Analyzing Timed Systems Using Tree Automata

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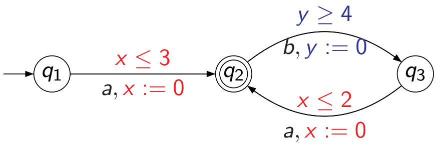
Timed automata and timed runs

$$q_1 \xrightarrow[a,\ x := 0]{x \leq 3} q_2$$

Timed automata and timed runs



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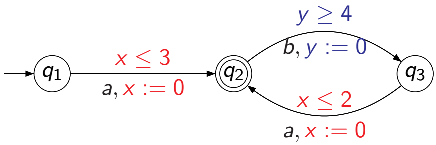

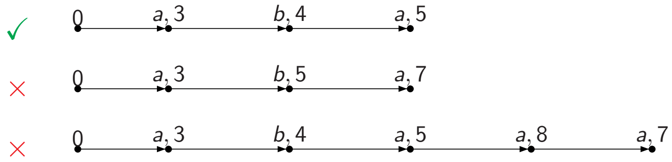

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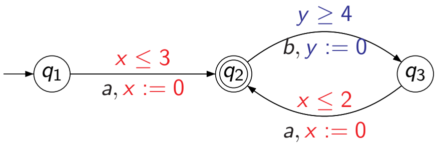

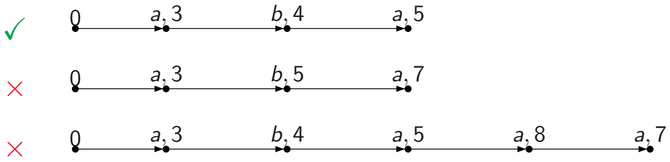

- The timed language $\mathcal{L}_T(\mathcal{A})$ = set of such good timed words
- Emptiness problem : Given $\mathcal{A}$, is $\mathcal{L}_T(\mathcal{A}) = \emptyset$?

Emptiness for timed automata

A well-studied problem with a now standard approach

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- Timed automata: Region construction [Alur-Dill'90], and many optimizations since...

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- An orthogonal approach: [Clemente-Lasota 2015]

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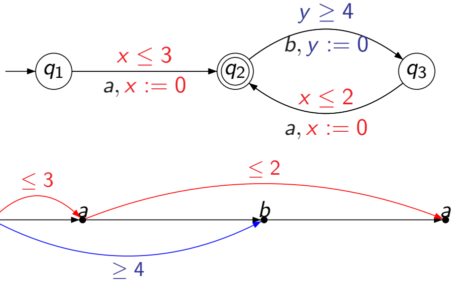
Our point-de-depart

- represent behaviors as graphs with timing constraints
- use tree interpretations to reduce to tree automata
  - A higher level and more powerful formalism
  - Yields simpler proofs for more complicated systems
  - A new technique which does not depend on regions/zones

Outline

1. Timed behaviours as graphs
2. Checking realizability
3. Interpreting graphs into trees
4. Bounding the (split-)width of graphs
5. Conclusion & future work

Abstracting paths of a timed system as graphs



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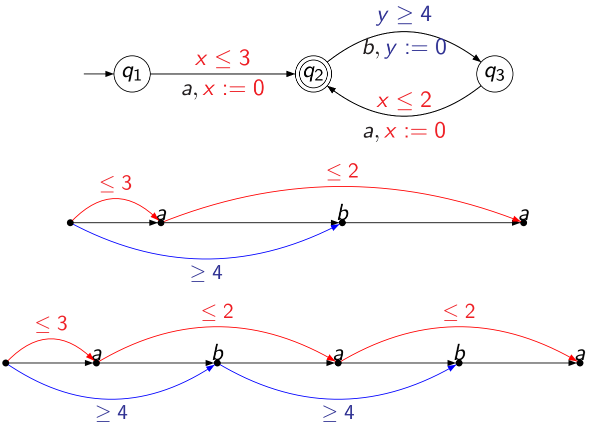

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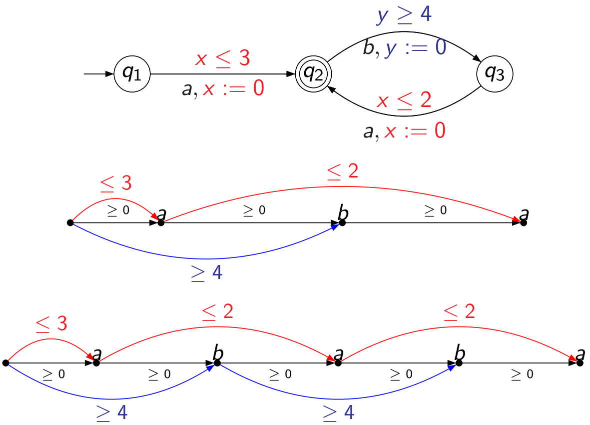

