Computer Programming

Dr. Deepak B Phatak
Dr. Supratik Chakraborty
Department of Computer Science and Engineering
IIT Bombay

Session: More on Two's Complement Representation

Quick Recap of Relevant Topics

IIT Bombay

- Representation of integers in a computer
  - Unsigned integers
  - Signed integers

Overview of This Lecture

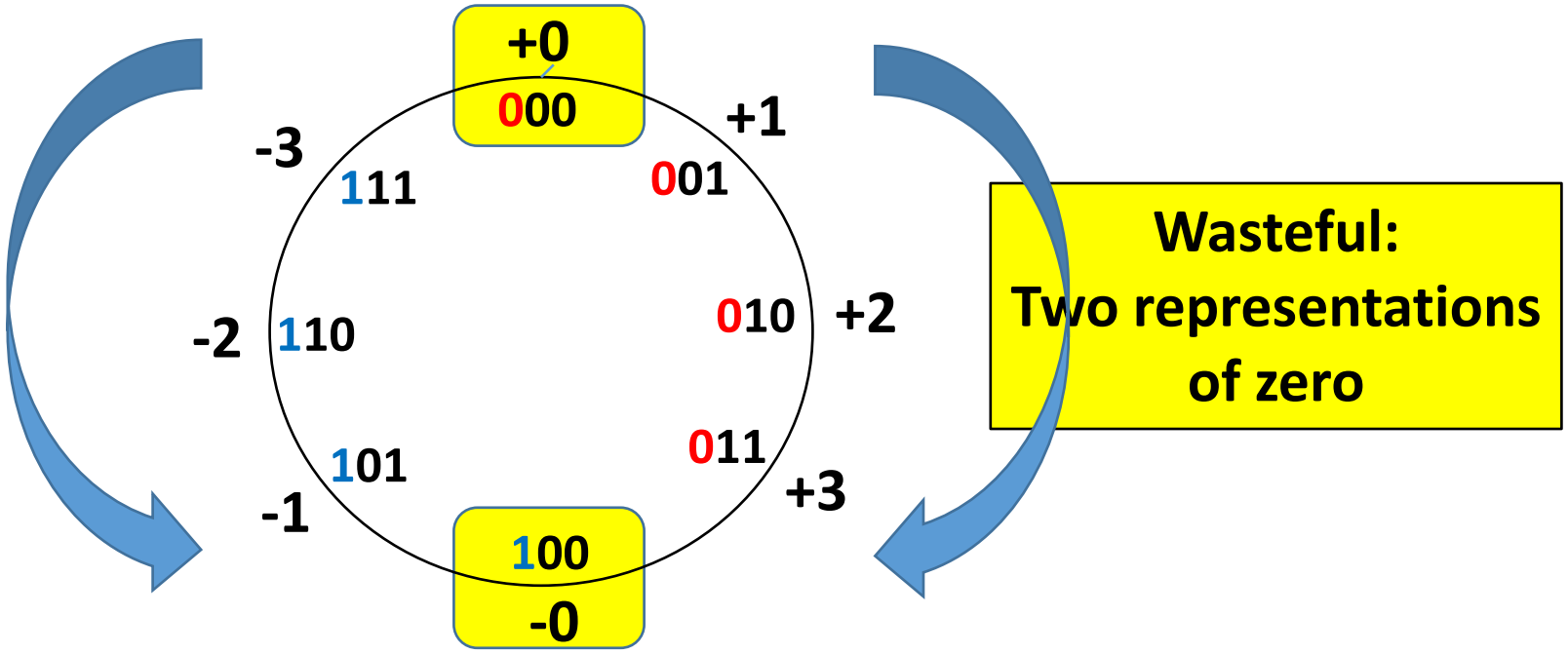
IIT Bombay

- A closer look at two's complement representation
- Magnitude of negative integers in two's complement representation

