

PROJECT REPORT

STAGE -II

SCIENTIFIC CALCULATOR

GROUP – 14

SLOT – 6

TEAM MEMBERS:-

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DISRIBUTION OF WORK:-

- T SAI GOUTAM - Entire coding part apart from graphics, prepare SRS and Project Report.
- TRISHALA BOTHRA - Improvement of code and additional code. Preparation of Draft User Manual.
- SANSKAR JAIN - Entire graphics code.

INTRODUCTION :-

Scientific Calculator is basically an electronic calculator which is basically designed to calculate problems in science, engineering and mathematics. They have applications in almost all the current fields of science and technology and widely used in both education and professional settings.

BASIS OF THE PROJECT-

The project is mainly based on most of the operations which are done by a scientific calculator, without using the cmath library. Simplecpp is used in the graphics part of this project.

APPLICATIONS OF A SCIENTIFIC CALCULATOR-

Scientific calculators are used widely in any situation where quick access to certain mathematical functions is needed such as trigonometric functions, inverse functions, etc. which were once traditionally looked into tables assigned to them. Scientific calculator helps greatly in solving complex numericals which if done manually take up a whole lot of time.

BASIC FUNCTIONS OF A CALCULATOR-

The functions which can be done using this scientific calculator are:-

1. Basic operations like Addition, Subtraction, Multiplication and Division:

Basic operations can be carried out easily and directly on real numbers.

2. Trigonometric functions like Sinx, Cosx and Tanx:

Sine, Cosine function values are found out using Taylor series expansion and $\text{Sin}x/\text{Cos}x$ gives $\text{Tan}x$. The Taylor series for sine function is valid only between $-\pi$ to π . Since the functions sine, cosine are periodic we can use these properties to find the sine value. Cosine value is found using the standard trigonometric identity.

$$(\text{Sin}x)^2 + (\text{Cos}x)^2 = 1$$

$\text{Tan}x$ is found by dividing $\text{Sin}x$ and $\text{Cos}x$.

$$\text{Tan}x = \frac{\text{Sin}x}{\text{Cos}x}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

3. Inverse trigonometric functions like arc sin, arc cos and arc tan:-

Arc sin function is found out using Taylor's series expansion and the values of x must remain in the domain.

$$\arcsin z = z + \left(\frac{1}{2}\right) \frac{z^3}{3} + \left(\frac{1 \cdot 3}{2 \cdot 4}\right) \frac{z^5}{5} + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right) \frac{z^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{\binom{2n}{n} z^{2n+1}}{4^n (2n+1)}$$

arc cosx is found by the expression $\arccos x = \pi/2 - \arcsin x$

arc tanx is found using Taylor's series expression.

$$\arctan z = z - \frac{z^3}{3} + \frac{z^5}{5} - \frac{z^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{2n+1}; \quad |z| \leq 1 \quad z \neq i, -i$$

4. Hyperbolic functions like sinh, cosh, tanh:-

Sinhx, coshx and tanhx can be found using the fundamental definition given below.

$$\text{Sinh}x = \frac{e^x - e^{-x}}{2}$$

$$\text{Cosh}x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{Tanh}x = \frac{\operatorname{Sinh}x}{\operatorname{Cosh}x}$$

5. Inverse hyperbolic functions like arc sinh, arc tanh, arc cosh:-

arcsinh , arccosh and arctanh are obtained by equations given below.

$$\operatorname{arsinh} z = \ln(z + \sqrt{z^2 + 1})$$

$$\operatorname{arcosh} z = \ln(z + \sqrt{z + 1}\sqrt{z - 1})$$

$$\operatorname{artanh} z = \frac{1}{2} \ln \left(\frac{1 + z}{1 - z} \right)$$

$$\operatorname{arcoth} z = \frac{1}{2} \ln \left(\frac{z + 1}{z - 1} \right)$$

$$\operatorname{arsch} z = \ln \left(\frac{1}{z} + \sqrt{\frac{1}{z^2} + 1} \right)$$

$$\operatorname{arsech} z = \ln \left(\frac{1}{z} + \sqrt{\frac{1}{z} + 1} \sqrt{\frac{1}{z} - 1} \right)$$

6. Exponential functions –

Exponential function is found out using Taylor expansion which approximated at higher powers.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, \quad -\infty < x < \infty$$

7. Logarithmic function –

Any positive real number is written in the form of mantissa, base and exponent where the base is 10. Logarithmic function is In function divided by ln 10.