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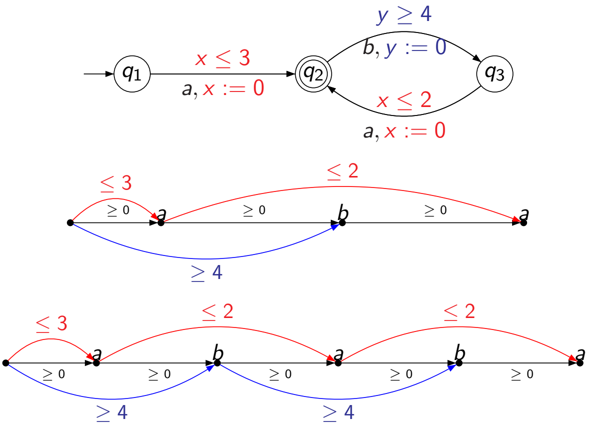

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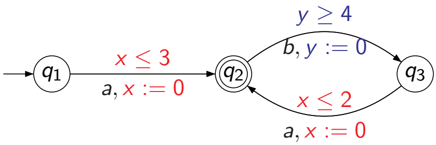

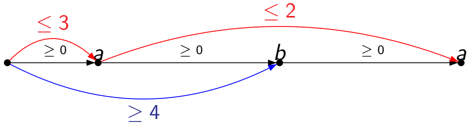

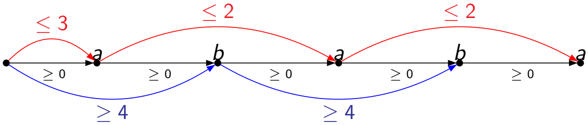

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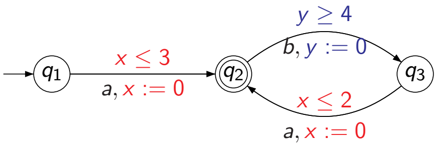

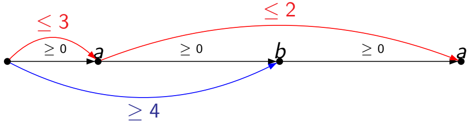

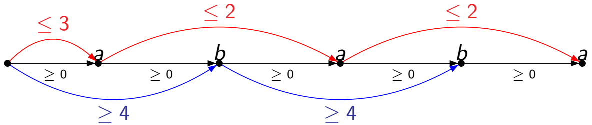

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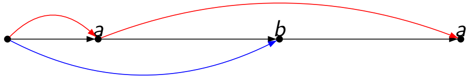
  - What are some properties of such graphs?
  - What is the link between $\mathcal{L}_{TCW}(\mathcal{A})$ and $\mathcal{L}_T(\mathcal{A})$?

TC-words and their relation to timed words

Properties of TC-words and timed words

1. Not all (linearly-ordered) graphs are TC-words



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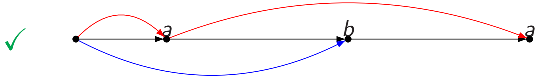

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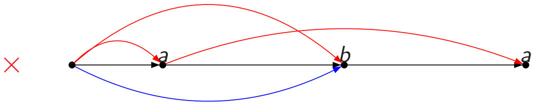

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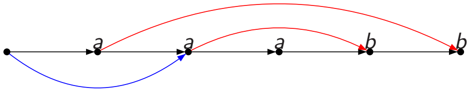

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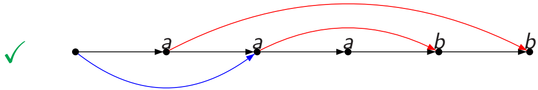

This graph cannot be generated by any timed automaton.
But, it can be generated by a timed pushdown automaton!.

