings for all depths of recursion without having to construct them



Equivalence of The Two Methods

- For non-recursive programs, equivalence is obvious
- For recursive program, we prove equivalence using staircase diagrams

Call Strings for Recursive Contexts



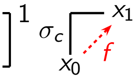
Let σ_c and σ_r be the call site sequences for RCS and RRS

- $\sigma_c \equiv c_j c_r c_k c_p c_i c_q$ (flow function f)
- $\sigma_r \equiv r_q r_i r_p r_k r_r r_j$ (flow function g)

Assume that we allow upto m occurrences of σ_c

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times

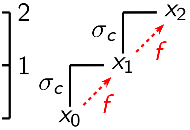


$$x_1 = f(x_0)$$

Data flow value at C_q

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times

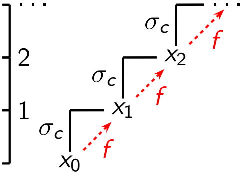


$$x_2 = f^2(x_0)$$

Data flow value at C_q

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times

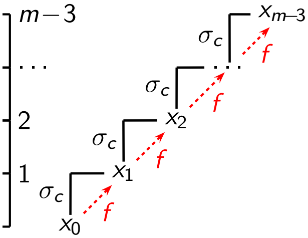


$$x_i = f^i(x_0)$$

Data flow value at C_q

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times

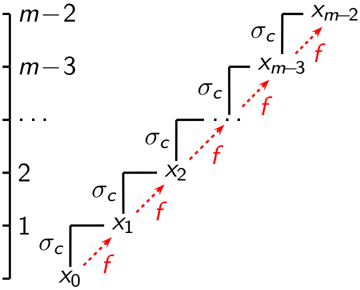


$$x_i = f^i(x_0)$$

Data flow value at C_q

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times

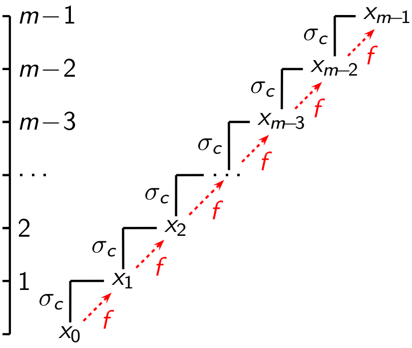


$$x_i = f^i(x_0)$$

Data flow value at C_q

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times

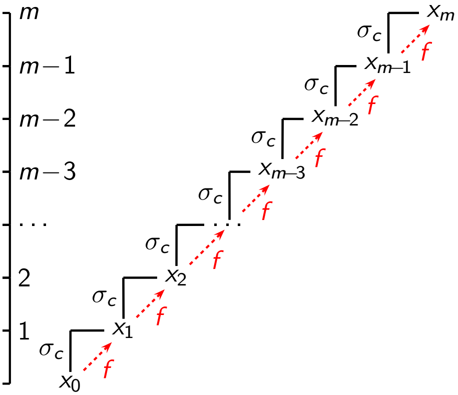


$$x_i = f^i(x_0)$$

Data flow value at C_q

Staircase Diagram of Computation along Recursive Paths

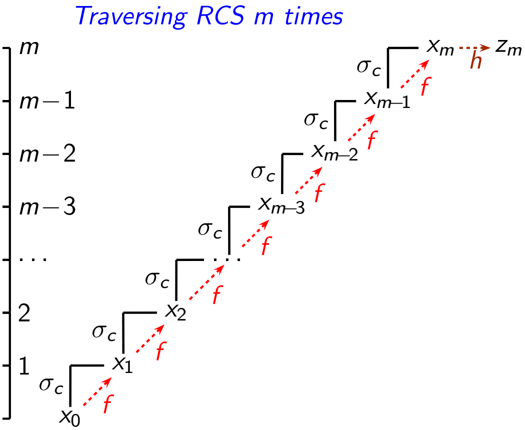
Traversing RCS m times



$x_i = f^i(x_0)$

Data flow value at C_q

Staircase Diagram of Computation along Recursive Paths



$x_i = f^i(x_0)$

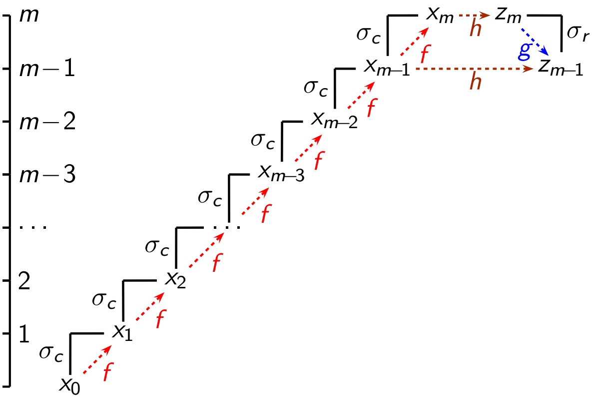
Data flow value at C_q

$z_m = h(x_m)$

Data flow value at R_q

Staircase Diagram of Computation along Recursive Paths

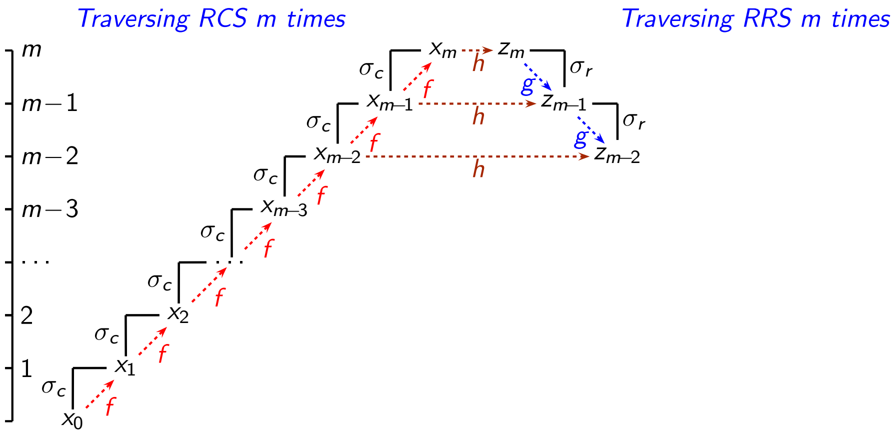
Traversing RCS m times *Traversing RRS m times*



