

Computer Programming

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Session: Recap of Recursive Functions

VIDEO LECTURE RECAP QUIZ

Q1. A function that calls itself is an example of _____ function

- A. Recursive**
- B. Iterative**
- C. Non-terminating**
- D. Mirror**

Q2. Which of the following is/are FALSE about recursive functions?

- A. Must have at least one parameter**
- B. Can have only call-by-value parameters**
- C. May not terminate for some input parameters**
- D. Cannot call any function other than itself**

Q3. For recursion to terminate, the values of parameters

- A. Can change in any order**
- B. Must move monotonically towards a termination case**
- C. Must stay unchanged in all calls to the function**
- D. Must never become negative**

Q4. Virahanka numbers can be computed

- A. Recursively but not iteratively**
- B. Iteratively but not recursively**
- C. Both iteratively and recursively**
- D. Neither iteratively nor recursively**

Video Lecture Recap Quiz



- Q5. When a function recursively calls itself**
- A. The activation record on top of the call stack is popped out**
 - B. A new activation record is pushed in the call stack**
 - C. The activation record on top of the call stack is overwritten**
 - D. No activation records are pushed/popped**

Q6. Specifying a termination case

- A. Guarantees that a recursive function terminates for all inputs**
- B. May not cause a recursive function to terminate for all inputs**
- C. Terminates a function if it calls itself**
- D. Helps the compiler avoid generating code for recursive functions**

VIDEO LECTURE RECAP SLIDES

Calling a Function From Itself

- Same mechanism of function calls and returns we studied earlier works perfectly !!!

Recursive Function: One that can call itself
Elegant and natural way to solve several problems

Mutually recursive functions
func1 calls func2, which calls func3,
which calls func1

A Program With A Recursive Function



```
#include <iostream>
using namespace std;
int newEnc(int q1Marks,int q2Marks);
int main() { ...
    for ( ... ) { ...
        cipher = newEnc(q1Marks, q2Marks);
    ...}
    ...
    return 0;
}
```

```
// PRECONDITION: ...
int newEnc(int q1Marks,
           int q2Marks)
{ switch(q2Marks) {
    case 1:
        if (q1Marks == 1) {return 6;}
        else {return
            2*newEnc(q1Marks – 1, 1);
        }
        break;
    default: ... }
}
// POSTCONDITION: ...
```

Caveats Using Recursive Functions

- Must specify how to terminate the recursion
Otherwise, recursion (calling a function from itself) can go on forever

- Must ensure parameters in recursion terminates

Changing parameters in an orderly way to ensure termination

changes eventually

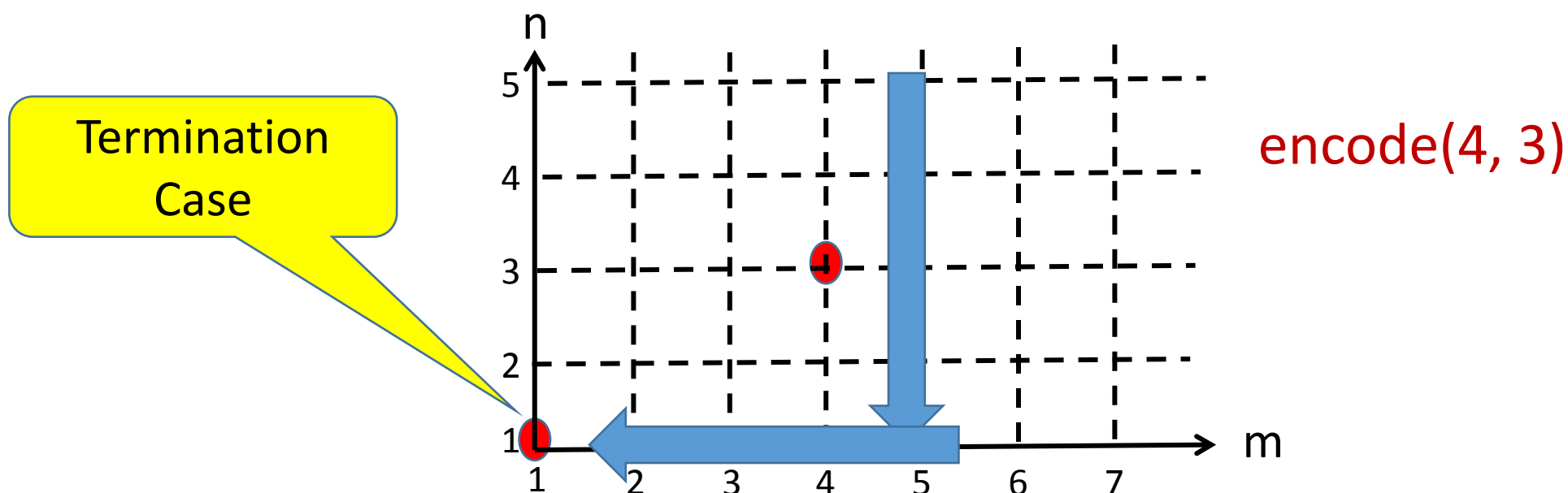
$\text{encode}(m, n) = \text{encode}(m, n-1) \times 3, \text{ if } m, n > 1$
 $= \text{encode}(m-1, 1) \times 2, \text{ if } m > 1, n=1$

$= 2 \times 3 = 6, \text{ if } m=1, n=1$

Termination case

Caveats Using Recursive Functions

- Think of all possible valuations of parameters as ordered with a fixed end (termination case)
- Recursion must change values of parameters so that we move along this order monotonically towards fixed end



Recursion vs Iteration



- Recursive formulation usually clean, intuitive and succinct
 - Need to worry about recursion termination (well-founded ordering of parameter values)
 - Need to worry about number of recursive calls
- Iterative formulation may be less clean or intuitive (not always!)
 - Need to worry about loop invariants, loop variants and termination
 - Can be very efficient if formulated correctly
- Best practice: Judicious mix of iteration and recursion

Practice Problem

Let's build on the problem discussed in last class. Recall the game of tic-tac-toe.