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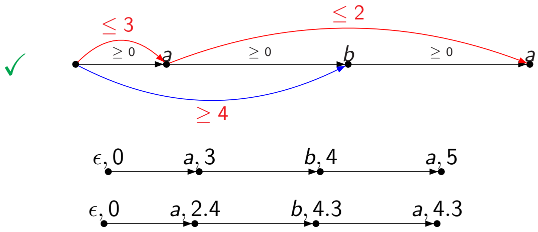

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Properties of TC-words and timed words

1. Not all (linearly-ordered) graphs are TC-words
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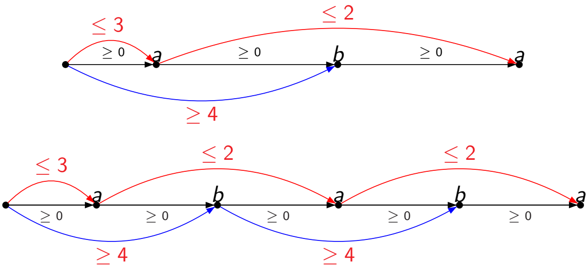
3. However, a TC-word may be realized by no timed word too!



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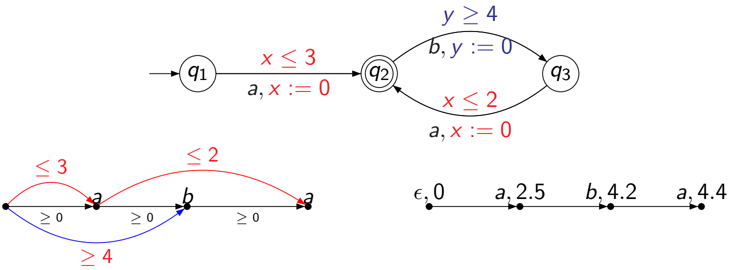
Difference between $\mathcal{L}_{TCW}(\mathcal{A})$ and $\mathcal{L}_T(\mathcal{A})$:



- $\mathcal{L}_{TCW}(\mathcal{A})$ is over a finite alphabet, while $\mathcal{L}_T(\mathcal{A})$ is not.

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For a given timed (pushdown) automaton $\mathcal{A}$,
$\mathcal{L}_T(\mathcal{A}) \neq \emptyset$ iff there exists a realizable TC-word in $\mathcal{L}_{TCW}(\mathcal{A})$.

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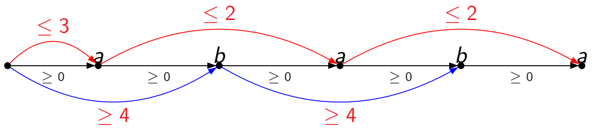
Thus, the question is: how to reason about these graphs?

Checking realizability of a single TC-word



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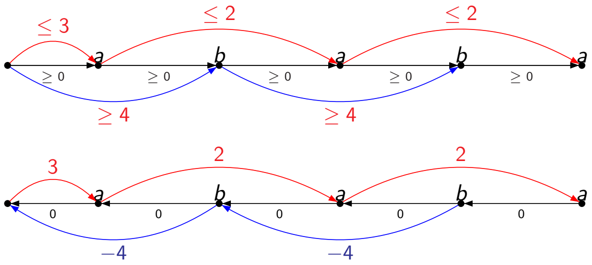

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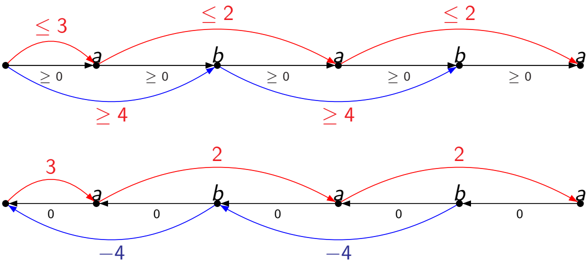

A simple exercise

A TC-word is realizable iff its directed graph has no negative cycle.

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For a given timed (pushdown) automaton $\mathcal{A}$,

Does there exist a TC-word in $\mathcal{L}_{TCW}(\mathcal{A})$, whose directed graph has no negative cycle?

- How to reason about the set of graphs $\mathcal{L}_{TCW}(\mathcal{A})$?

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  Same strategy as [Madhusudan & Parlato'11, Aiswarya et al '12] for untimed pushdown systems.

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- Step 1: graphs from T(PD)A have a bounded (split-)width.

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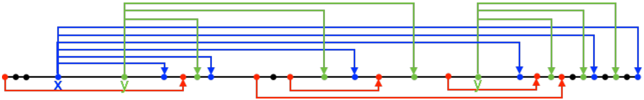

Graphs from timed systems are different!