Representing Signed Integers

IIT Bombay

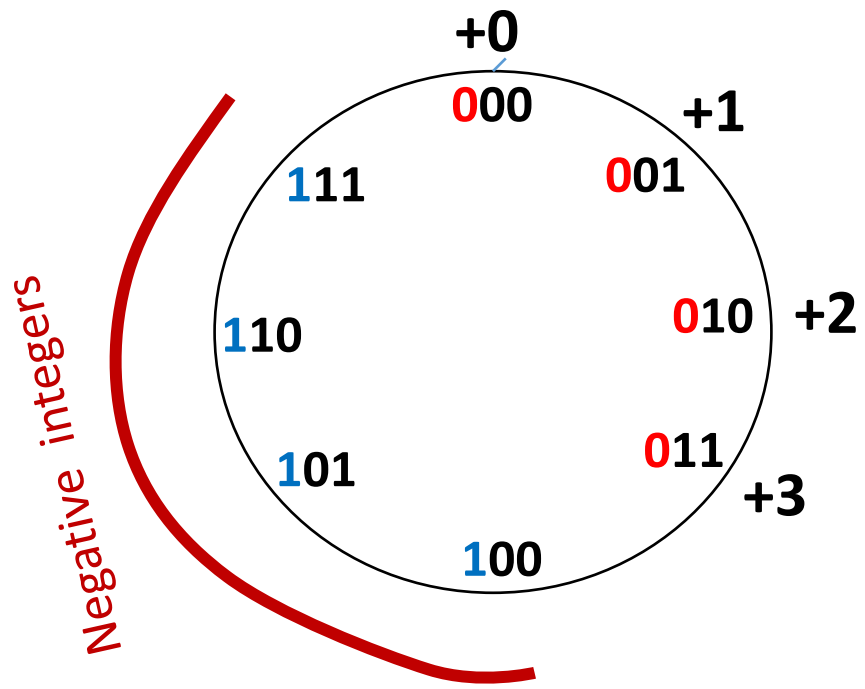
- Treat MSB as sign bit: negative if MSB is 1, positive if MSB is 0
  - **Sign-magnitude representation**
  - Consider integers represented using 3 bits



How Else Could We Represent?

IIT Bombay

- Using MSB to represent sign is convenient



What negative number should 100 represent?

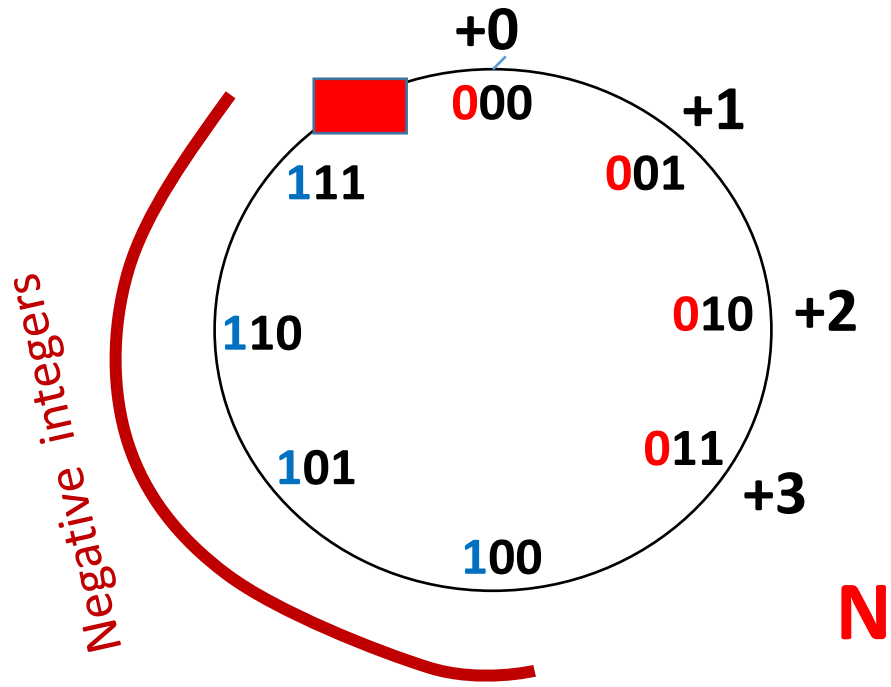
How about 101, 111, …?