8. In function –

Any positive real number is written in the form of mantissa, base and exponent where the base is e. The Ln of the mantissa is found using the expansion of $\ln(1-x)$. Then the Ln value of required positive real number is $\ln(1-x) + \text{exponent}$.

$$\log(1 - x) = - \sum_{n=1}^{\infty} \frac{x^n}{n} \quad \text{for } |x| < 1$$

9. Power function –

x power n can be also written as e to the power $n \cdot \ln x$. $\ln x$ and exponential functions are found out already. From this we can find out the power of any real number.

Let $y = x^n$ i.e. $\ln y = n \cdot \ln x$

$$\Rightarrow e^{\ln y} = e^{n \cdot \ln x}$$

$$\Rightarrow y = e^{n \cdot \ln x}$$

10. Basic operations on complex numbers –

Addition and subtraction can be done by adding or subtracting the respective real and imaginary parts.

Real part of multiplication of two complex numbers can be obtained by multiplying the two imaginary parts subtracted from the product of the two real parts. The imaginary part is obtained by multiplying the real part of first complex number and the imaginary part of second complex number and adding with the product of the real part of second complex number and the imaginary part of the first complex number.

Division is done by multiplying the complex number in the numerator by conjugate of second complex number.

Conjugate is obtained by reversing the sign of the imaginary part of the complex number.

Square root of the complex number is found out by using the standard formula.

$$\sqrt{z} = \sqrt{\frac{|z| + \operatorname{Re}(z)}{2}} + i \operatorname{sgn}(\operatorname{Im}(z)) \sqrt{\frac{|z| - \operatorname{Re}(z)}{2}}$$

11. Modulus functions –

For a complex number $z = x+iy$, the modulus is essentially given by square root of (x square + y square).

12. Inverse functions –

Inverse function of a given real number is given by 1 by x. Inverse of zero is not defined.

13. Conversion of degrees into radians –

One degree is equal to $\pi/180$ radians. According to this relation, any angle in degrees can be correspondingly converted to radians.

14. Factorial of whole numbers-

Factorial function is essentially found using the recursive call of function.

$$F(n)=n \cdot F(n-1)$$

$$F(1)=1$$

$$F(0)=1$$

15. Permutations (nPr) and combinations (nCr)-

nPr, nCr are found using the formulae given below

$$nPr = \frac{n!}{(n-r)!}$$

$$nCr = \frac{n!}{r!(n-r)!}$$

16. Solutions to quadratic equations:-

For the equation, $ax^2+bx+c=0$

Case(i):When $a=0$ This reduces the quadratic equation to a single variable linear equation.

$$bx+c=0$$

$$\Rightarrow x=-c/b \quad \text{when } b \neq 0 \quad \text{but when } b=0 \quad (=) \quad c=0$$

$$\text{Case(ii):When } a \neq 0 \quad x = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a} \quad \&$$

$$x = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}$$

When $(b^2 - 4ac) = 0$ we get two equal roots given by

$$x = -b/(2a).$$

17. Nature of roots of cubic equations-

For a given cubic equation with real co-efficients $ax^3+bx^2+cx+d=0$.

We define a expression t as shown below

$$t = (18abcd) - (4b^3d) + (b^2c^2) - (4ac^3) - (27a^2d^2)$$

If $t > 0$ then the cubic equation has 3 real distinct roots

If $t = 0$ then the equation has equal roots and all the roots of the cubic equation are real.

If $t < 0$ then the equation has 1 real root and two non real roots (complex roots).

18. Properties of triangle -

When the co-ordinates of a triangle are given we can find the sides of triangle using the distance between two points formula.

The centroid is essentially given by

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

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Incentre is given by $I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$

Orthocentre is

$O = \left(\frac{\tan A x_1 + \tan B x_2 + \tan C x_3}{\tan A + \tan B + \tan C}, \frac{\tan A y_1 + \tan B y_2 + \tan C y_3}{\tan A + \tan B + \tan C} \right)$

We define $s = (a+b+c)/2$

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Circumradius $R = \text{Area}/4abc$

Inradius $r = \text{Area}/s$

Length of median is given by $\sqrt{(2b^2 + 2c^2 - a^2)/4}$ and so on.