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- Step 1: graphs from T(PD)A have a bounded (split-)width.
- Step 2: directly build a finite bottom-up tree automaton.

Complexity bounds

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Lift to timed multi-pushdown systems with bounded rounds

- Easy generalization, new decidability result & complexity too!

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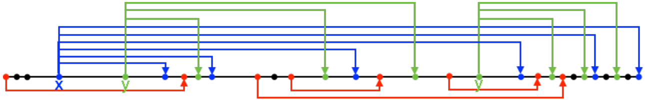

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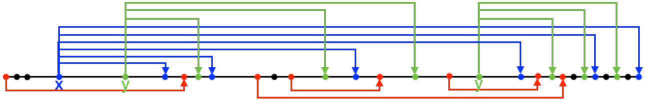

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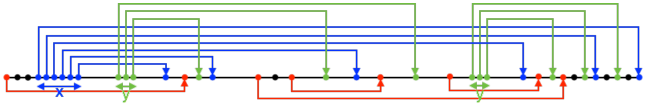

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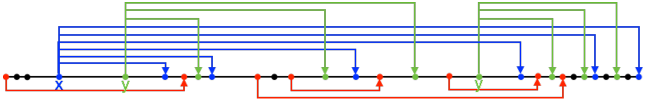

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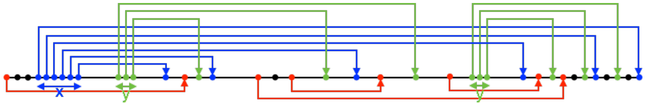
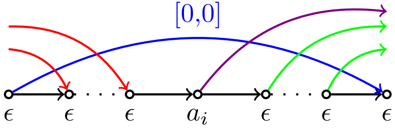
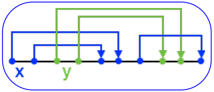


- To maintain atomicity, we use a single extra clock & add a constraint to each event:



Step 1: Split-width for timed systems

Now, define split game (see [Aiswarya et. al.'12, '15])...



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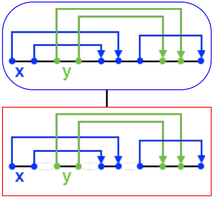

- Eve tries to disconnect the graph by cutting process edges.
- Positions are simple TC-words with holes.

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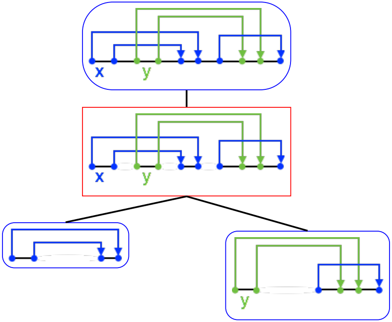

- Eve tries to disconnect the graph by cutting process edges.
- Positions are simple TC-words with holes.
- Adam chooses which connected component to continue.

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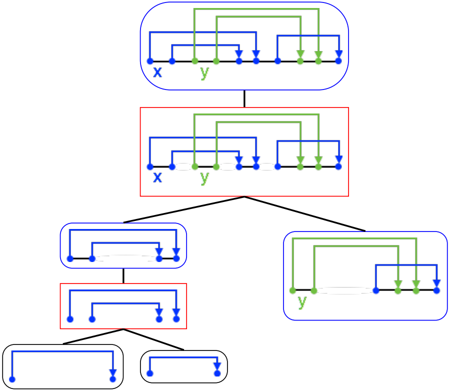

- Game ends at atomic nodes (no process edges left).

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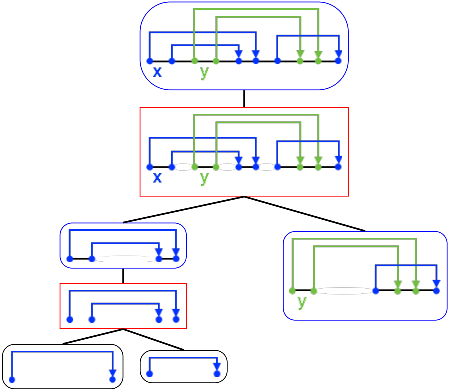

- Width of such a split simple TC-word = no. of blocks in it.
- Cost of play = max width of split TC-word seen along play.
- Split-width = min cost that Eve can achieve.

