Computer Programming

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Session: Recursive Functions – Part B

Quick Recap of Relevant Topics

- Use of simple functions in programs
- Contract-centric view of programming with functions
- Flow of control in function call and return
- Activation records and call stack
- Parameter passing by value and reference
- Recursive functions

Overview of This Lecture

- Designing recursive functions
    - Termination and recursive changing of parameters
- Recursion vs iteration

Acknowledgment

- Some examples in this lecture are from

  An Introduction to Programming Through C++

  by Abhiram G. Ranade

  McGraw Hill Education 2014

- All such examples indicated in slides with the citation **AGRBook**

Recall: Encoding Example

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- We want to store quiz 1 and quiz 2 marks of CS101 students in an encoded form
- Encoding strategy: encode(m, n) = $2^m \times 3^n$
- Assume all marks are integers in {1, 2, … 10}

Observe: encode(m, n) = encode(m, n-1) x 3, if m, n > 1
= encode(m-1, 1) x 2, if m > 1, n=1
= 2 x 3 = 6, if m=1, n=1

A Recursive Solution

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```
#include <iostream>
using namespace std;
int new
int mai
 for ( …
  ciphe
 …}
…
return 0;
}
```

```
// PRECONDITION: …
int newEnc(int q1Marks,
           int q2Marks)
         (q2Marks) {
