CS 105: Department Introductory Course on Discrete Structures

Instructor : S. Akshay

Aug 25, 2025

Lecture 08 – Basic structures: Countable and Uncountable sets

Recap and Logistics

Last 07 lectures: What are Discrete Structures?

- Chapter 1: Proofs and mathematical reasoning (5 lectures, 3 problemsheets)
  - Propositions: statements that can be assigned a truth value
  - Predicates and Quantifiers
  - Theorems and types of proofs: contradiction, contrapositive
  - Induction, WOP, Strong Induction
- Chapter 2: Basic Mathematical Structures (2 lectures, 1 problemsheet)
  - Sets: Finite and infinite
  - Functions: injections, surjections and bijections

Recap and Logistics

Last 07 lectures: What are Discrete Structures?

- Chapter 1: Proofs and mathematical reasoning (5 lectures, 3 problemsheets)
  - Propositions: statements that can be assigned a truth value
  - Predicates and Quantifiers
  - Theorems and types of proofs: contradiction, contrapositive
  - Induction, WOP, Strong Induction
- Chapter 2: Basic Mathematical Structures (2 lectures, 1 problemsheet)
  - Sets: Finite and infinite
  - Functions: injections, surjections and bijections

Quiz 1 on Aug 28th, 3.45pm

- VENUE: LH 301, 302, LT 305 (Seating Arrangement on Piazza!)
- Syllabus: Everything till now!

Recap and Logistics

Last 07 lectures: What are Discrete Structures?

- Chapter 1: Proofs and mathematical reasoning (5 lectures, 3 problemsheets)
  - Propositions: statements that can be assigned a truth value
  - Predicates and Quantifiers
  - Theorems and types of proofs: contradiction, contrapositive
  - Induction, WOP, Strong Induction
- Chapter 2: Basic Mathematical Structures (2 lectures, 1 problemsheet)
  - Sets: Finite and infinite
  - Functions: injections, surjections and bijections

Quiz 1 on Aug 28th, 3.45pm

- VENUE: LH 301, 302, LT 305 (Seating Arrangement on Piazza!)
- Syllabus: Everything till now!
- Can assume what was taught in class & slides, but should state result clearly

Recap and Logistics

Last 07 lectures: What are Discrete Structures?

- Chapter 1: Proofs and mathematical reasoning (5 lectures, 3 problemsheets)
  - Propositions: statements that can be assigned a truth value
  - Predicates and Quantifiers
  - Theorems and types of proofs: contradiction, contrapositive
  - Induction, WOP, Strong Induction
- Chapter 2: Basic Mathematical Structures (2 lectures, 1 problemsheet)
  - Sets: Finite and infinite
  - Functions: injections, surjections and bijections

Quiz 1 on Aug 28th, 3.45pm

- VENUE: LH 301, 302, LT 305 (Seating Arrangement on Piazza!)
- Syllabus: Everything till now!
- Can assume what was taught in class & slides, but should state result clearly (unless question is exactly same!).

Recap and Logistics

Last 07 lectures: What are Discrete Structures?

- Chapter 1: Proofs and mathematical reasoning (5 lectures, 3 problemsheets)
  - Propositions: statements that can be assigned a truth value
  - Predicates and Quantifiers
  - Theorems and types of proofs: contradiction, contrapositive
  - Induction, WOP, Strong Induction
- Chapter 2: Basic Mathematical Structures (2 lectures, 1 problemsheet)
  - Sets: Finite and infinite
  - Functions: injections, surjections and bijections

Quiz 1 on Aug 28th, 3.45pm

- VENUE: LH 301, 302, LT 305 (Seating Arrangement on Piazza!)
- Syllabus: Everything till now!
- Can assume what was taught in class & slides, but should state result clearly (unless question is exactly same!). Stuff in problemsheets can't be assumed/must be reproved!

Recap: Sets, Functions and Countability

Similarities between finite and infinite sets

1. $\exists$ **bij** from A to B and B to C, implies $\exists$ **bij** from A to C.
2. $\exists$ **bij** from A to B, then $\exists$ **bij** from B to A.
3. (Schröder-Bernstein Theorem:) $\exists$ **inj** from A to B and $\exists$ **inj** B to A, implies $\exists$ **bij** from A to B.

