

CS344: Introduction to Artificial Intelligence

(associated lab: CS386)

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Lecture—5 and 6: Propositional Calculus
and Co-operative Puzzle Solving

Propositional Calculus and Puzzles

Propositions

- Stand for facts/assertions
- Declarative statements
 - As opposed to interrogative statements (questions) or imperative statements (request, order)

Operators

AND($\wedge$), *OR*($\vee$), *NOT*($\neg$), *IMPLICATION*($\Rightarrow$)

- $\Rightarrow$ and $\neg$ form a minimal set (can express other operations)
- Prove it.

Tautologies are formulae whose truth value is always T, whatever the assignment is

Model

In propositional calculus any formula with n propositions has 2^n models (assignments)

- Tautologies evaluate to T in all models.

Examples:

$$1) \quad P \vee \neg P$$

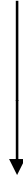
$$2) \quad \neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$$

.e Morgan with AND

Semantic Tree/Tableau method of proving tautology

Start with the negation of the formula

$$\neg[\neg(P \wedge Q) \Rightarrow (\neg P \vee \neg Q)] \quad - \alpha - \text{formula}$$



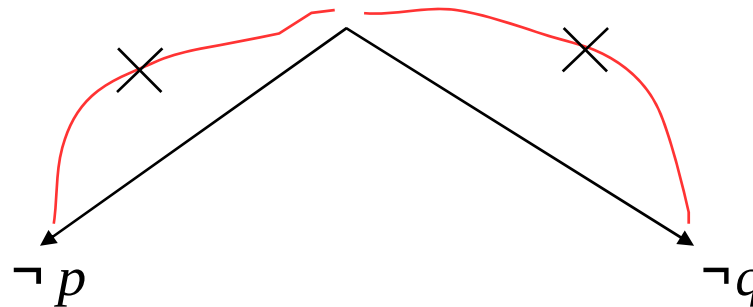
$$\neg (P \wedge Q) \quad - \beta - \text{formula}$$