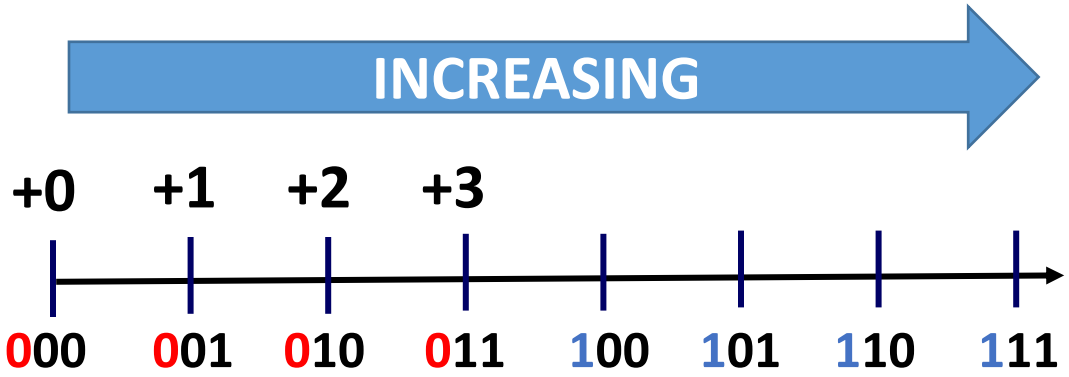
Can we think of the circle as a wrapped-around number line?

How Else Could We Represent?

IIT Bombay

- Where do we break the circle?