$x_i = f^i(x_0)$ $z_{m-1} = h(x_{m-1}) \sqcap g(z_m)$

Data flow value at C_q *Data flow value at R_q*

Staircase Diagram of Computation along Recursive Paths



$x_i = f^i(x_0)$

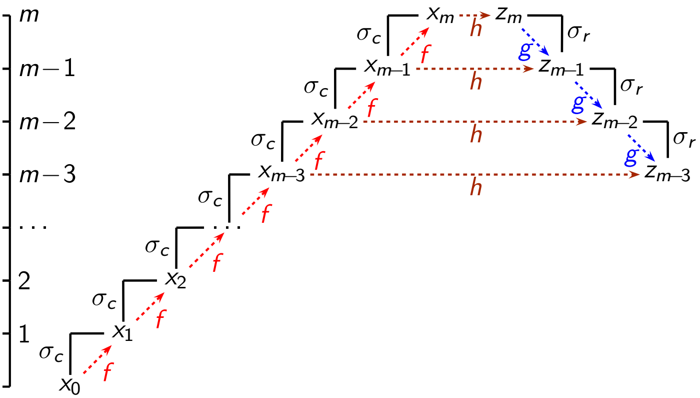
Data flow value at C_q

$z_{m-2} = h(x_{m-2}) \sqcap g(z_{m-1})$

Data flow value at R_q

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times *Traversing RRS m times*



$x_i = f^i(x_0)$

Data flow value at C_q

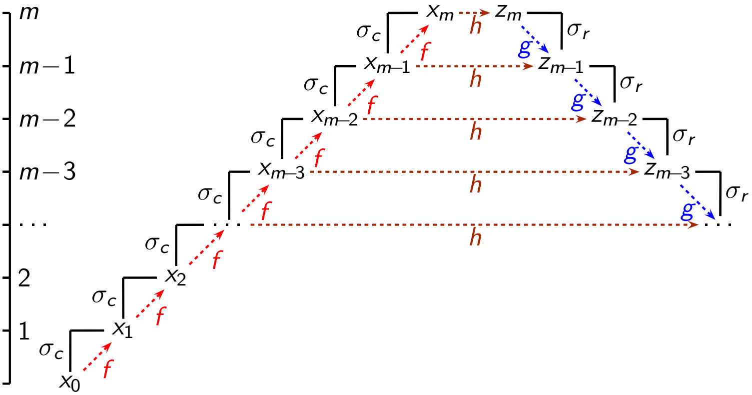
$z_{m-j} = h(x_{m-j}) \sqcap g(z_{m-j+1})$