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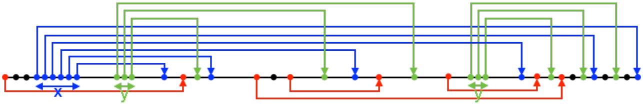
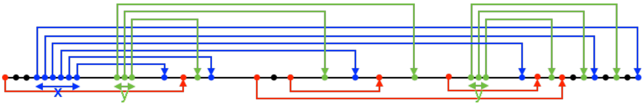
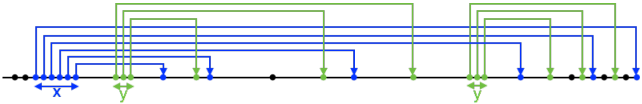


Step 1: Split-width for timed systems

- To bound: split-width of any well-formed simple TC-word, i.e., graph from a timed (pushdown) automaton.
- Let's play the game...



Split-width for timed automata



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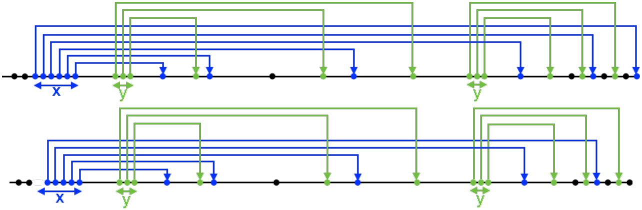

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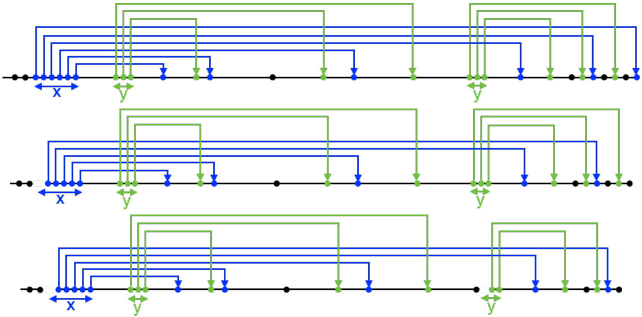

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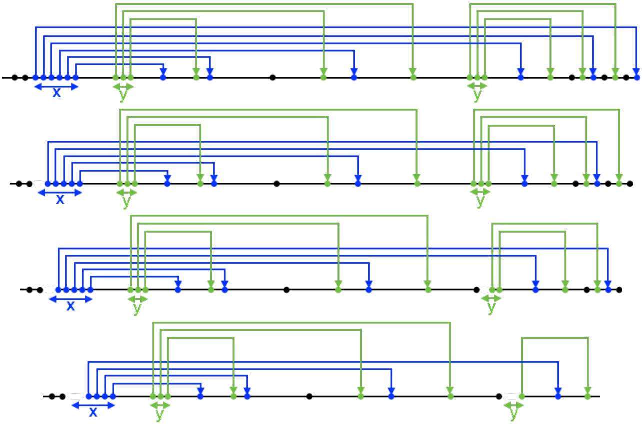

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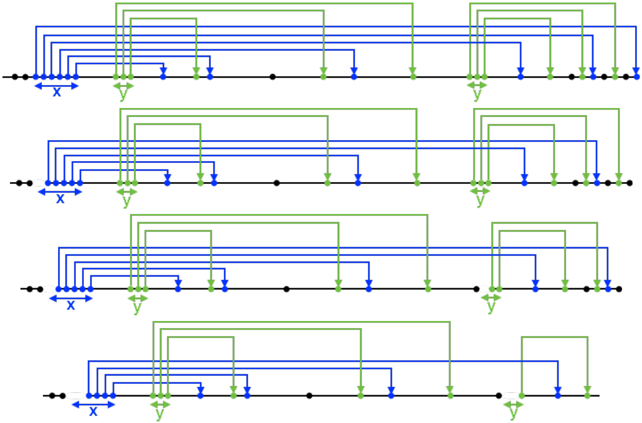

For any TC-word of a timed automaton

In any move of the game, we have:

- Each hole is attached to last reset of a clock, holes only widen!