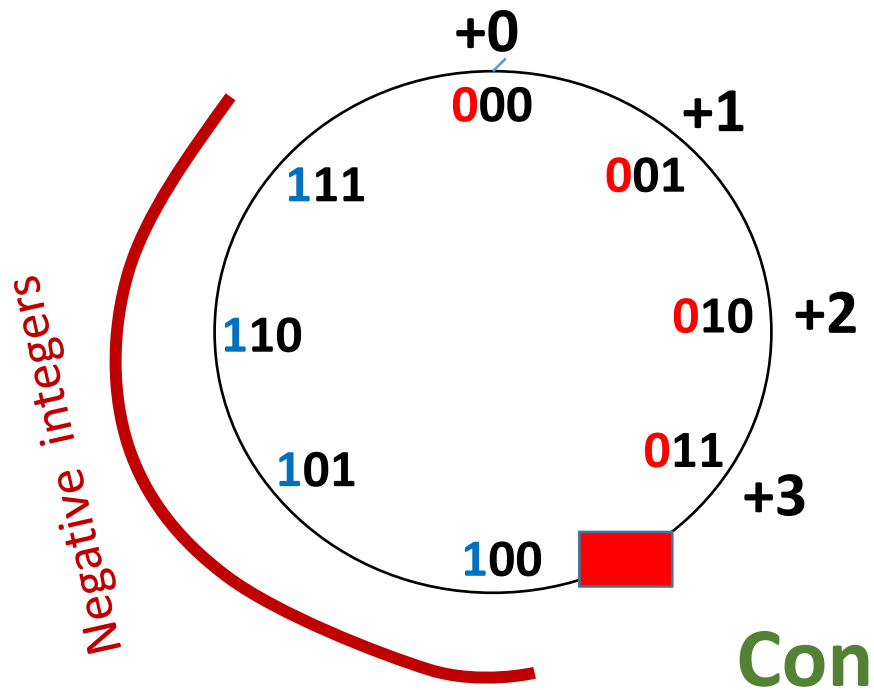


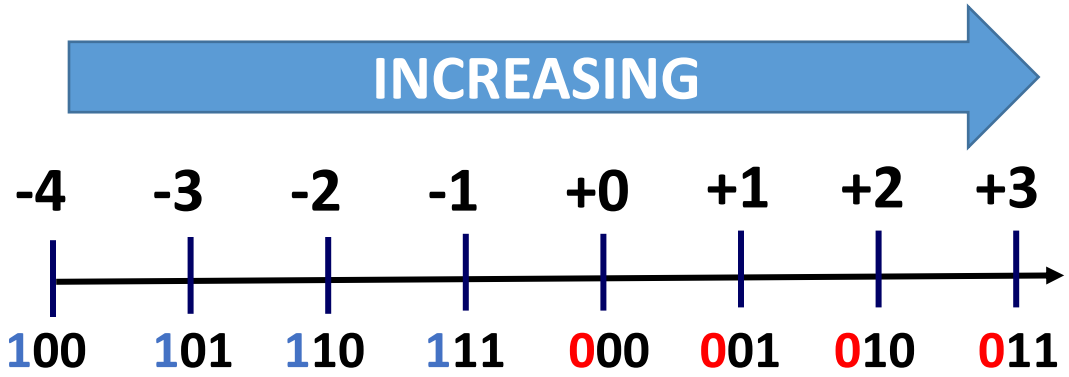
No space for negative integers !!!

How Else Could We Represent?

IIT Bombay

- Where do we break the circle?