0	1	0
	1	
0	1	

Practice Problem (continued)

A configuration of the tic-tac-toe grid is represented by a sequence of 9 integer valued variables x_1, x_2, \dots, x_9

x_1	x_2	x_3
x_4	x_5	x_6
x_7	x_8	x_9

Practice Problem (continued)

A configuration of the tic-tac-toe grid is represented by a sequence of 9 integer valued variables x_1, x_2, \dots, x_9

0	1	0
	1	
0	1	

$$x_1 = 0, x_2 = 1, x_3 = 0$$

$$x_4 = -1, x_5 = 1, x_6 = -1$$

$$x_7 = 0, x_8 = 1, x_9 = -1$$

Practice Problem: Subproblem 1



Write a C++ function that takes as input an input configuration and determines who ("0" or "1") should move next.

```
int nextTurn(int x1, int x2, ... int x9)
{ // Check if configuration is valid
  // Count no. of 0's and 1's
  // Determine who moves next
}
```

Practice Problem (continued)



Given a configuration of tic-tac-toe, we want to determine if there is a **winning/losing move** of the next player.

Winning Move of 0: A move of 0 from which there is at least one way for 0 to win no matter how 1 plays.

Winning Move of 1 similarly defined

Practice Problem (continued)

Example: tic-tac-toe configuration

$x_1 = x_3 = 0$, $x_2 = x_6 = 1$, Rest are -1

x_1	x_2	x_3	0	1	0
x_4	x_5	x_6			1
x_7	x_8	x_9	0		

Next move: 0

**Winning move of 0:
 $x_7 = 0$**

Guaranteed win for 0 no matter how 1 plays

Practice Problem (continued)

Example: tic-tac-toe configuration

$x_1 = x_3 = 0$, $x_2 = x_5 = 1$, Rest are -1

x_1	x_2	x_3	0	1	0
x_4	x_5	x_6		1	
x_7	x_8	x_9			

Next move: 0

No winning move of 0

Cannot guarantee win for 0 from any next move

Practice Problem (continued)



Losing Move: A move from which there is at least one winning move of the opponent

Practice Problem (continued)

Example: tic-tac-toe configuration

$x_1 = x_3 = 0, x_2 = 1$, Rest are -1

x_1	x_2	x_3	0	1	0
x_4	x_5	x_6			1
x_7	x_8	x_9	0		

Next move: 1

**Losing move of 1:
 $x_6 = 1$**

Winning move of 0 exists after this

Useful Observation:

If 0 has a winning move from a configuration, then after this move is taken, 1 cannot have a winning move from the new configuration.

Similarly, with roles of 0 and 1 reversed.

Practice Problem (continued)



Useful Observation:

If 0 has a losing move from a configuration, then after this move is taken, 1 has a winning move from the new configuration.

Similarly, with roles of 0 and 1 reversed.

Practice Problem: Subproblem 2



Write mutually recursive C++ functions

winMove and **loseMove**,

such that each takes as inputs

(i) a configuration, and (ii) next player (0 or 1)
and determines

**If next player has at least one winning move
and**

If all moves of next player are losing moves

Practice Problem: Subproblem 3



If there is a winning move for the next player, `winMove(...)` should return the position for the winning move, else it should return -1

If all moves for the next player are losing moves, `loseMove(...)` should return true, else it should return false.

Practice Problem (continued)

Recall example: tic-tac-toe configuration

$x_1 = x_3 = 0$, $x_2 = x_6 = 1$, Rest are -1

x_1	x_2	x_3	0	1	0
x_4	x_5	x_6			1
x_7	x_8	x_9	0		

Next move: 0

Winning move of 0:
 $x_7 = 0$

$\text{winMove}(0,1,0,-1,-1,1,-1,-1,-1, 0)$ returns 7

Practice Problem (continued)

Example: tic-tac-toe configuration

$x_1 = x_3 = 0$, $x_2 = x_5 = 1$, Rest are -1

x1	x2	x3
x4	x5	x6
x7	x8	x9

0	1	0
	1	

Next move: 0

No winning move

`winMove(0,1,0,-1,1,-1,-1,-1,-1, 0)` returns -1

Practice Problem (continued)

Recall example: tic-tac-toe configuration

$x_1 = x_3 = 0$, $x_2 = x_6 = 1$, Rest are -1

x1	x2	x3	0	1	0
x4	x5	x6			1
x7	x8	x9	0		

Next move: 1

**All moves of 1:
Losing**

loseMove(0,1,0,-1,-1,1,0,-1,-1, 1) returns true

Practice Problem: Subproblem 3



```
int winMove(int x1, ... int x9, int nextPlayer)  
{ // Validate inputs  
    // Determine if winning move exists for  
    // nextPlayer  
}
```

[Hint: Check if opponent has only losing moves after nextPlayer takes a move. What are the termination cases?]

Practice Problem: Subproblem 3



```
int loseMove(int x1, ... int x9, int nextPlayer)  
{ // Validate inputs  
    // Determine if all moves of nextPlayer  
    // are losing moves  
}
```

**[Hint: Check if opponent has a winning move
for every next move of nextPlayer]**