         .:
        1Marks == 1) {return 6;}
         {return
          2*newEnc(q1Marks – 1, 1);
      }
      break;
   default: … }
}
// POSTCONDITION: …
```

Are we really sure that every call to newEnc that satisfies precondition eventually terminates?

Caveats Using Recursive Functions

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- Must specify how to terminate the recursion

  Otherwise, recursion (calling a function from itself) can go on forever

- Must ensure ... anges parameters i... entually terminates

Changing parameters in an orderly way to ensure termination

encode(m, n) = encode(m, n-1) x 3, if m, n > 1

= encode(m-1, 1) x 2, if m > 1, n=1

= 2 x 3 = 6, if m=1, n=1

Termination case

Caveats Using Recursive Functions

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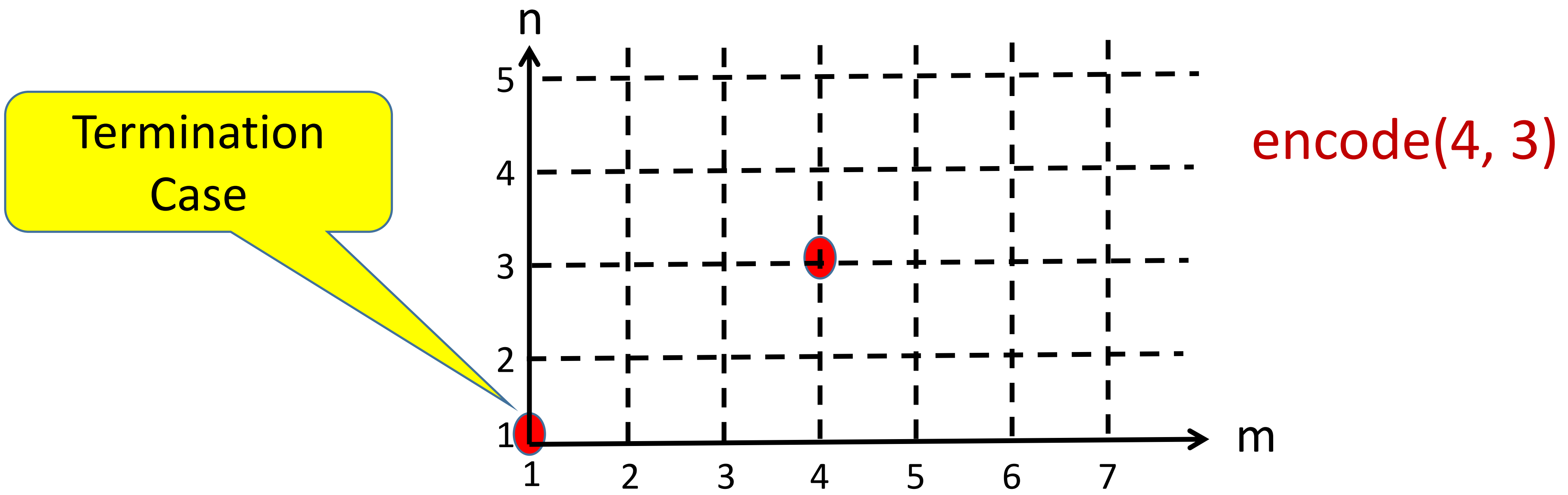
- Think of all possible valuations of parameters as ordered with a fixed end (termination case)
- Recursion must change values of parameters so that we move along this order monotonically towards fixed end



Caveats Using Recursive Functions

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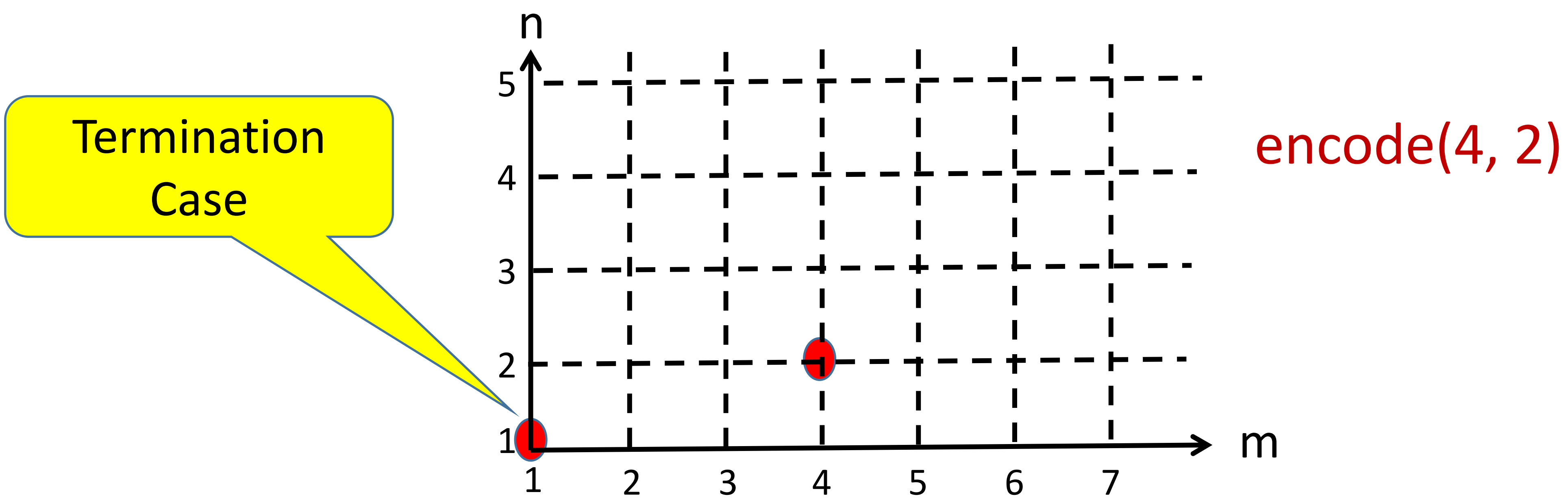

Caveats Using Recursive Functions

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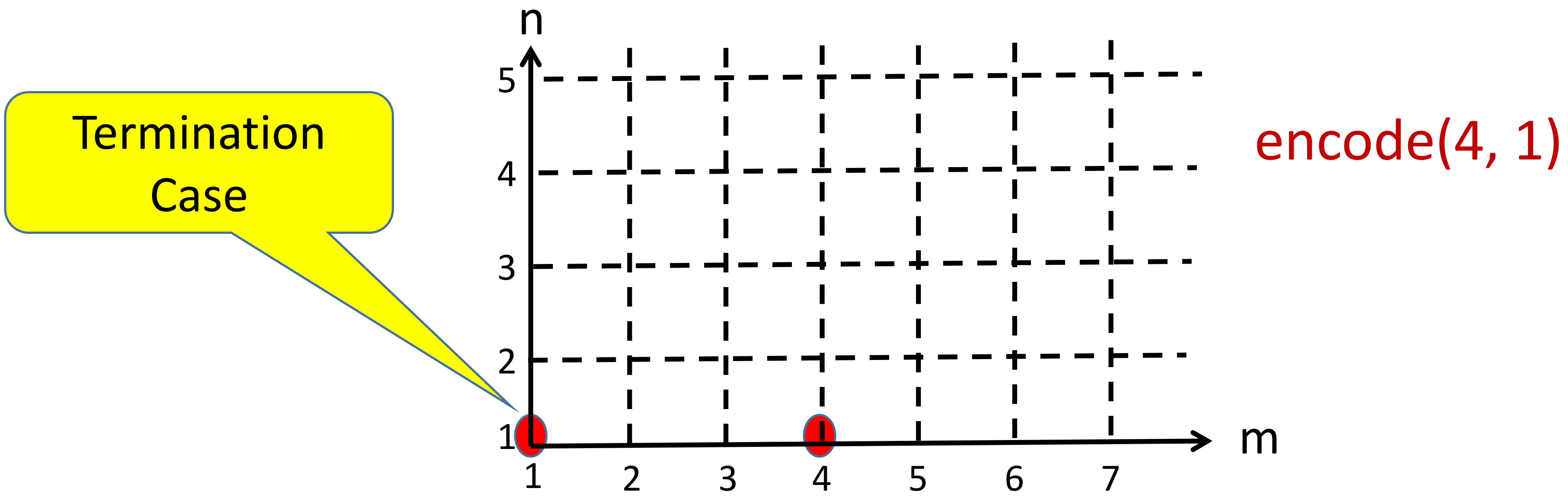

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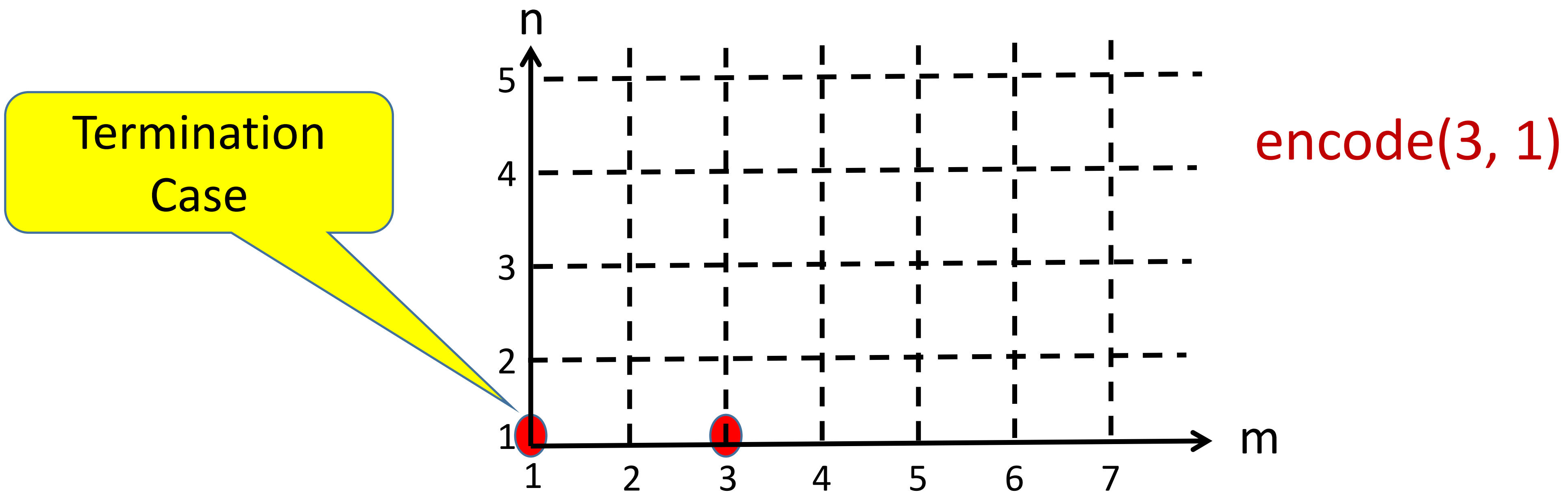