Data flow value at R_q

Staircase Diagram of Computation along Recursive Paths

Traversing RCS m times

Traversing RRS m times



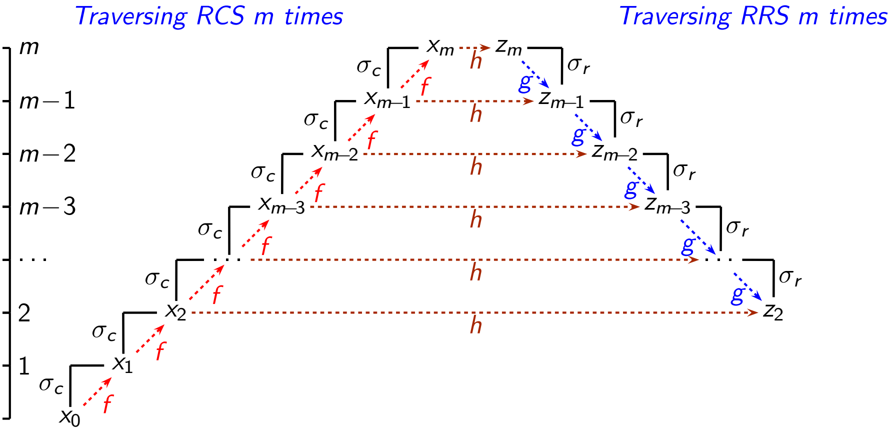
$x_i = f^i(x_0)$

Data flow value at C_q

$z_{m-j} = h(x_{m-j}) \sqcap g(z_{m-j+1})$

Data flow value at R_q

Staircase Diagram of Computation along Recursive Paths



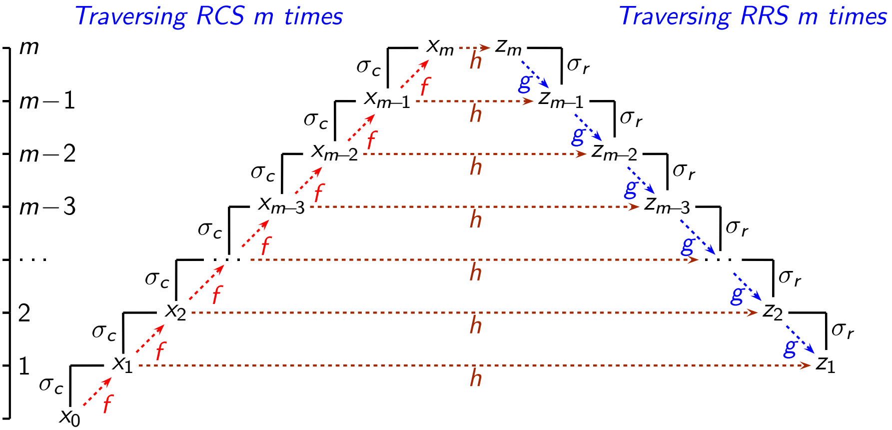
$x_i = f^i(x_0)$

Data flow value at C_q

$z_{m-j} = h(x_{m-j}) \sqcap g(z_{m-j+1})$

Data flow value at R_q

Staircase Diagram of Computation along Recursive Paths



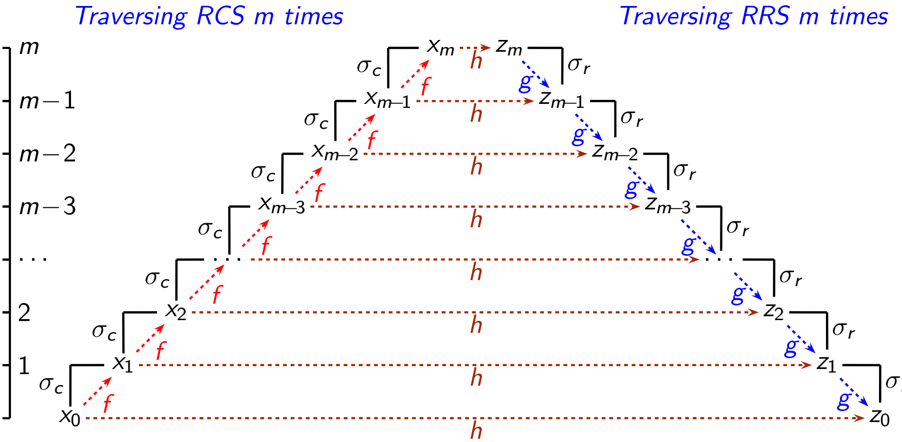
$x_i = f^i(x_0)$

Data flow value at C_q

$z_{m-j} = h(x_{m-j}) \sqcap g(z_{m-j+1})$

Data flow value at R_q

Staircase Diagram of Computation along Recursive Paths



$x_i = f^i(x_0)$

Data flow value at C_q

$z_{m-j} = h(x_{m-j}) \sqcap g(z_{m-j+1})$

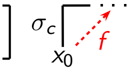
Data flow value at R_q

Fixed Bound Closure Bound of Flow Function

- $n > 0$ is the fixed point closure bound of $h : L \mapsto L$ if it is the smallest number such that

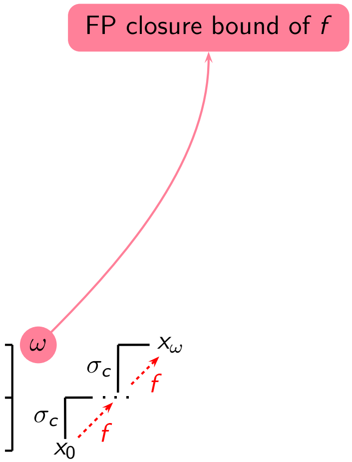
$$\forall x \in L,\ h^{n+1}(x) = h^n(x)$$

Computation of Data Flow Values along Recursive Paths



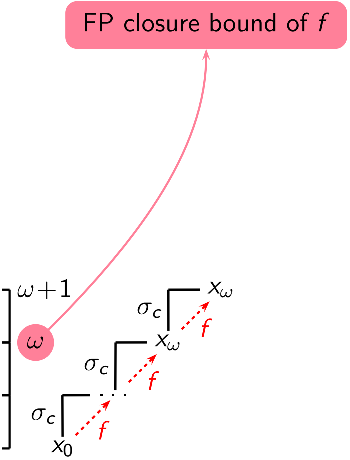
$$x_1 = f(x_0)$$

Computation of Data Flow Values along Recursive Paths



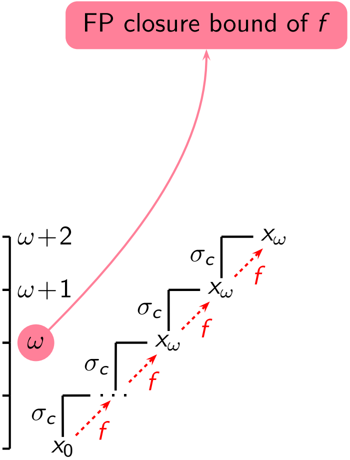
$$x_2 = f^2(x_0)$$

Computation of Data Flow Values along Recursive Paths



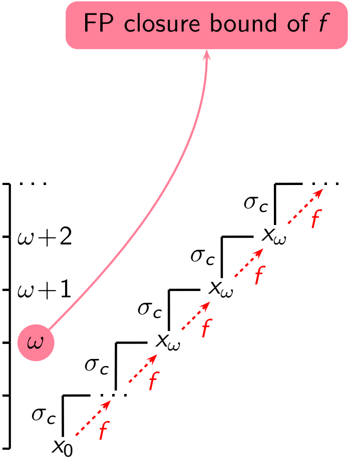
$$x_\omega = f^\omega(x_0)$$

Computation of Data Flow Values along Recursive Paths



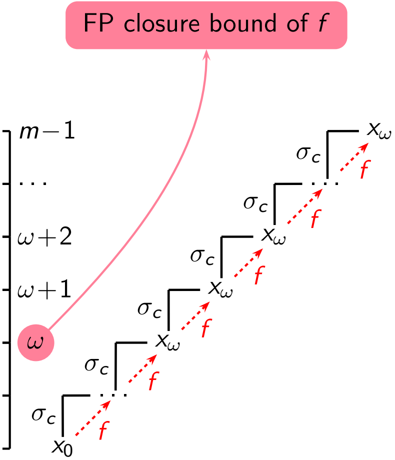
$$x_i = \begin{cases} f^i(x_0) & i < \omega \\ f^\omega(x_0) & \text{otherwise} \end{cases}$$