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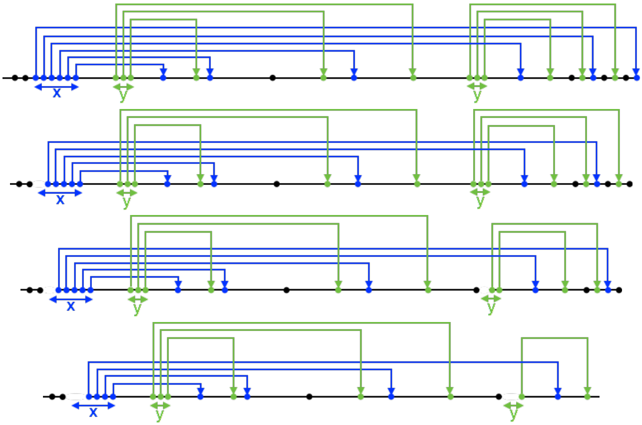

For any TC-word of a timed automaton

In any move of the game, we have:

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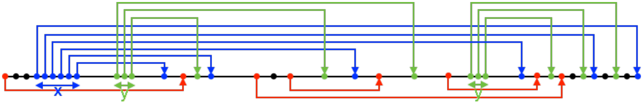
- Thus, no. of blocks $\leq$ No. of clocks $+ 4$.

Split-width for timed pushdown automata



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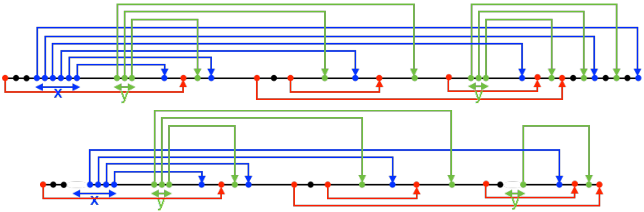

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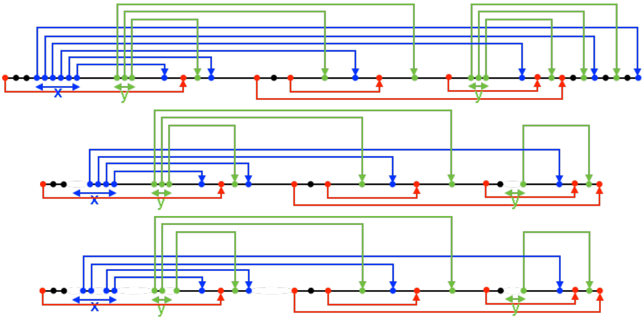

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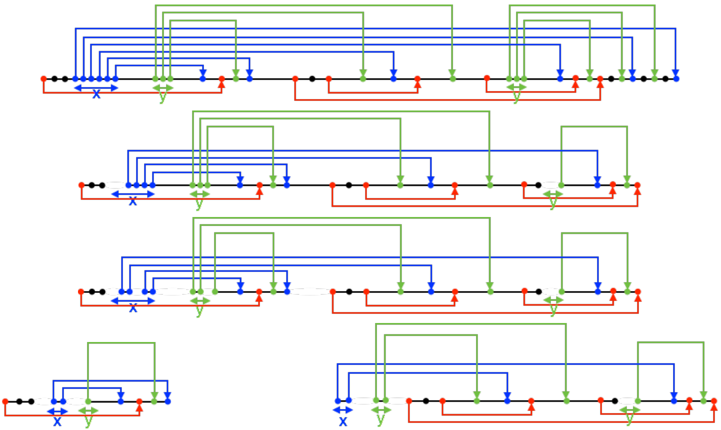

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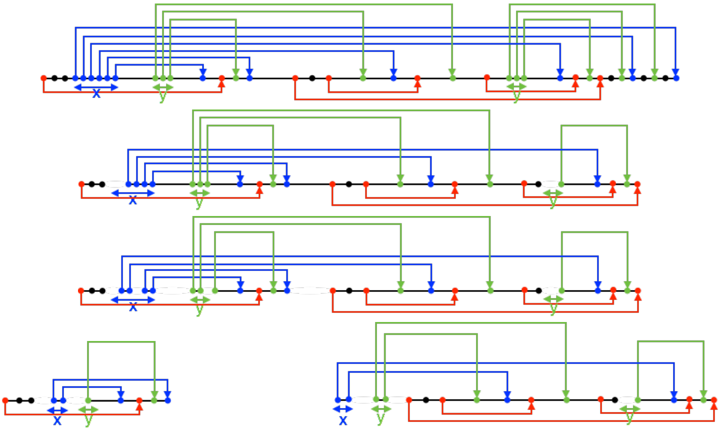

For any TC-word of a timed pushdown automaton

In any move of the game, we have:

- Number of blocks $\leq 4\cdot$ No. of clocks $+ 6$.

Conclusion and Future work

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Future work

- Concurrent recursive timed programs
- MSO definability of realizability
- Going beyond emptiness. What about model-checking?