Recap: Sets, Functions and Countability

Similarities between finite and infinite sets

1. $\exists$ **bij** from A to B and B to C, implies $\exists$ **bij** from A to C.
2. $\exists$ **bij** from A to B, then $\exists$ **bij** from B to A.
3. (Schröder-Bernstein Theorem:) $\exists$ **inj** from A to B and $\exists$ **inj** B to A, implies $\exists$ **bij** from A to B.

Differences between finite and infinite sets

1. Let A be a set, $b \notin A$. A is infinite iff there is a bijection from A to $A \cup \{b\}$.
2. There exist infinite sets A, B, $A \subsetneq B$, s.t there is a bijection from A to B.
3. From any infinite set A, there is a surjection from A to $\mathbb{N}$.

Recap: Sets, Functions and Countability

Similarities between finite and infinite sets

1. $\exists$ **bij** from A to B and B to C, implies $\exists$ **bij** from A to C.
2. $\exists$ **bij** from A to B, then $\exists$ **bij** from B to A.
3. (Schröder-Bernstein Theorem:) $\exists$ **inj** from A to B and $\exists$ **inj** B to A, implies $\exists$ **bij** from A to B.

Differences between finite and infinite sets

1. Let A be a set, $b \notin A$. A is infinite iff there is a bijection from A to $A \cup \{b\}$.
2. There exist infinite sets A, B, $A \subsetneq B$, s.t there is a bijection from A to B.
3. From any infinite set A, there is a surjection from A to $\mathbb{N}$.

Countable and countably infinite sets

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Recap: Properties of countable sets

Definition

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Recap: Properties of countable sets

Definition

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Properties of countable sets

1. To show that an (infinite) set S is countable, it is enough to show:
   - there is an injection from S to $\mathbb{N}$

Recap: Properties of countable sets

Definition

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Properties of countable sets

1. To show that an (infinite) set S is countable, it is enough to show:
   - there is an injection from S to $\mathbb{N}$
   - or there is a surjection from $\mathbb{N}$ (or any countable set) to S.

Recap: Properties of countable sets

Definition

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Properties of countable sets

1. To show that an (infinite) set S is countable, it is enough to show:
   - there is an injection from S to $\mathbb{N}$
   - or there is a surjection from $\mathbb{N}$ (or any countable set) to S.
2. The union of two countable sets is countable.

Recap: Properties of countable sets

Definition

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Properties of countable sets

1. To show that an (infinite) set S is countable, it is enough to show:
   - there is an injection from S to $\mathbb{N}$
   - or there is a surjection from $\mathbb{N}$ (or any countable set) to S.
2. The union of two countable sets is countable.
3. The cartesian product of two countable sets is countable.

Recap: Properties of countable sets

Definition

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Properties of countable sets

1. To show that an (infinite) set S is countable, it is enough to show:
   - there is an injection from S to $\mathbb{N}$
   - or there is a surjection from $\mathbb{N}$ (or any countable set) to S.
2. The union of two countable sets is countable.
3. The cartesian product of two countable sets is countable.

Today's Lecture

Are all infinite sets countable?

Recap: Properties of countable sets

Definition

- Set C is countably infinite iff there exists a bijection from C to $\mathbb{N}$. It is countable if it is finite or countably infinite.

Properties of countable sets

1. To show that an (infinite) set S is countable, it is enough to show:
   - there is an injection from S to $\mathbb{N}$
   - or there is a surjection from $\mathbb{N}$ (or any countable set) to S.
2. The union of two countable sets is countable.
3. The cartesian product of two countable sets is countable.

Today's Lecture

Are all infinite sets countable?

But before that...

Pop Quiz?

Exercises

1. Show that set of primes P is countable.

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Pop Quiz?

Exercises

1. Show that set of primes P is countable.

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Pop Quiz?