Computation of Data Flow Values along Recursive Paths



$$x_i = \begin{cases} f^i(x_0) & i < \omega \\ f^\omega(x_0) & \text{otherwise} \end{cases}$$

Computation of Data Flow Values along Recursive Paths



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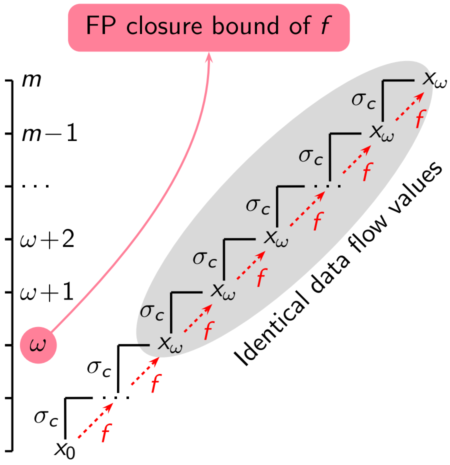
Computation of Data Flow Values along Recursive Paths



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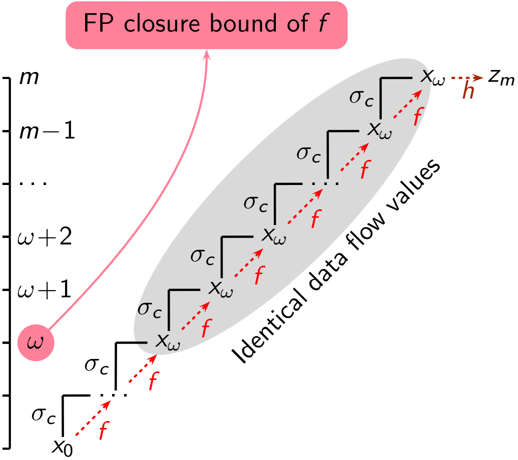

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$$z_m = h(x_\omega)$$

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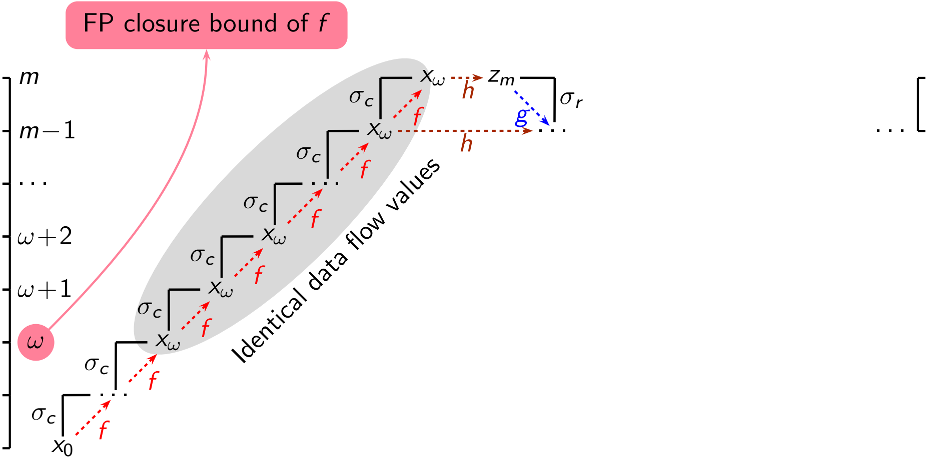

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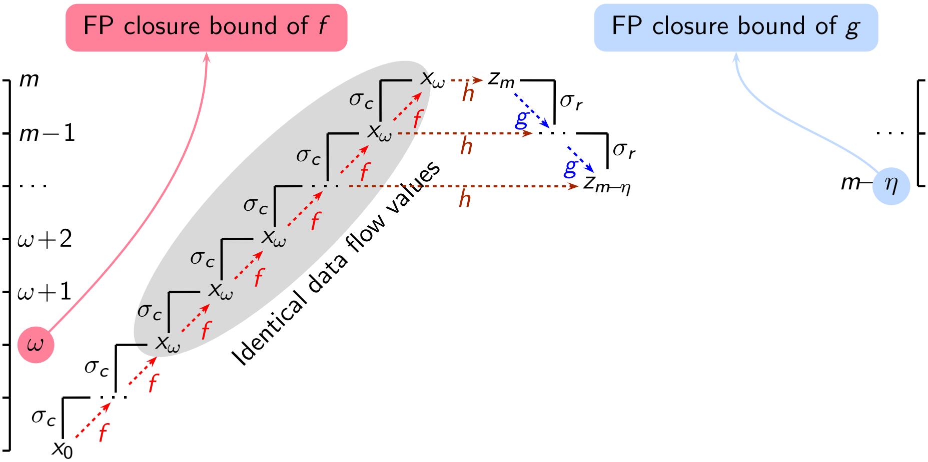