19. Differentiation-

The code was based on fundamental formulae in differentiation.

$$d(x^n)/dx = n \cdot x^{n-1}$$

$$d(e^{ax})/dx = ae^{ax}$$

$$d(\sin ax)/dx = a \cos x$$

$$d(e^{-ax})/dx = -ae^{-ax}$$

$$d(\cos bx)/dx = -b \sin x$$

$$d(c)/dx = 0$$

20. Integration-

Using the fact that integration was the reverse of differentiation.

$$d(x^n)/dx = x^{n+1} / (n+1) + k$$

$$d(e^{ax})/dx = e^{ax}/a + k$$

$$d(\sin ax)/dx = -\cos x/a + k$$

$$d(e^{-ax})/dx = -e^{-ax}/a + k$$

$$d(\cos bx)/dx = \sin x/b + k$$

$$d(c)/dx = cx + k$$

where k is the integration constant

21. Number of prime numbers below a given integer-

The basic idea of this function was discussed as a practice problem in class.

The number of prime numbers below a given positive number is given by this function. A number is said to be prime if and only if the remainder when it is divided by any number less than it but greater than 1 is not equal to zero. 1 is neither prime nor composite.

22. Solving simultaneous linear equations using Gauss Jordan equations-

This code is based on the lectures of Gaussian Elimination method. In this the co-efficient matrix is transformed so that all elements of the matrix $A[i][j]=0$ when $i>j$ and $A[i][j]=1$. While transformation we have to adjust the constant matrix also. By the method of back substitution we can find the variables.

23. Operations on vectors-

Addition and subtraction of vectors is done by adding or subtracting the respective the respective i, j and k components.

Dot product is the sum of product of respective i, j, k parts of two vectors.

$$\mathbf{A} \cdot \mathbf{B} = \sum_{i=1}^n A_i B_i = A_1 B_1 + A_2 B_2 + \dots + A_n B_n$$

Vector product or cross product is obtained by using the det-method.

$$\mathbf{u} = u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k}$$

$$\mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}$$

$$\begin{aligned} \mathbf{u} \times \mathbf{v} &= u_1 v_1 \mathbf{0} + u_1 v_2 \mathbf{k} - u_1 v_3 \mathbf{j} - \\ &u_2 v_1 \mathbf{k} - u_2 v_2 \mathbf{0} + u_2 v_3 \mathbf{i} + \\ &u_3 v_1 \mathbf{j} - u_3 v_2 \mathbf{i} - u_3 v_3 \mathbf{0} \\ &= (u_2 v_3 - u_3 v_2) \mathbf{i} + (u_3 v_1 - u_1 v_3) \mathbf{j} + (u_1 v_2 - u_2 v_1) \mathbf{k} \end{aligned}$$

The magnitude of a vector is found using the equation

If the vector is $v=v_xi +v_yj +v_zk$

$$\|v\|=\sqrt{(v_x^2+v_y^2+v_z^2)}$$

Angles made by a vector with X, Y and Z axes is given by these :
 $\text{Cos}^{-1}(v_x/\|v\|)$, $\text{Cos}^{-1}(v_y/\|v\|)$ and $\text{Cos}^{-1}(v_z/\|v\|)$ respectively with X,Y and Z axes.

24. Operations on matrices-

Addition and subtraction on matrices can be performed iff both the matrices on which the operation is to be done have the same order. The operation can be done by adding or subtracting the corresponding elements of the two matrices.

Multiplication of two matrices can be done only when number of columns of first matrix and number of rows of second matrix are equal. Division on matrices is not defined.

INDIVIDUAL WORK:-

- T SAI GOUTAM - Listed the functions to be written in the code and made the functions code and wrote SRS and Project report.
- TRISHALA BOTHRA - Improved some of the functions code. Prepared the Draft User Manual.
- SANSKAR JAIN - Worked on graphics. Functions were put into the graphics code.

IDEAS FOR FUTURE WORK:-

- We had initially planned to make a calculator with buttons but due to lack of time , we haven't completed (though tried) it. The Graphics can be improved.
- Some more functions can be introduced in the calculator such as inverse of a matrix.

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THANK YOU