Exercises

1. Show that set of primes P is countable.
   - Proof 1

     1.1 In Lecture 1, we showed that set of primes is infinite.

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Pop Quiz?

Exercises

1. Show that set of primes P is countable.
   - Proof 1
     1.1 In Lecture 1, we showed that set of primes is infinite.
     1.2 So there is a surjection f from P to $\mathbb{N}$.

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Pop Quiz?

Exercises

1. Show that set of primes P is countable.
   - Proof 1
     1.1 In Lecture 1, we showed that set of primes is infinite.
     1.2 So there is a surjection f from P to $\mathbb{N}$.
     1.3 Which implies there is an injection g from $\mathbb{N}$ to P (why?)

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Pop Quiz?

Exercises

1. Show that set of primes P is countable.
   - Proof 1
     1.1 In Lecture 1, we showed that set of primes is infinite.
     1.2 So there is a surjection f from P to $\mathbb{N}$.
     1.3 Which implies there is an injection g from $\mathbb{N}$ to P (why?)
     1.4 Now, $P \subseteq \mathbb{N}$. So $Id : P \to \mathbb{N}$ is an injection.

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Pop Quiz?

Exercises

1. Show that set of primes P is countable.
   - Proof 1
     1.1 In Lecture 1, we showed that set of primes is infinite.
     1.2 So there is a surjection f from P to $\mathbb{N}$.
     1.3 Which implies there is an injection g from $\mathbb{N}$ to P (why?)
     1.4 Now, $P \subseteq \mathbb{N}$. So $Id : P \to \mathbb{N}$ is an injection.
     1.5 So by Schroder-Bernstein Theorem there is a bijection from P to $\mathbb{N}$.

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Pop Quiz?

Exercises

1. Show that set of primes P is countable.
   - Proof 1
     1.1 In Lecture 1, we showed that set of primes is infinite.
     1.2 So there is a surjection f from P to $\mathbb{N}$.
     1.3 Which implies there is an injection g from $\mathbb{N}$ to P (why?)
     1.4 Now, $P \subseteq \mathbb{N}$. So $Id : P \to \mathbb{N}$ is an injection.
     1.5 So by Schroder-Bernstein Theorem there is a bijection from P to $\mathbb{N}$.
   - Proof 2 Show $f : P \to \mathbb{N}$ by f maps i^{th} prime to i is a bijection
2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Countable sets and functions

Are the following sets countable?

- the set of all integers $\mathbb{Z}$
- $\mathbb{N} \times \mathbb{N}$
- $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$
- the set of rationals $\mathbb{Q}$
- the set of all (finite and infinite) subsets of $\mathbb{N}$
- the set of all real numbers $\mathbb{R}$

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

- Proving existence just needs one to exhibit a function
- But how do we prove non-existence?

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

- Proving existence just needs one to exhibit a function
- But how do we prove non-existence? Try contradiction.

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

Proof by contradiction: Suppose there is such a bijection, say f. This would imply that each $i \in \mathbb{N}$ maps to some set $f(i) \subseteq \mathbb{N}$.

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

Proof by contradiction: Suppose there is such a bijection, say f. This would imply that each $i \in \mathbb{N}$ maps to some set $f(i) \subseteq \mathbb{N}$.

	0	1	2	3	...
$f(0)$	✓ ×	×	×	×	...
$f(1)$	✓	✗ ✓	✓	✓	...
$f(2)$	×	×	✗ ✓	×	...
$f(3)$	×	✓	×	✓ ×	...

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

Proof by contradiction: Suppose there is such a bijection, say f. This would imply that each $i \in \mathbb{N}$ maps to some set $f(i) \subseteq \mathbb{N}$.

	0	1	2	3	...
$f(0)$	✓ ×	×	×	×	...
$f(1)$	✓	✗ ✓	✓	✓	...
$f(2)$	×	×	✗ ✓	×	...
$f(3)$	×	✓	×	✓ ×	...

- Consider the set $S \subseteq \mathbb{N}$ obtained by switching the diagonal elements, i.e., $S = \{i \in \mathbb{N} \mid i \notin f(i)\}$.