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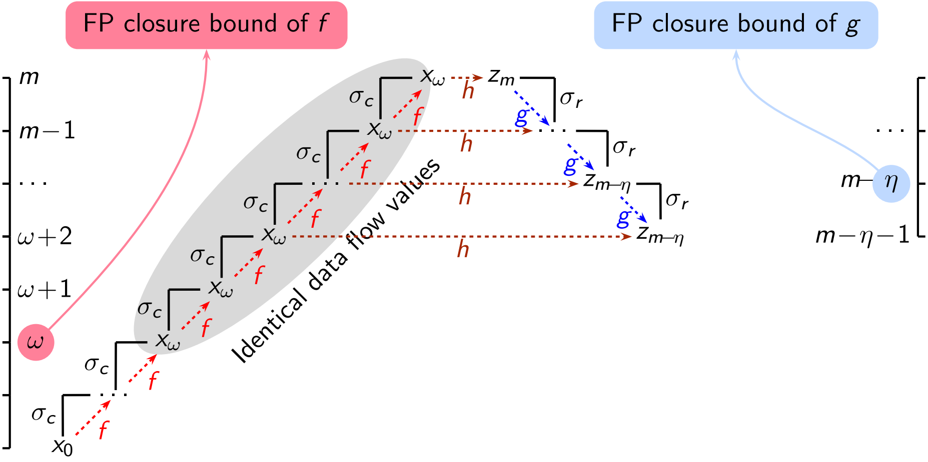

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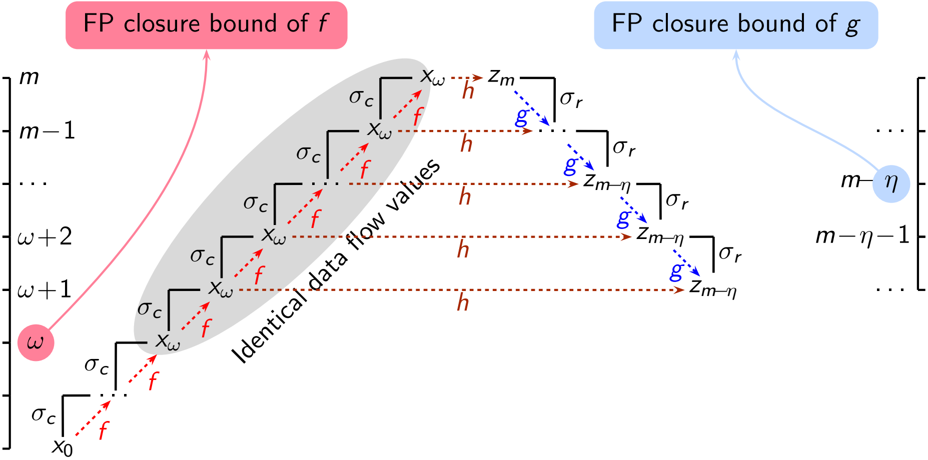

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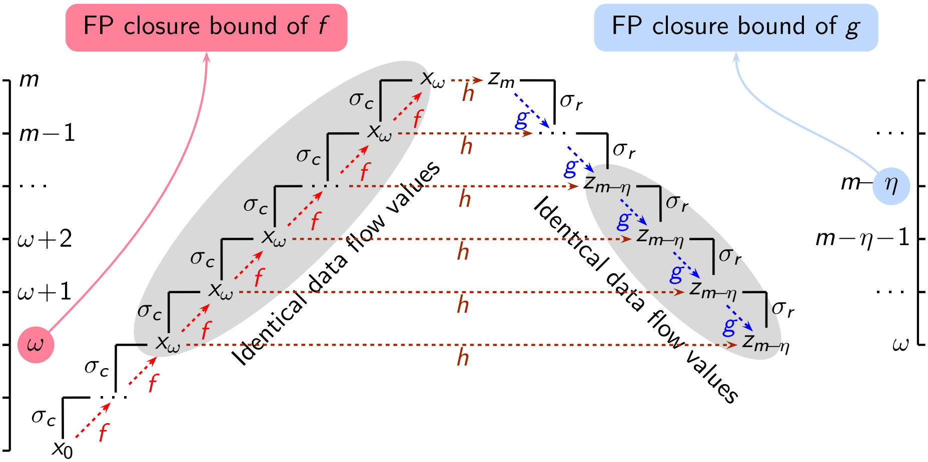

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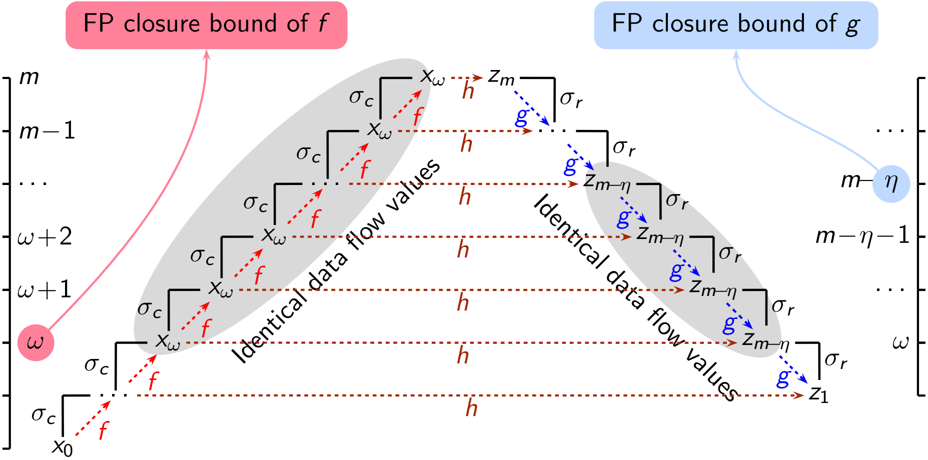

