CS 105: Department Introductory Course on Discrete Structures

Instructor : S. Akshay

Aug 01, 2025

Lecture 02 – Propositions, Predicates, Boolean Algebra

Recap: Logistics

Course hours: Slot 10;

Tue 14:00-15:25, Thu 14:00-15:25

Office hours: To be announced.

Problem Solving/Help Session (Optional): One hour per week, run by teaching assistants. (Time and Venue to be decided)

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- One problem sheet will be posted (almost) every week!

Attendance
As per Institute rules: SAFE

Recap: More Logistics

Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- Other {participation, pop quizzes, assignments}: 5%

Minimum requirements (Tentative)

10/40 in endsem + 15/60 in remaining.

How to reach me after class?

- Send a message on piazza
- Drop by my office...
  - CS 507 (5th floor of New CSE/CC building)
  - Temporarily CC 313 (3rd floor!)

Course Outline

What we will cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory

Textbooks

- Discrete Mathematics and its Applications with Combinatorics and Graph Theory, by Kenneth H Rosen.
- Introduction to Graph theory by Douglas B West.
- Discrete Mathematics by Norman Biggs.
- More will be listed on webpage as we go along.

Chapter 1: Proofs and Logical reasoning

Outline of next few classes

- Propositions, statements
- What/why of proofs and some generic proof strategies
- Mathematical induction

Propositional calculus and Boolean algebra

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Combining propositions

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- p, q are called atomic propositions (basic propositions that can't be further divided) and $\vee, \neg, \wedge, \rightarrow, \leftrightarrow$ are Boolean connectives.

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Truth Tables

p	$\neg p$
T	F
F	T

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Logical Equivalence: Truth tables are identical!

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- You will pass this course iff you score $\geq 10/40$ in endsem and $\geq 15/60$ in rest.

Some more exercises

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Exercise: Show that the following are equivalent

1. De-Morgan's Laws
   - 1.1 $\neg(p \wedge q) \equiv \neg p \vee \neg q$
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2. $\neg\neg p \equiv p$ (Double negation)
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So idea: just use previous proved equivalences! When in doubt, fall back to truth tables.

Recap: Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled.

1. What is its converse?
2. What is its contra-positive?
3. What is its negation?

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Which of these are equivalent to the original proposition? Why?

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Exercise

Consider the proposition: If I will eat samosa or bhel puri, then I will not eat rice.

1. Write it as a Boolean combination of atomic propositions.
2. Write its converse, negation and contra-positive, both in plain english and formally.

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Consider again...

$$(n+1)(n-1) = (n^2 - 1)$$

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- $\forall n \qquad (n+1)(n-1) = (n^2-1)$
- $\forall x, \exists y, \qquad x = y + 8$

- $\forall n$ stands for all values of n in a given domain
- $\exists n$ stands for exists n

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Can all mathematical statements be written as above?

Consider again...

- $\forall n \in \mathbb{N}\ (n+1)(n-1) = (n^2-1)$
- $\forall x, \exists y, x, y \in \mathbb{Z}\ x = y + 8$

- $\forall n$ stands for all values of n in a given domain
- $\exists n$ stands for exists n
- $\in$ is the element of symbol
- $\mathbb{N}$ stands for all natural numbers
- $\mathbb{Z}$ stands for all integers
- $\mathbb{R}$, $\mathbb{Q}$, ...

Some propositions are not so easy to "determine"...
– e.g., $2^{67} - 1$ is not a prime.

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  - $\forall x(L(x) \vee \exists y(L(y) \wedge F(x, y)))$ Fix Domain = students in this class; $L(x) = x$ has a laptop; $F(x, y) = x, y$ are friends.

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    - All students in this class have a laptop or have a friend who has a laptop.

A bit more about quantifiers

Negating Quantifiers

- What is the negation of “All students have MA105”?

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  There exists a student who does not have MA105.

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[1] $\equiv$ stands for "is logically equivalent to".

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  - $\neg\forall x P(x)$ is true iff $\forall x P(x)$ is false
  - $\forall x P(x)$ is false iff there is some x s.t. $\neg P(x)$ is true.
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Exercises

1. What is the negation of $\forall x(x^2 > x)$?
2. Show that $\neg\forall x(P(x) \to Q(x)) \equiv \exists x(P(x) \wedge \neg Q(x))$.

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1. What is the negation of $\forall x(x^2 > x)$? $\exists x(x^2 \leq x)$
2. Show that $\neg\forall x(P(x) \to Q(x)) \equiv \exists x(P(x) \wedge \neg Q(x))$.
3. Use predicates and quantifiers to say "Every email larger than 10MB will be compressed."

[1] $\equiv$ stands for "is logically equivalent to".

A bit more about quantifiers

Negating Quantifiers

- What is the negation of "All students have MA105"?

  There exists a student who does not have MA105.
- $\neg\forall x P(x) \equiv \exists x \neg P(x)$ [1]
  - $\neg\forall x P(x)$ is true iff $\forall x P(x)$ is false
  - $\forall x P(x)$ is false iff there is some x s.t. $\neg P(x)$ is true.
  - i.e., $\exists x \neg P(x)$.
- $\neg\exists x P(x) \equiv \forall x \neg P(x)$

Exercises

1. What is the negation of $\forall x(x^2 > x)$? $\exists x(x^2 \leq x)$
2. Show that $\neg\forall x(P(x) \rightarrow Q(x)) \equiv \exists x(P(x) \wedge \neg Q(x))$.
3. Use predicates and quantifiers to say "Every email larger than 10MB will be compressed." $\forall M(S(M, 10) \rightarrow C(M))$

[1] $\equiv$ stands for "is logically equivalent to".