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

Proof by contradiction: Suppose there is such a bijection, say f. This would imply that each $i \in \mathbb{N}$ maps to some set $f(i) \subseteq \mathbb{N}$.

	0	1	2	3	...
$f(0)$	✓ ×	×	×	×	...
$f(1)$	✓	✗ ✓	✓	✓	...
$f(2)$	×	×	✗ ✓	×	...
$f(3)$	×	✓	×	✓ ×	...

- Consider the set $S \subseteq \mathbb{N}$ obtained by switching the diagonal elements, i.e., $S = \{i \in \mathbb{N} \mid i \notin f(i)\}$.
- As f is bij, $\exists j \in \mathbb{N}, f(j) = S$.

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

Proof by contradiction: Suppose there is such a bijection, say f. This would imply that each $i \in \mathbb{N}$ maps to some set $f(i) \subseteq \mathbb{N}$.

	0	1	2	3	...
$f(0)$	✓ ×	×	×	×	...
$f(1)$	✓	✗ ✓	✓	✓	...
$f(2)$	×	×	✗ ✓	×	...
$f(3)$	×	✓	×	✗ ×	...

- Consider the set $S \subseteq \mathbb{N}$ obtained by switching the diagonal elements, i.e., $S = \{i \in \mathbb{N} \mid i \notin f(i)\}$.
- As f is bij, $\exists j \in \mathbb{N}, f(j) = S$.
- S and $f(j)$ differ at position j, for any j.

Comparing $\mathbb{N}$ and set of all subsets of $\mathbb{N}$

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

Proof by contradiction: Suppose there is such a bijection, say f. This would imply that each $i \in \mathbb{N}$ maps to some set $f(i) \subseteq \mathbb{N}$.

	0	1	2	3	...
$f(0)$	✓ ×	×	×	×	...
$f(1)$	✓	✗ ✓	✓	✓	...
$f(2)$	×	×	✗ ✓	×	...
$f(3)$	×	✓	×	✓ ×	...

- Consider the set $S \subseteq \mathbb{N}$ obtained by switching the diagonal elements, i.e., $S = \{i \in \mathbb{N} \mid i \notin f(i)\}$.
- As f is bij, $\exists j \in \mathbb{N}, f(j) = S$.
- S and $f(j)$ differ at position j, for any j.
- Thus, $S \neq f(j)$ for all $j \in \mathbb{N}$, which is a contradiction! □

Cantor's diagonalization

Does this proof look familiar??

Cantor's diagonalization

Does this proof look familiar??

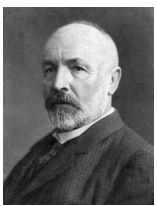
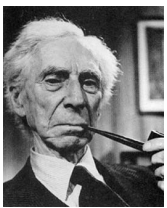



Figure: Cantor and Russell

Cantor's diagonalization

Does this proof look familiar??

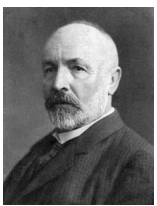
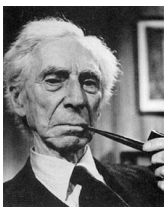



Figure: Cantor and Russell

- $S = \{i \in \mathbb{N} \mid i \notin f(i)\}$ is like the one from Russell's paradox.

Cantor's diagonalization

Does this proof look familiar??

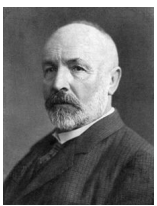
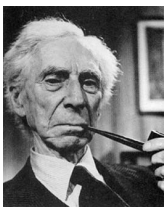



Figure: Cantor and Russell

- $S = \{i \in \mathbb{N} \mid i \notin f(i)\}$ is like the one from Russell's paradox.
- If $\exists j \in \mathbb{N}$ such that $f(j) = S$, then we have a contradiction.
  - If $j \in S$, then $j \notin f(j) = S$.
  - If $j \notin S$, then $j \notin f(j)$, which implies $j \in S$.

Cantor's diagonalization

Does this proof look familiar??