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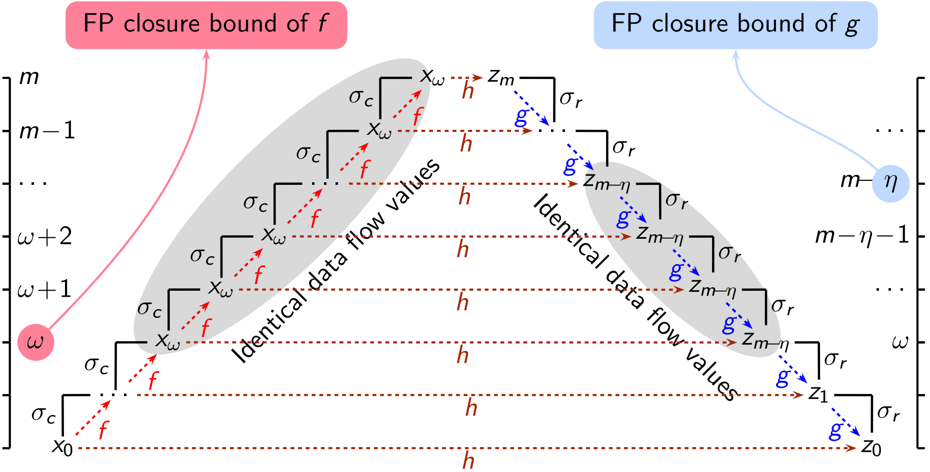

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- In the cyclic call sequence,

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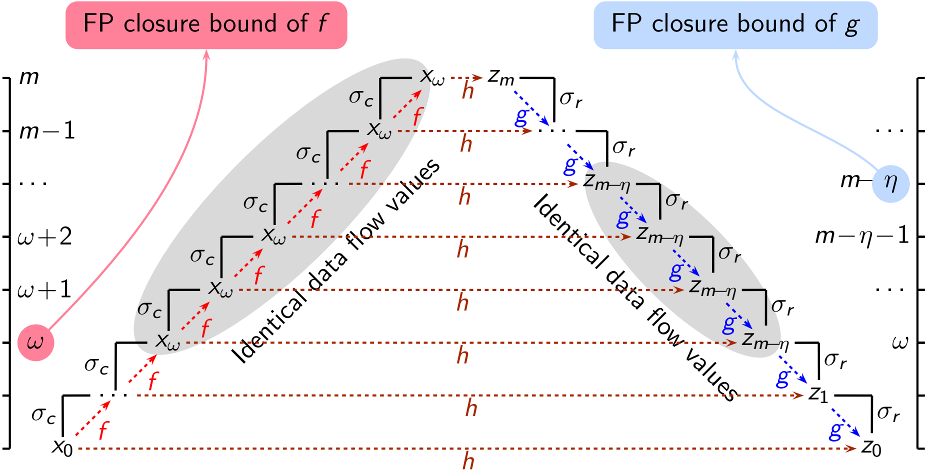

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Bounding the Call String Length Using Data Flow Values

FP closure bound of f

FP closure bound of g

Theorem: Data flow values $z_{m-i}, 0 \leq i \leq \omega$ (computed along σ_r) follow a strictly descending chain.

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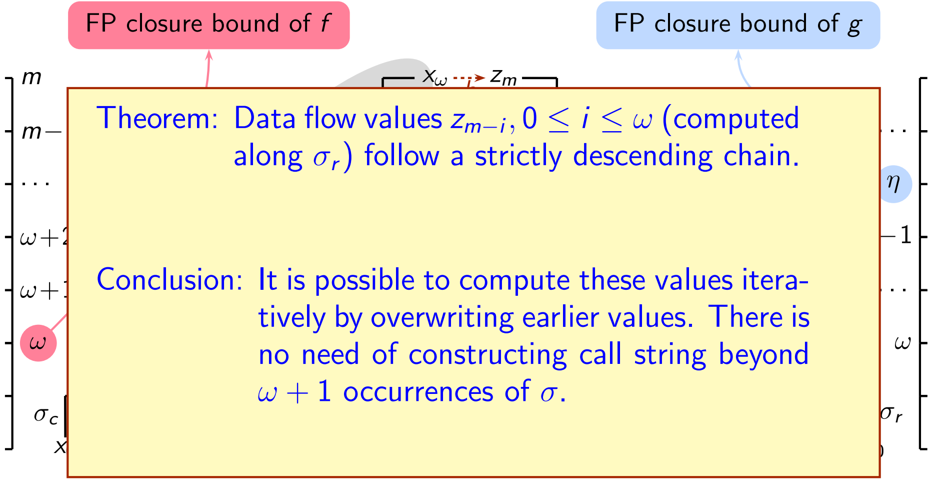

Theorem: Data flow values $z_{m-i}, 0 \leq i \leq \omega$ (computed along σ_r) follow a strictly descending chain.

Conclusion: It is possible to compute these values iteratively by overwriting earlier values. There is no need of constructing call string beyond $\omega + 1$ occurrences of σ.

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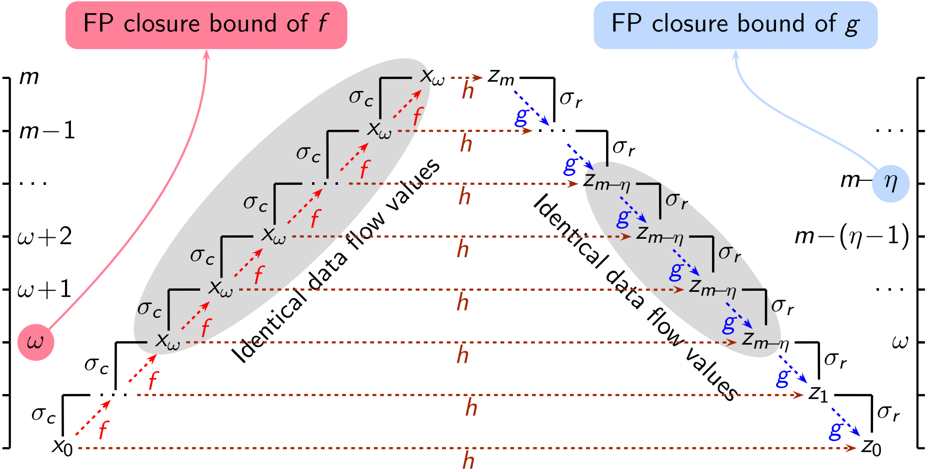

