CS310 : Automata Theory 2019

Lecture 23: Turing Machines and Computability

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04-03-2019

Topics covered till date

I. Finite state automata

1. Deterministic finite-state automata (DFA)
2. Non-determinism, the subset construction, ϵ-transitions.
3. Regular expressions and equivalence with DFA
4. Regular languages and their properties
5. Pumping lemma
6. Myhill-Nerode and minimization

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II. Finite state automata with a stack

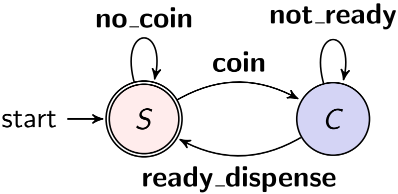
1. Pushdown automata (PDA)
2. Context-free grammars (CFG) and Parse trees
3. Equivalence of PDA and CFG, deterministic PDA
4. Chomsky normal form, Pumping lemma for CFLs
5. Applications of CFGs to parsing.

Today's lecture

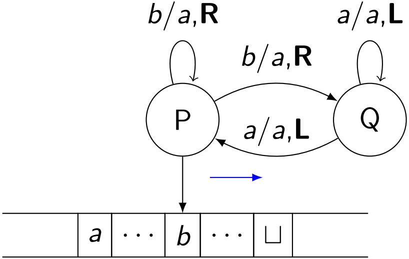
Turing Machines

From finite to infinite memory

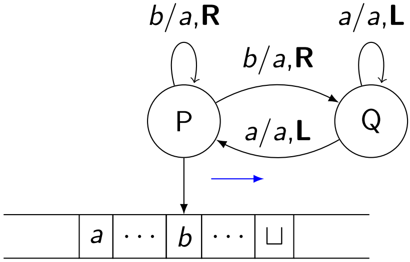
Finite instruction machine with finite memory (Finite State Automata)



Finite instruction machine with unbounded memory (Turing machine)



From NFA and PDA to Turing Machines



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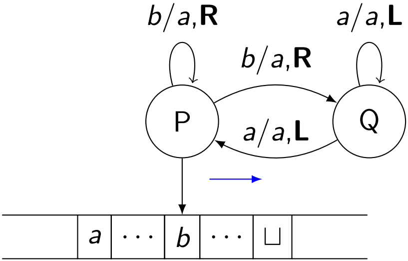

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4. The read-write head can move both to left and right.

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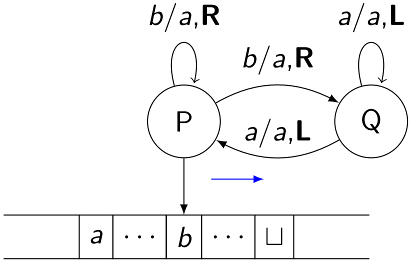

1. PDA are obtained from NFA by adding a stack
2. Turing machines are obtained from NFA by adding an infinite tape.
3. A Turing machine can both write on the tape and read from it.
4. The read-write head can move both to left and right.
5. Initially all cells have special blank symbol, except where input is written.
6. Once accept/reject states are reached, computation terminates.

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In general, we use the finite control to process the symbols that are being read on tape. How does one formalize this?

Formal definition of a Turing Machine

Definition

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$ where

1. Q is a finite set of states,
2. Σ is a finite input alphabet,
3. Γ is a finite tape alphabet where $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$,
4. q_0 is the start state,
5. q_{acc} is the accept state,
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7. $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the transition function.

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For a $q \in Q$, $a \in \Gamma$ if $\delta(q, a) = (q', b, L)$, then

- q' is the new state of the machine,
- b is the letter which replaces a on the tape,
- the head moves to Left of the current position.

Example: TM for $B = \{w\#w \mid w \in \{0,1\}^*\}$?

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From Sipser's book: What is $Q, \Sigma, \Gamma, \delta$, etc.?