Caveats Using Recursive Functions

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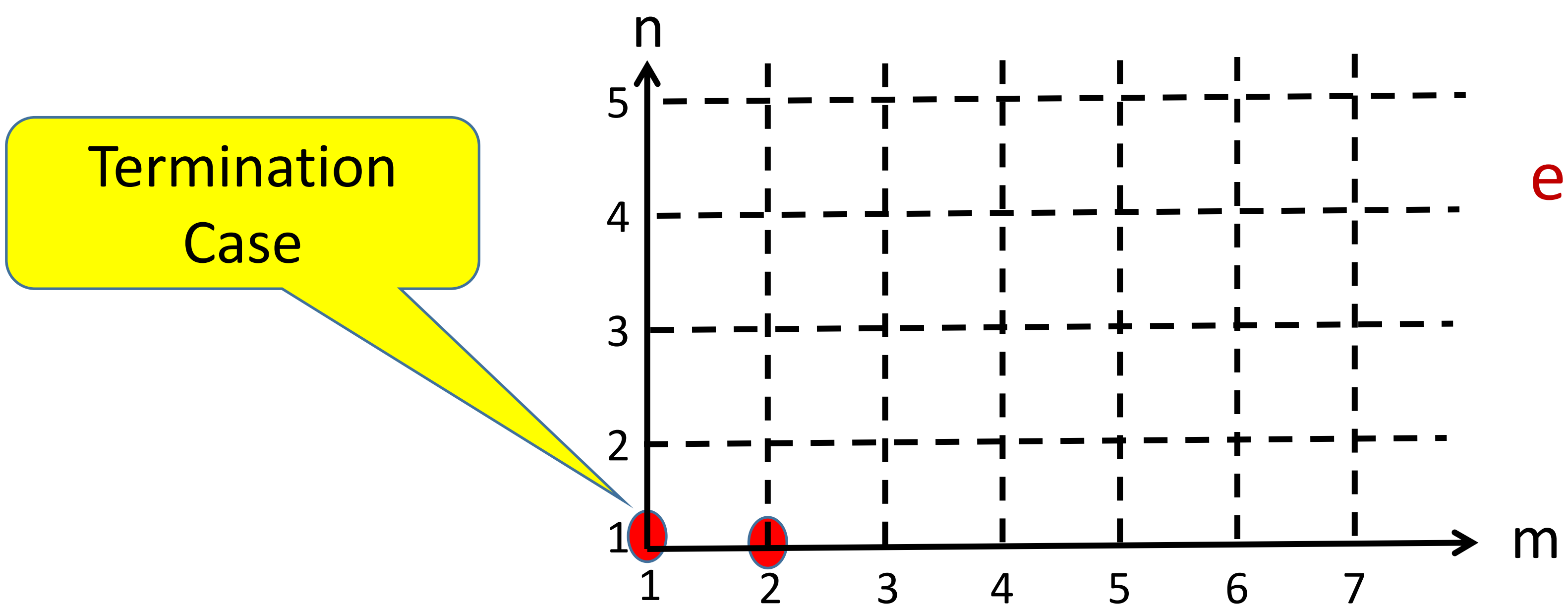

encode(2, 1)

Caveats Using Recursive Functions

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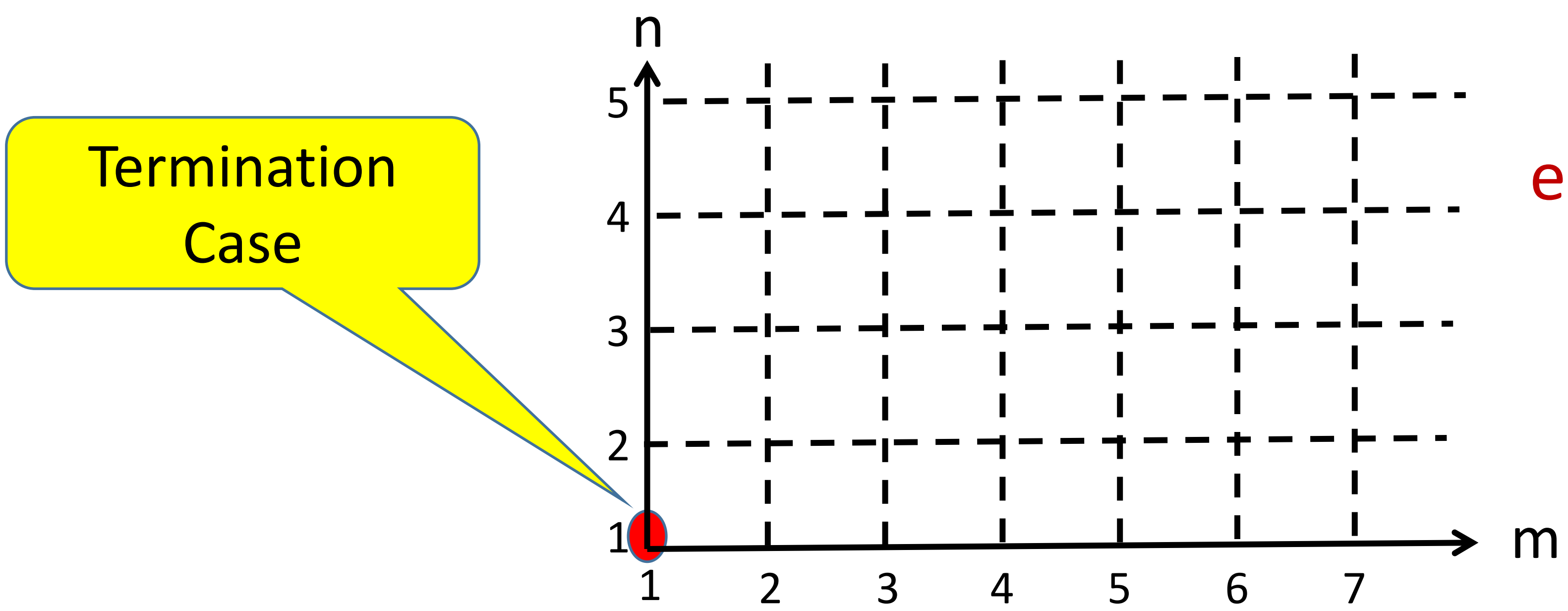

encode(1, 1)

Caveats Using Recursive Functions

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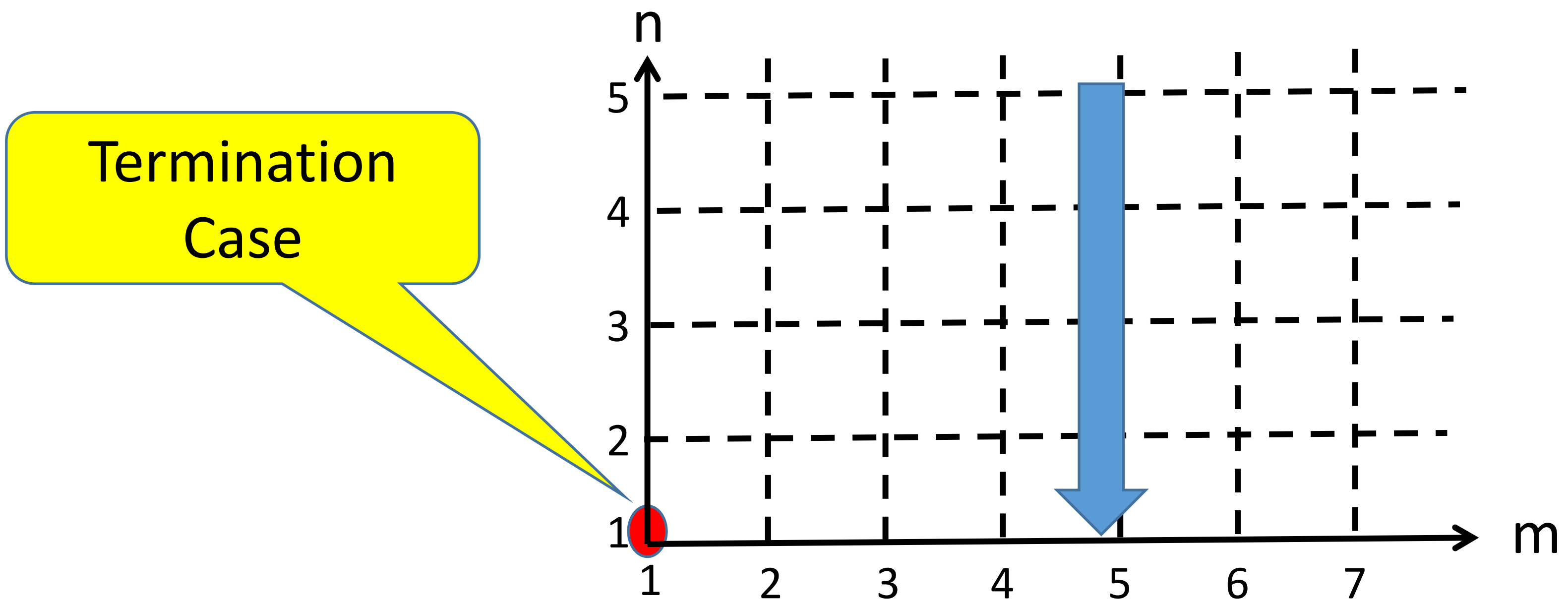
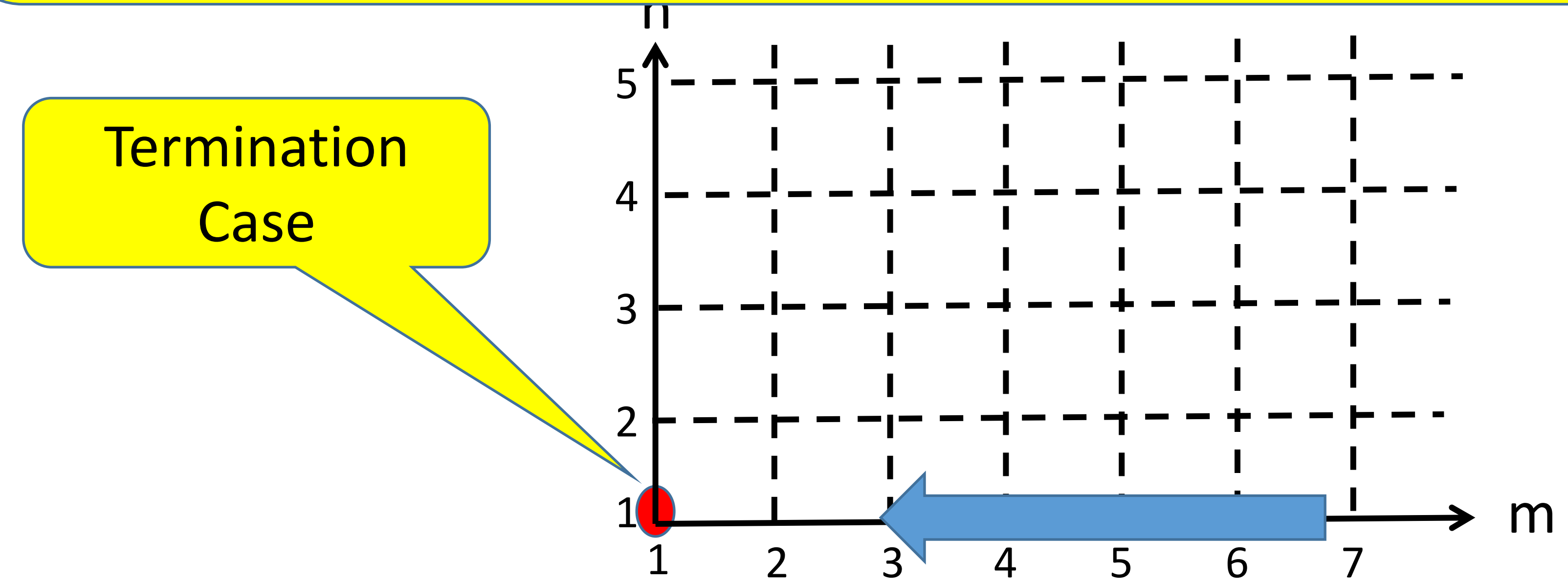


Caveats Using Recursive Functions

Well-founded ordering of parameters with a "least" element

Move monotonically along order towards "least" element



Caveats Using Recursive Functions