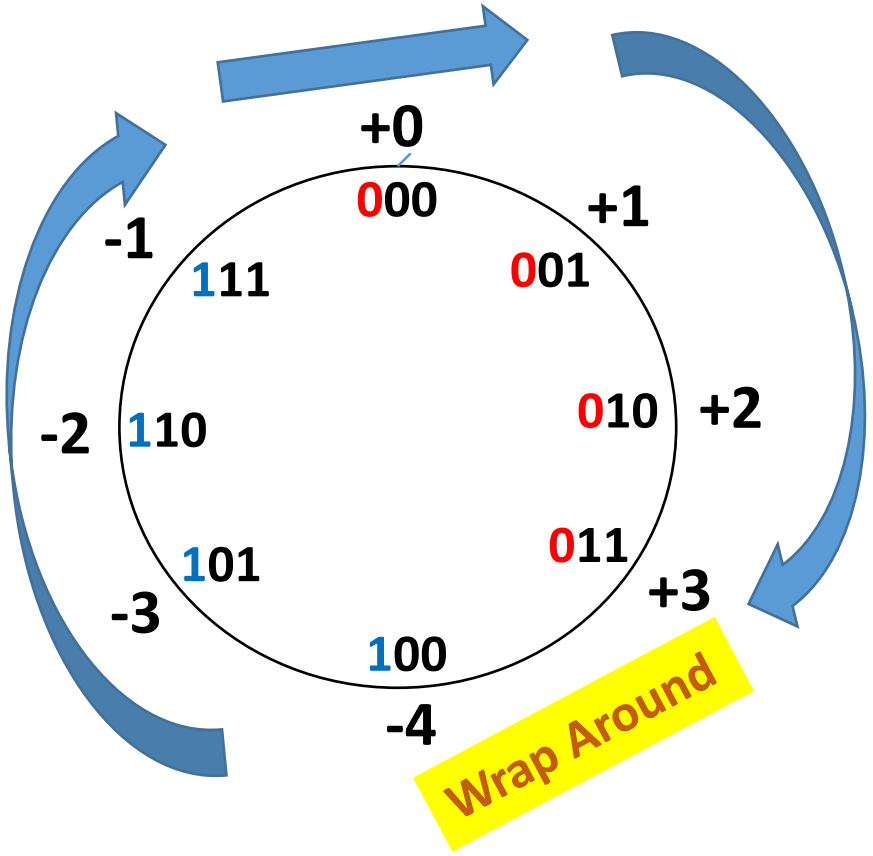




Consistent with increasing direction

Two's Complement Representation

IIT Bombay

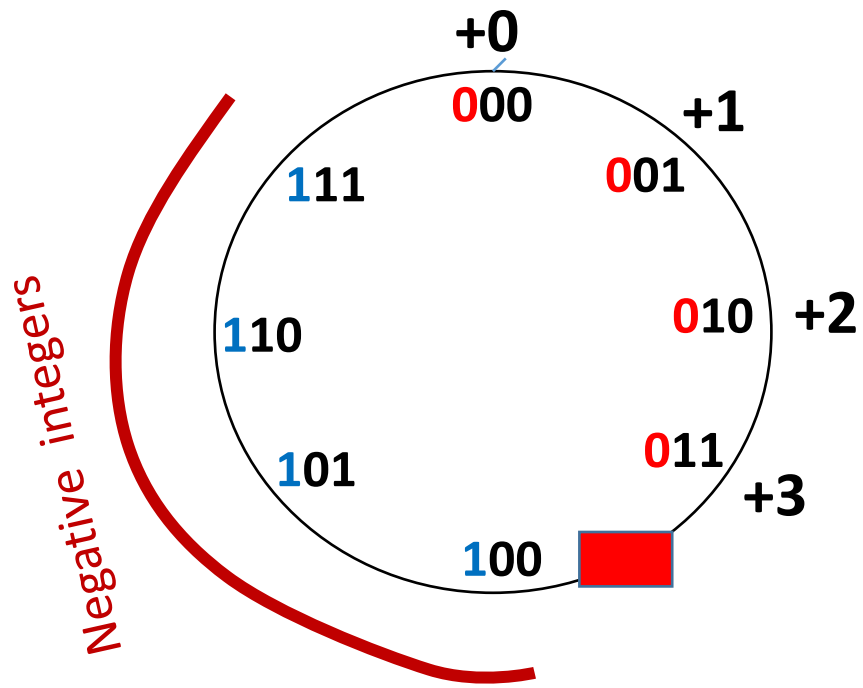


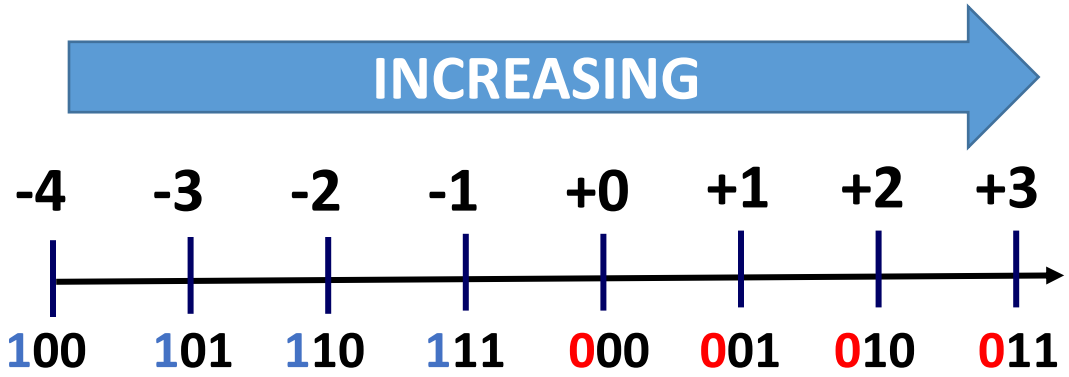
8 numbers represented:
-4 through +3

Only one representation of 0

Magnitudes of Negative Integers

IIT Bombay

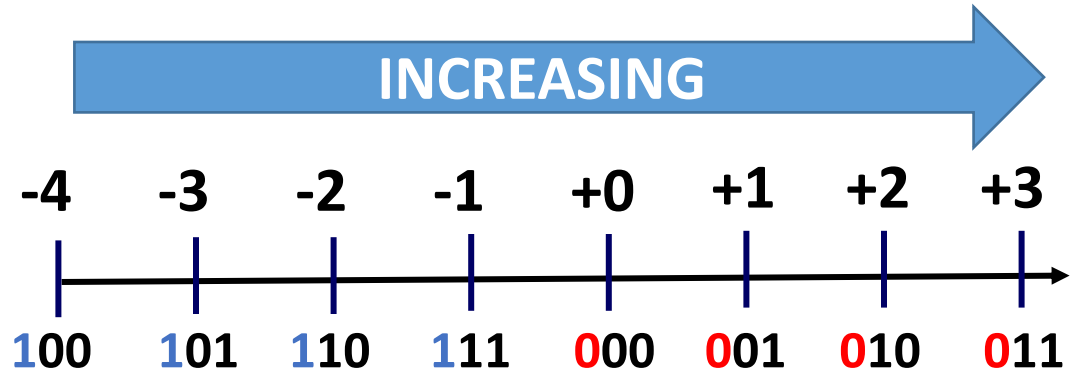




Need a way to map binary representation to magnitudes of negative integers

Magnitudes of Negative Integers

IIT Bombay



Desired map: $11 \rightarrow 1,\ 10 \rightarrow 2,\ 01 \rightarrow 3,\ 00 \rightarrow 4$

Observation:

11 (represents unsigned 3) $\rightarrow 2^2 - 3 = 1$

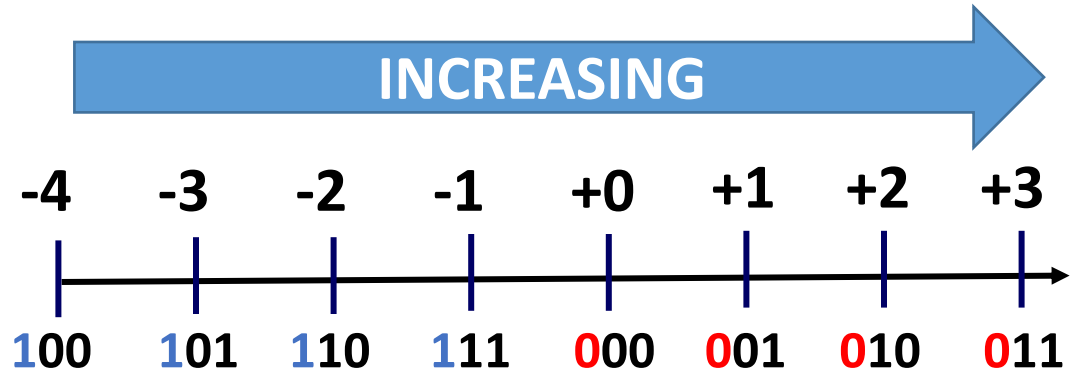
10 (represents unsigned 2) $\rightarrow 2^2 - 2 = 2$

01 (represents unsigned 1) $\rightarrow 2^2 - 1 = 3$

00 (represents unsigned 0) $\rightarrow 2^2 - 0 = 4$

Magnitudes of Negative Integers

IIT Bombay



Desired map: 11 ⟶ 1, 10 ⟶ 2, 01 ⟶ 3, 00 ⟶ 4

Observation:

11 (represents unsigned 3) ⟶ 00 (= 0) + 1 = 1

10 (represents unsigned 2) ⟶ 01 (= 1) + 1 = 2

01 (represents unsigned 1) ⟶ 10 (= 2) + 1 = 3

00 (represents unsigned 0) ⟶ 11 (= 3) + 1 = 4

Magnitude of Negative Integers

IIT Bombay

- Is there an easy way to figure out the magnitude of what 10111 represents in 2's complement?
  - 10111 has MSB 1: Negative integer
  - To get absolute value of 10111
    - Ignore MSB: 10111
    - Flip every bit in 0111: 1000 (decimal 8)
    - Add 1: decimal 9
    - Absolute value is 9
  - Answer: -9

Two's Complement Representation

IIT Bombay

- Is there an easy way to figure out the magnitude of what 10111 represents in 2's complement?
  - 10111 has MSB 1: Negative integer
  - To get absolute value of 10111
    - Ignore MSB: 10111 (decimal 7)
    - Magnitude: $2^4 - 7 = 16 - 7 = 9$
  - Answer: -9

No. of bits in magnitude = No. of bits - 1

Summary

IIT Bombay

- Rationale behind two's complement representation
- Simple ways of getting magnitude of negative integers from two's complement representation