$$\neg(\neg P \vee \neg Q) \quad - \alpha - \text{formula}$$

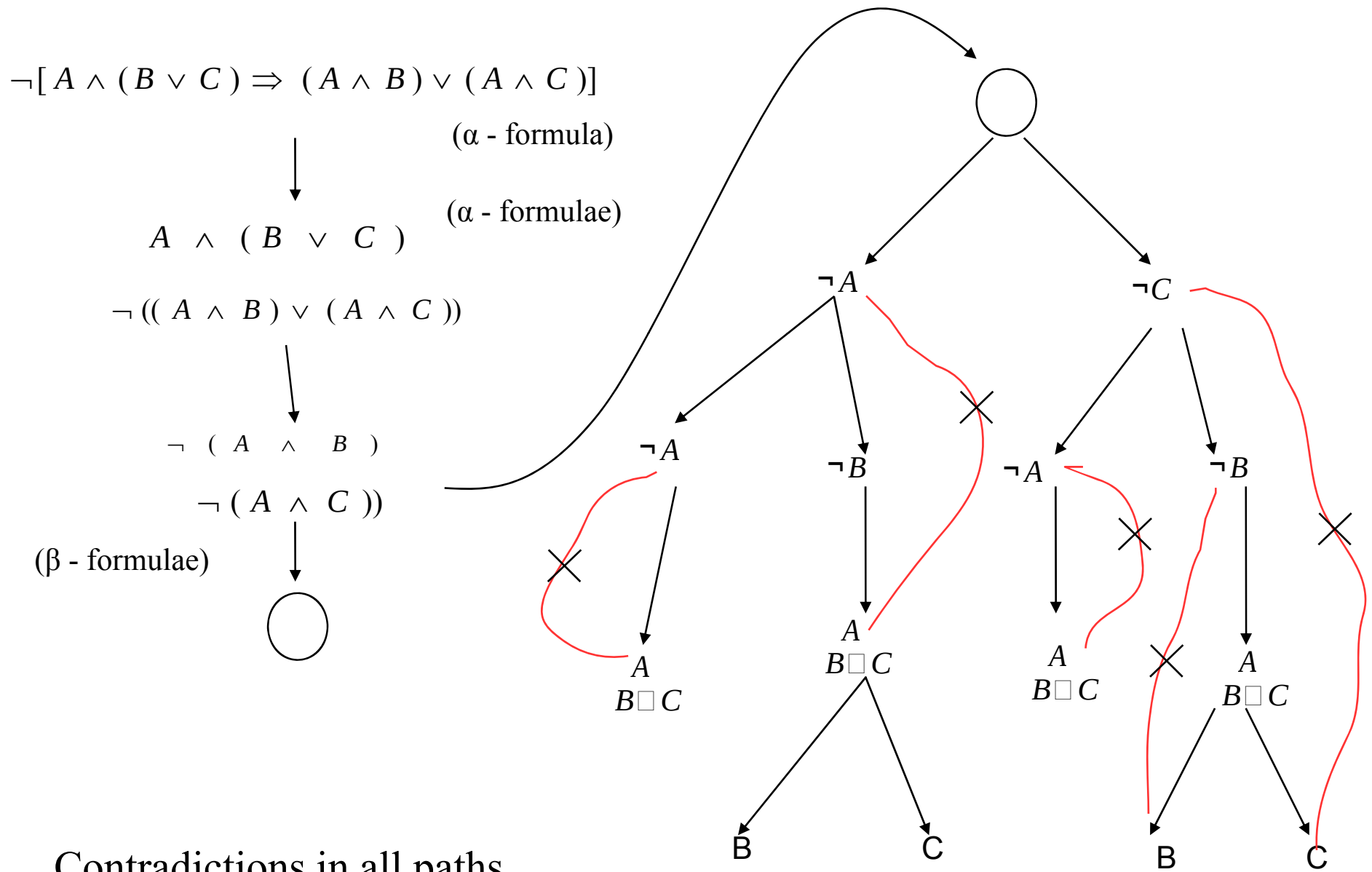


p

q



Example 2:



A puzzle

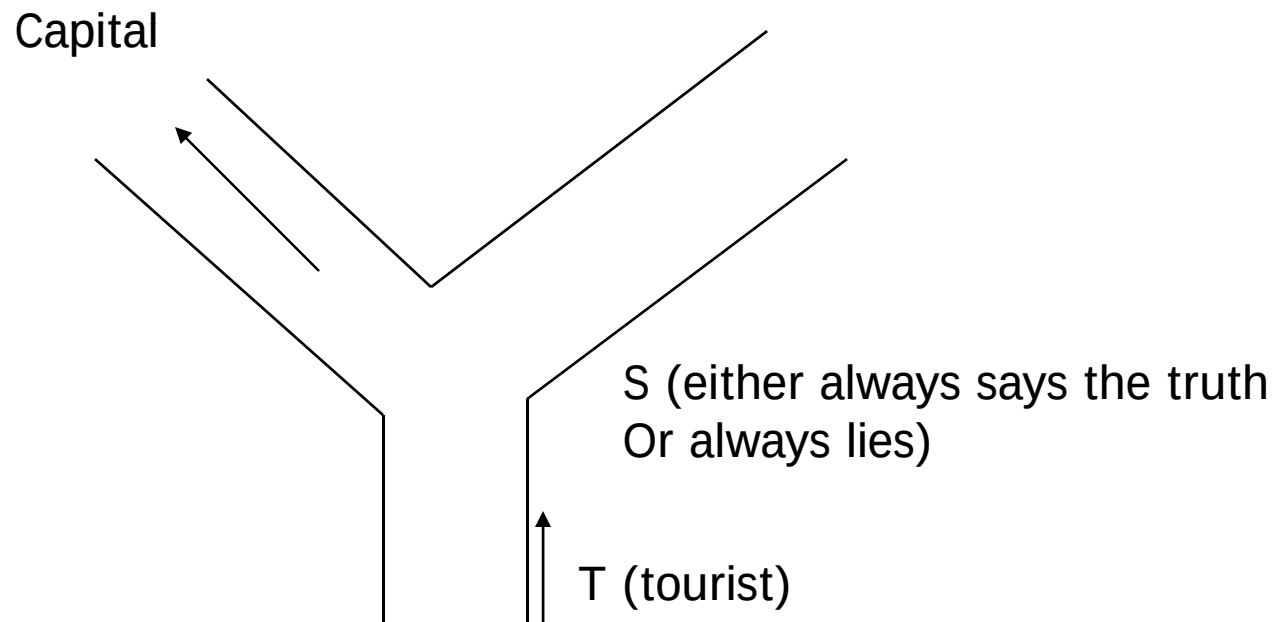
(Zohar Manna, Mathematical Theory of Computation, 1974)

From Propositional Calculus

Tourist in a country of truth-sayers and liars

- Facts and Rules: In a certain country, people **either always** speak the truth **or always** lie. A tourist T comes to a junction in the country and finds an inhabitant S of the country standing there. One of the roads at the junction leads to the capital of the country and the other does not. S can be asked only **yes/no** questions.
- Question: What **single** yes/no question can T ask of S, so that the direction of the capital is revealed?