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encode(m, n) = $2^m \times 3^n$ can also be thought as

Changing parameters in this way doesn't ensure termination

encode(m, n) = encode(m, n+1)/3, if m, n > 1

= encode(m+1, 1)/2, if m > 1, n = 1

= 2 x 3 = 6, if m = 1, n = 1

Termination case

Caveats Using Recursive Functions

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- Think of all possible valuations of parameters as ordered with a fixed end (termination case)
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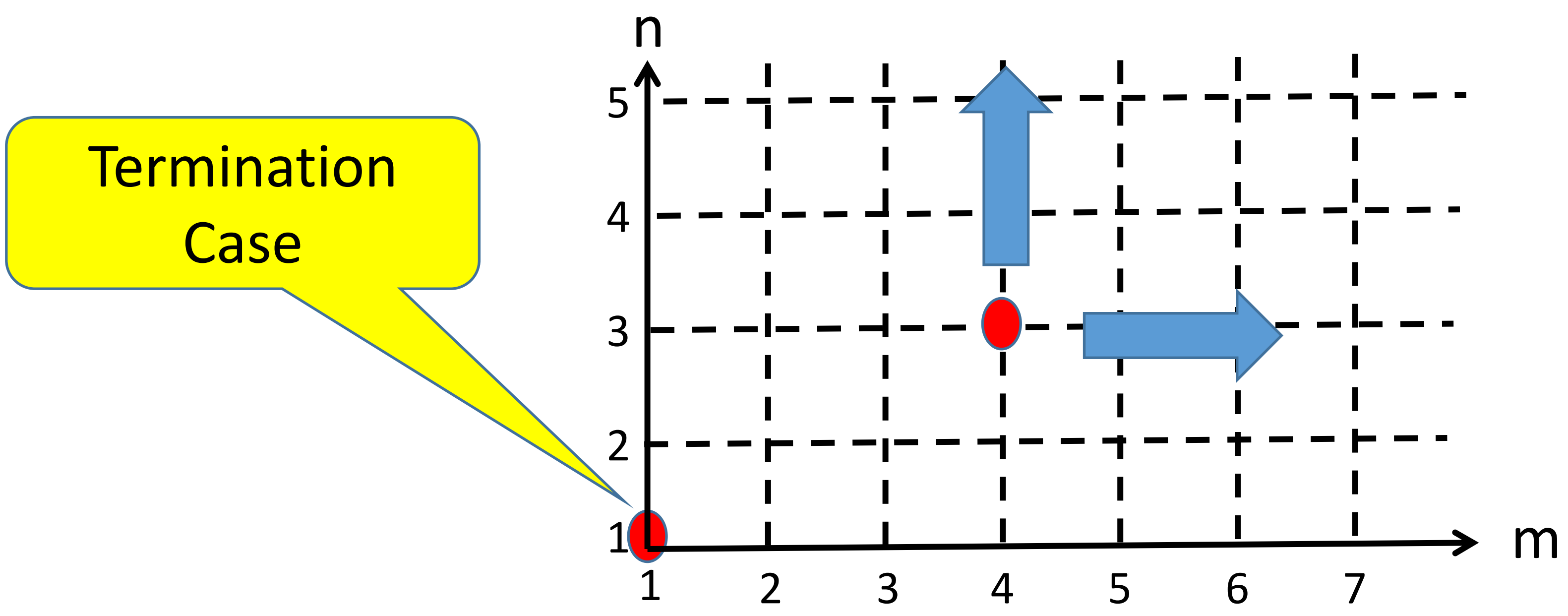

A Second Example of Recursion

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Given n, compute factorial(n) = 1 x 2 x … x n