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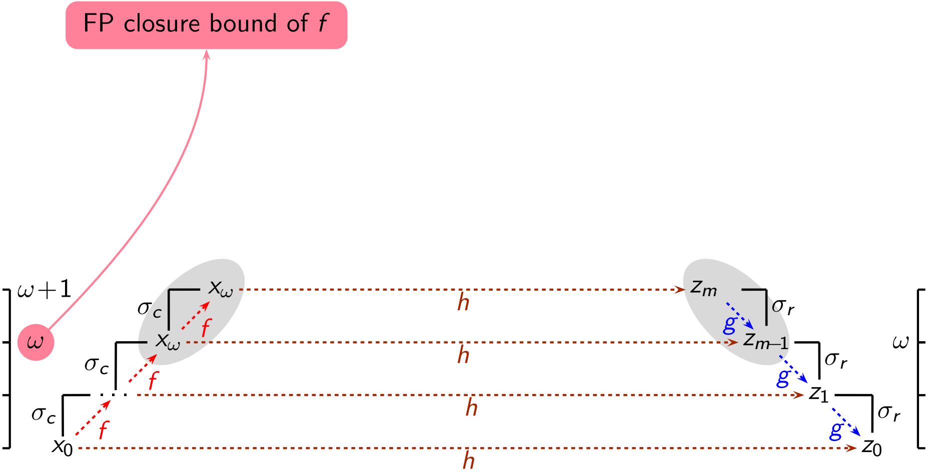

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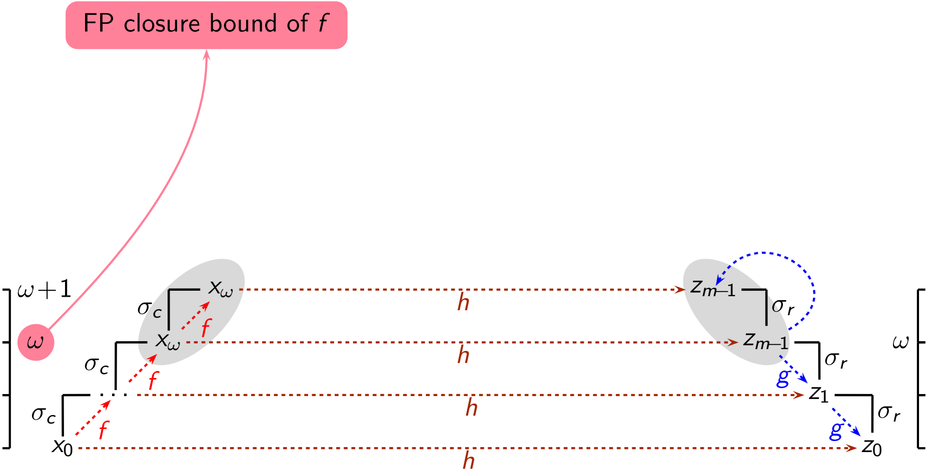

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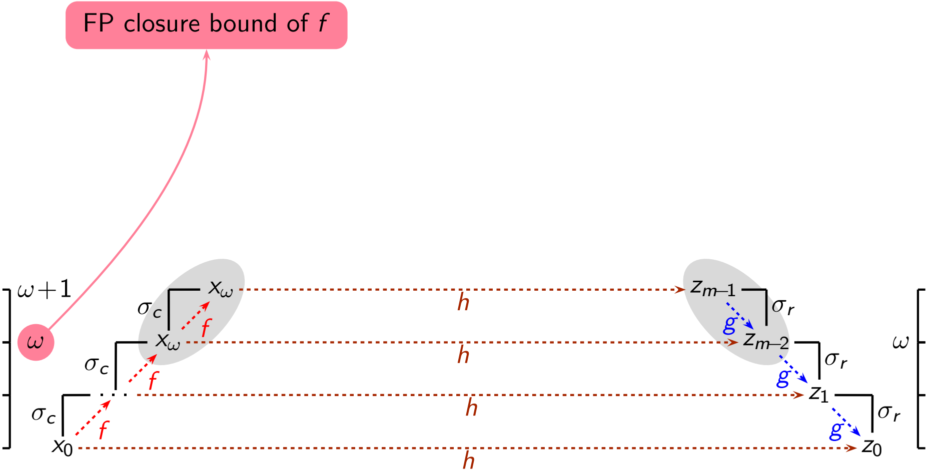

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- This amounts to simulating all call strings that would have otherwise been constructed while traversing RCS.
- It can also be seen as virtually climbing up the steps in RRS as much as needed and then climbing down.
- This is possible only because
  - all these call strings would have the same data flow value associated with them, and
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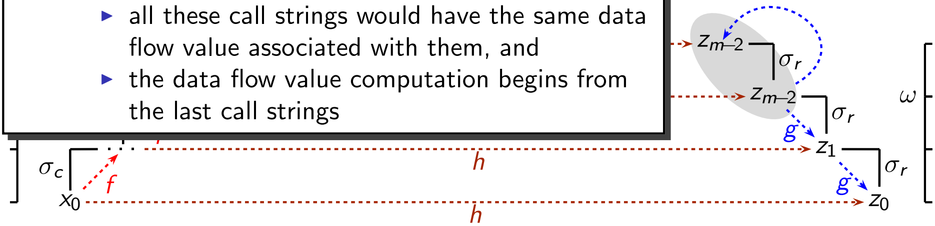