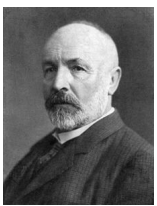
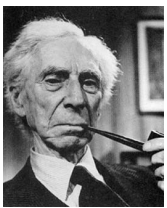



Figure: Cantor and Russell

In fact, using diagonalization Cantor showed that...

- There cannot be a bijection between any set and its power set (i.e., its set of subsets).(H.W)
- So there is an infinite hierarchy of "larger" infinities...

Cantor's diagonalization

Does this proof look familiar??

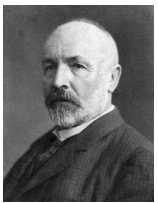
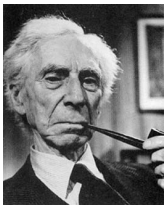



Figure: Cantor and Russell

In fact, using diagonalization Cantor showed that...

- There cannot be a bijection between any set and its power set (i.e., its set of subsets).(H.W)
- So there is an infinite hierarchy of "larger" infinities...
- There is no bijection from $\mathbb{R}$ to $\mathbb{N}$ (H.W). Moreover, there is a bijection from $\mathbb{R}$ to set of subsets of $\mathbb{N}$.

One infinity is “strictly” bigger than another!

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

One infinity is "strictly" bigger than another!

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

- But, there is a surjection from set of all subsets of $\mathbb{N}$ to $\mathbb{N}$.
- Thus, the "size" of $\mathcal{P}(\mathbb{N})$ is strictly greater that $\mathbb{N}$!

One infinity is "strictly" bigger than another!

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

- But, there is a surjection from set of all subsets of $\mathbb{N}$ to $\mathbb{N}$.
- Thus, the "size" of $\mathcal{P}(\mathbb{N})$ is strictly greater that $\mathbb{N}$!

Can there be some set whose "size" is in between the two?

One infinity is "strictly" bigger than another!

Theorem (Cantor, 1891)

There is no bijection between $\mathbb{N}$ and the set of all subsets of $\mathbb{N}$.

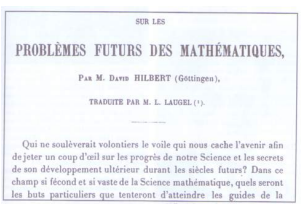
- But, there is a surjection from set of all subsets of $\mathbb{N}$ to $\mathbb{N}$.
- Thus, the "size" of $\mathcal{P}(\mathbb{N})$ is strictly greater that $\mathbb{N}$!

Can there be some set whose "size" is in between the two?

Cantor's Continuum hypothesis

There is no set whose "cardinality" is strictly between $\mathbb{N}$ and $\mathcal{P}(\mathbb{N})$





SUR LES

PROBLÈMES FUTURS DES MATHÉMATIQUES,

PAR M. DAVID HILBERT (Göttingen),

TRADUITE PAR M. L. LAUGEL (1).

Qui ne soulèverait volontiers le voile qui nous cache l'avenir afin de jeter un coup d'œil sur les progrès de notre Science et les secrets de son développement ultérieur durant les siècles futurs? Dans ce champ si fécond et si vaste de la Science mathématique, quels seront les buts particuliers que tenteront d'atteindre les guides de la

Hilbert's lecture from the Paris Proceedings

Figure: 1st of Hilbert's 23 problems for the 20th century in 1900.

What did the world think about these proofs (in 1890s?)



(a) Kronecker



(b) Poincare



(c) Theologians

- Kronecker: Only constructive proofs are proofs! "Scientific Charlatan", "Corruptor of youth"!
- Poincare: Set theory is a "disease" from which mathematics will be cured.
- Christian Theologians: God=Uniqueness of an absolute infinity. So, what is all this different infinities...?!

What did the world think about these proofs (in 1890s?)



(a) Kronecker



(b) Poincare



(c) Theologians

- Kronecker: Only constructive proofs are proofs! "Scientific Charlatan", "Corruptor of youth"!
- Poincare: Set theory is a "disease" from which mathematics will be cured.
- Christian Theologians: God=Uniqueness of an absolute infinity. So, what is all this different infinities...?!
- Hilbert: No one can expel us from the paradise that Cantor has created for us.