```
// PRECONDITION:  integer n >= 0
int factorial(int n)
{
  if ( n == 0 ) {return 1;}  // factorial(0) = 1 – Termination case
  else {
    return (n * factorial(n-1));  // Reduce parameter monotonically
                                  //  to 0, and use recursion
  }
 }
// POSTCONDITION: return value = factorial(n)
```

A Third Example [Sec 10.3 of AGRBook]

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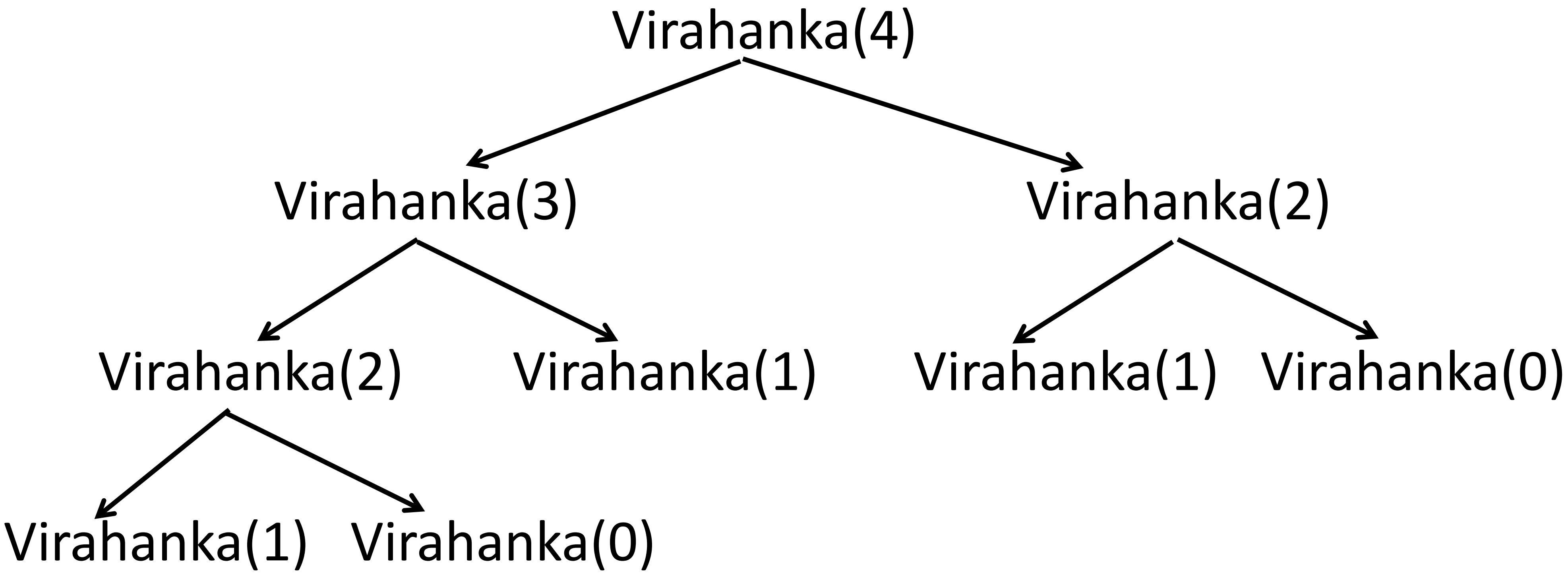
Virahanka numbers: $V_0 = V_1 = 1$, and $V_n = V_{n-1} + V_{n-2}$ for $n >= 2$

Also known as Fibonacci numbers

(although Virahanka studied these in the context of counting specific types of poetic meters before Fibonacci!)