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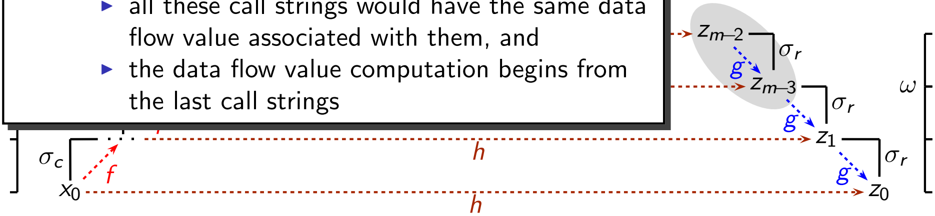

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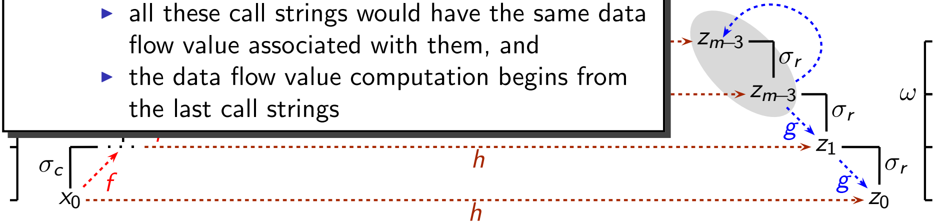

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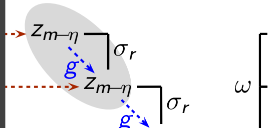
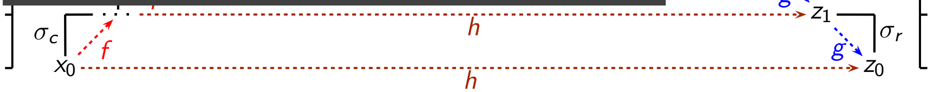
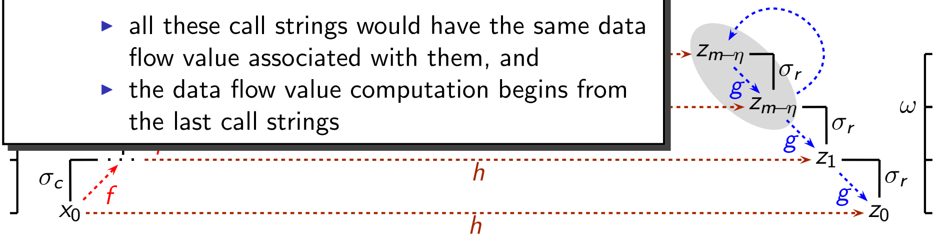




$$x_i = \begin{cases} f^i(x_0) & i < \omega \\ f^\omega(x_0) & \text{otherwise} \end{cases} \qquad z_{m-j} = \begin{cases} h(x_\omega) \sqcap g(z_{m-j+1}) & 0 \le j \le \eta \\ h(x_\omega) \sqcap g(z_{m-\eta}) & \eta < j \le (m-\omega) \\ h(x_j) \sqcap g(z_{m-j+1}) & \text{otherwise} \end{cases}$$

Bounding the Call String Length Using Data Flow Values

- This amounts to simulating all call strings that would have otherwise been constructed while traversing RCS.
- It can also be seen as virtually climbing up the steps in RRS as much as needed and then climbing down.
- This is possible only because
  - all these call strings would have the same data flow value associated with them, and
  - the data flow value computation begins from the last call strings



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  where $(c_i)_j$ denotes the j^{th} occurrence of c_i
  Let $j \geq |L| + 1$
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- Original required length $= K \times (|L| + 1)^2$

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  Context insensitive beyond that.
- Assumption: Height of the lattice is finite.

Reaching Definitions Analysis in GCC 4.0

Program	LoC	$\#F$	$\#C$	$3K$ length bound				Proposed Approach		
				K	#CS	Max	Time	#CS	Max	Time
hanoi	33	2	4	4	100000+	99922	3973×10^3	8	7	2.37
bit_gray	53	5	11	7	100000+	31374	2705×10^3	17	6	3.83
analyzer	288	14	20	2	21	4	20.33	21	4	1.39
distray	331	9	21	6	96	28	322.41	22	4	1.11
mason	350	9	13	8	100000+	22143	432×10^3	14	4	0.43
fourinarow	676	17	45	5	510	158	397.76	46	7	1.86
sim	1146	13	45	8	100000+	33546	1427×10^3	211	105	234.16
181_mcf	1299	17	24	6	32789	32767	484×10^3	41	11	5.15
256_bzip2	3320	63	198	7	492	63	258.33	406	34	200.19

- LoC is the number of lines of code,
- $\#F$ is the number of procedures,
- $\#C$ is the number of call sites,
- $\#CS$ is the number of call strings
- Max denotes the maximum number of call strings reaching any node.
- Analysis time is in milliseconds.

(Implementation was carried out by Seema Ravandale.)

Some Observations

- Compromising on precision may not be necessary for efficiency.
- Separating the necessary information from redundant information is much more significant.
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$$\#\text{ distinct tagged values} = \text{Min}\ (\#\text{ actual contexts},\ \#\text{ actual data flow values})$$

Tutorial Problem on Interprocedural Points-to Analysis

```
main()
{  x = &y;
   z = &x;
   y = &z;
   p(); /* C1 */
}

p()
{  if (...)
   {  p(); /* C2 */
      x = *x;
   }
}
```

- Number of distinct call sites in a call chain $K = 2$.
- Number of variables: 3
- Number of distinct points-to pairs: $3 \times 3 = 9$
- L is powerset of all points-to pairs
- $\mid L \mid = 2^9$
- Length of the longest call string in Sharir-Pnueli method $2 \times (|L| + 1)^2 = 2^{19} + 2^{10} + 1 = 5,25,313$
- All call strings upto this length must be constructed by the Sharir-Pnueli method!

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- Modified call strings method requires only three call strings: λ, c_1, and $c_1 c_2$