Diagrammatic representation



Deciding the Propositions: a very difficult step- needs human intelligence

- P: Left road leads to capital
- Q: S always speaks the truth

Meta Question: What question should the tourist ask

- The **form** of the question
- Very difficult: needs human intelligence
- The tourist should ask
 - *Is R true?*
 - *The answer is “yes” if and only if the left road leads to the capital*
 - *The structure of R to be found as a function of P and Q*

A more mechanical part: use of truth table

P	Q	S's Answer	R
T	T	Yes	T
T	F	Yes	F
F	T	No	F
F	F	No	T

Get form of R: quite mechanical

- From the truth table
 - ***R is of the form $(P \text{ x-nor } Q)$ or $(P \equiv Q)$***

Get R in English/Hindi/Hebrew...

- Natural Language Generation: non-trivial
- The question the tourist will ask is
 - ***Is it true that the left road leads to the capital if and only if you speak the truth?***
- Exercise: A more well known form of this question asked by the tourist uses the X-OR operator instead of the X-Nor. What changes do you have to incorporate to the solution, to get that answer?

Another Similar Problem

From Propositional Calculus

Another tourist example: this time in a restaurant setting in a different country (Manna, 1974)

- Facts: A tourist is in a restaurant in a country when the waiter tells him:
 - “do you see the three men in the table yonder? One of them is X who always speaks the truth, another is Y who always lies and the third is Z who sometimes speaks the truth and sometimes lies, i.e., answers yes/no randomly without regard to the question.
- Question: Can you (the tourist) ask three yes/no questions to these men, always indicating who should answer the question, and determine who of them is X, who y and who Z?

Solution: Most of the steps are doable by humans only

- Number the persons: 1, 2, 3
 - 1 can be X/Y/Z
 - 2 can be X/Y/Z
 - 3 can be X/Y/Z
- Let the first question be to 1
- One of 2 and 3 **has to be NOT Z.**
- Critical step in the solution: only humans can do?

Now cast the problem in the same setting as the tourist and the capital example

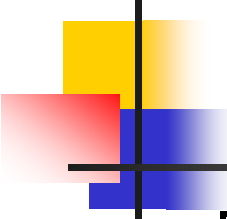
- Solving by analogy
- Use of previously solved problems
- Hallmark of intelligence

Analogy with the tourist and the capital problem

- Find the direction to the capital
 - → Find Z; who amongst 1, 2 and 3 is Z?
- Ask a single yes/no question to S (the person standing at the junction)
 - → Ask a single yes/no question to 1
- Answer forced to reveal the direction of the capital
 - → Answer forced to reveal who from 1,2,3 is Z

Question to 1

- Ask “Is R true” and the answer is yes if and only if 2 is not Z
- Propositions
 - P: 2 is not Z
 - Q: 1 always speaks the truth, i.e., 1 is X



Use of truth table as before

P	Q	1's Answer	R
T	T	Yes	T
T	F	Yes	F
F	T	No	F
F	F	No	T

Question to 1: the first question

- ***Is it true that 2 is not Z if and only if you are X?***

Analysis of 1's answer

- *Ans = yes*

- Case 1: 1 is X/Y (always speaks the truth or always lies)
 - 2 is indeed not Z (we can trust 1's answer)
- Case 2: 1 is Z
 - 2 is indeed not Z (we cannot trust 1's answer; but that does not affect us)

Analysis of 1's answer (contd)

- *Ans= no*

- Case 1: 1 is X/Y (always speaks the truth or always lies)
 - 2 is Z; hence 3 is not Z
- Case 2: 1 is Z
 - 3 is not Z

**Note carefully: how cleverly Z is identified.
Can a machine do it?**

Next steps: ask the 2nd
question to determine X/Y

- Once “Not Z” is identified- say 2, ask him a tautology
 - Is $P \equiv P$
 - If yes, 2 is X
 - If no, 2 is Y

Ask the 3rd Question

- Ask 2 “is 1 Z”
- If 2 is X
 - Ans=yes, 1 is Z
 - Ans=no, 1 is Y
- If 2 is Y (always lies)
 - Ans=yes, 1 is X
 - Ans=no, 1 is Z
- 3 is the remaining person

What do these examples show?

- Logic systematizes the reasoning process
- Helps identify what is mechanical/routine/automatable
- Brings to light the steps that only human intelligence can perform
 - These are especially of foundational and structural nature (e.g., deciding **what propositions to start with**)
- Algorithmizing reasoning is not trivial