```
// PRECONDITION: integer n >= 0
int Virahanka(int n)
{ if ((n == 0) || (n == 1)) { return 1; }
  else { return ( Virahanka(n-1) + Virahanka(n-2) ); }
}
// POSTCONDITION: return value = V_n
```

Watch Number of Recursive Calls



Number of calls required to compute Virahanka(n) grows exponentially with n

Is There A Better Way?

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An iterative solution is much better here

```
int Virahanka(int n)
{ int count, result;
  int prevVN = 1, prevPrevVN = 1;
  if ((n == 0) || (n == 1)) { return 1; }
  else {
     for(count = 2; count <= n; count++) {
        result = prevVN + prevPrevVN;
        prevPrevVN = prevVN;  prevVN = result;
     }
   return result; }
}
```

Recursion vs Iteration

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- Recursive formulation usually clean, intuitive and succinct
  - Need to worry about recursion termination (well-founded ordering of parameter values)
  - Need to worry about number of recursive calls
- Iterative formulation may be less clean or intuitive (not always!)
  - Need to worry about loop invariants, loop variants and termination
  - Can be very efficient if formulated correctly
- Best practice: Judicious mix of iteration and recursion

Summary

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- Recursive functions
  - Termination and ordering of parameter values
  - Recursion: Monotonically move towards termination case
- Recursion vs iteration