Summary and moving on...

- Finite and infinite sets.
- Using functions to compare sets: focus on bijections.
- Countable, countably infinite and uncountable sets.
- Cantor's diagonalization argument (A new powerful proof technique!).

Summary and moving on...

- Finite and infinite sets.
- Using functions to compare sets: focus on bijections.
- Countable, countably infinite and uncountable sets.
- Cantor's diagonalization argument (A new powerful proof technique!).

Next: Basic Mathematical Structures – Relations

Relations

Definition: Function

Let A, B be two sets. A function f from A to B is a subset R of $A \times B$ such that

(i) $\forall a \in A$, $\exists b \in B$ such that $(a, b) \in R$, and

(ii) if $(a, b) \in R$ and $(a, c) \in R$, then $b = c$.

- Now, suppose A is the set of all Btech students and B is the set of all courses. Clearly, we can assign to each student the courses he/she is taking. Is this a function?

Relations

Definition: Function

Let A, B be two sets. A function f from A to B is a subset R of $A \times B$ such that

(i) $\forall a \in A$, $\exists b \in B$ such that $(a, b) \in R$, and

(ii) if $(a, b) \in R$ and $(a, c) \in R$, then $b = c$.

- Now, suppose A is the set of all Btech students and B is the set of all courses. Clearly, we can assign to each student the courses he/she is taking. Is this a function?
- By removing the two extra assumptions in the defn, we get:

Definition: Relation

- A relation R from A to B is a subset of $A \times B$. If $(a, b) \in R$, we also write this as $a \ R \ b$.
- Thus, a relation is a way to relate the elements of two (not necessarily different) sets.

Examples and representations of relations

We write $R(A, B)$ for a relation from A to B and just $R(A)$ if $A = B$. Also if A is clear from context, we just write R.

Examples of relations

- All functions are relations.
- $R_1(\mathbb{Z}) = \{(a, b) \mid a, b \in \mathbb{Z}, a - b \text{ is even } \}$.
- $R_2(\mathbb{Z}) = \{(a, b) \mid a, b \in \mathbb{Z}, a \leq b\}$.
- Let S be a set, $R_3(\mathcal{P}(S)) = \{(A, B) \mid A, B \subseteq S, A \subseteq B\}$.
- Relational databases are practical examples.

Examples and representations of relations

We write $R(A, B)$ for a relation from A to B and just $R(A)$ if $A = B$. Also if A is clear from context, we just write R.

Examples of relations

- All functions are relations.
- $R_1(\mathbb{Z}) = \{(a, b) \mid a, b \in \mathbb{Z}, a - b \text{ is even } \}$.
- $R_2(\mathbb{Z}) = \{(a, b) \mid a, b \in \mathbb{Z}, a \leq b\}$.
- Let S be a set, $R_3(\mathcal{P}(S)) = \{(A, B) \mid A, B \subseteq S, A \subseteq B\}$.
- Relational databases are practical examples.

Representations of a relation from A to B.

- As a set of ordered pairs of elements, i.e., subset of $A \times B$.
- As a directed graph.
- As a (database) table.

Use of relations

Practical application in relational databases: IMDB, university records, etc.

Use of relations

Practical application in relational databases: IMDB, university records, etc.

But, why study relations in this course?

Use of relations

Practical application in relational databases: IMDB, university records, etc.

But, why study relations in this course?

- Functions were special kinds of relations that were useful to compare sets.

Use of relations

Practical application in relational databases: IMDB, university records, etc.

But, why study relations in this course?

- Functions were special kinds of relations that were useful to compare sets.
- Are there other special relations?What are they useful for?

Use of relations

Practical application in relational databases: IMDB, university records, etc.

But, why study relations in this course?

- Functions were special kinds of relations that were useful to compare sets.
- Are there other special relations?What are they useful for?
  - Equivalence relations
  - Partial orders