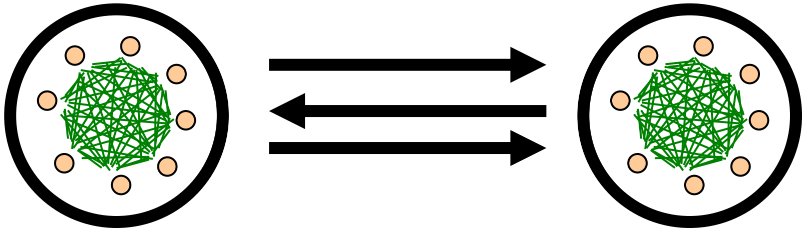
# “MPC in the Head”



Yuval Ishai

Technion and UCLA

# Back to the 1980s

- Zero-knowledge proofs for NP [GMR85,GMW86]
- Computational MPC with no honest majority [Yao86, GMW87]
- Unconditional MPC with honest majority [BGW88, CCD88, RB89]
- Unconditional MPC with no honest majority assuming ideal OT [Kilian88]

- Are these unrelated?

# Message of this talk

- Honest-majority MPC is useful even when there is no honest majority!
- Establishes unexpected relations between classical results
- New results for MPC with no honest majority
- New application domains for honest-majority tools and techniques

## Allison


Research interests:
- zero-knowledge proofs
- efficient two-party protocols

## Bernard


Research interests:
- information-theoretic cryptography
- honest-majority MPC

some relevance →

← no relevance?

Allison


Bernard


Research interests:
- zero-knowledge proofs
- efficient two-party protocols

Research interests:
- information-theoretic cryptography
- honest-majority MPC

Want to hear about my latest and coolest VSS protocol?

what a dork…

# Helping make the match

- Add to Allison's world a simple ideal functionality
  - Ideal commitment oracle for ZK (Com-hybrid model)
  - Ideal OT oracle for general protocols (OT-hybrid model)
- Makes unconditional (and UC) security possible
  - Analogous to secure channels in Bernard's world
- Why should Allison be happy?
  - Generality: Com or OT can be realized in a variety of models, under a variety of assumptions
  - Efficiency: Com or OT can be realized with little overhead
    - Essentially free given preprocessing [BG89]
    - Cheap preprocessing: fast OT […,PVW08,…], faster OT extension [Bea96,IKNP03…]
- Still: Why should Bernard's research be relevant?

# Helping make the match

- Add to Allison's world a simple ideal functionality
  - Ideal commitment oracle for ZK (Com-hybrid model)
  - Ideal OT oracle for general protocols (OT-hybrid model)
- Makes uncon… …rity possible
  - … world
- … …ariety of models,
  - … with little overhead
    - … …cessing [BG89]
    - Cheap preprocessing: fast OT […,PVW08,…], faster OT extension [Bea9… …P03…]

**A high level idea:**

- **Run MPC "in the head".**
- **Commit to generated views.**
- **Use consistency checks to ensure honest majority.**

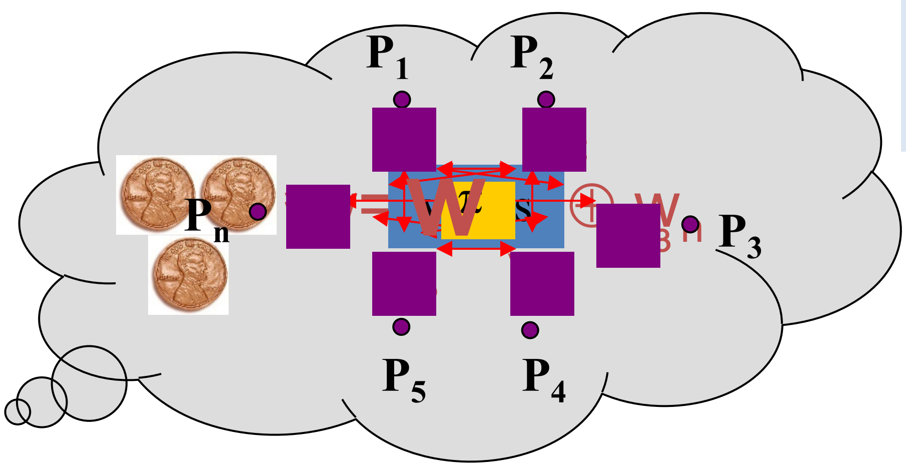
- Still: Why should Bernard's research be relevant?

# Zero-knowledge proofs

- Goal: ZK proof for an NP-relation $R(x,w)$
  - Completeness
  - Soundness
  - Zero-knowledge

- Towards using MPC:
  - define n-party functionality
    $g(x; w_1,\ldots,w_n) = R(x, w_1 \oplus \ldots \oplus w_n)$
  - use any 2-secure, perfectly correct protocol for g
    - security in semi-honest (passive adversary) model
    - honest majority when $n \geq 5$

# MPC → ZK [IKOS07]




Given MPC protocol $\pi$ for
$g(x; w_1,\ldots,w_n) = R(x, w_1 \oplus \ldots \oplus w_n)$

accept iff output=1
&
$V_i, V_j$ are consistent

Prover

Verifier

commit to views $V_1,\ldots,V_n$ →

← random $i,j$

open views $V_i$, $V_j$ →

# Analysis

Prover

Verifier

commit to views $V_1,\dots,V_n$ →

$w = w_1 \oplus \dots \oplus w_n$

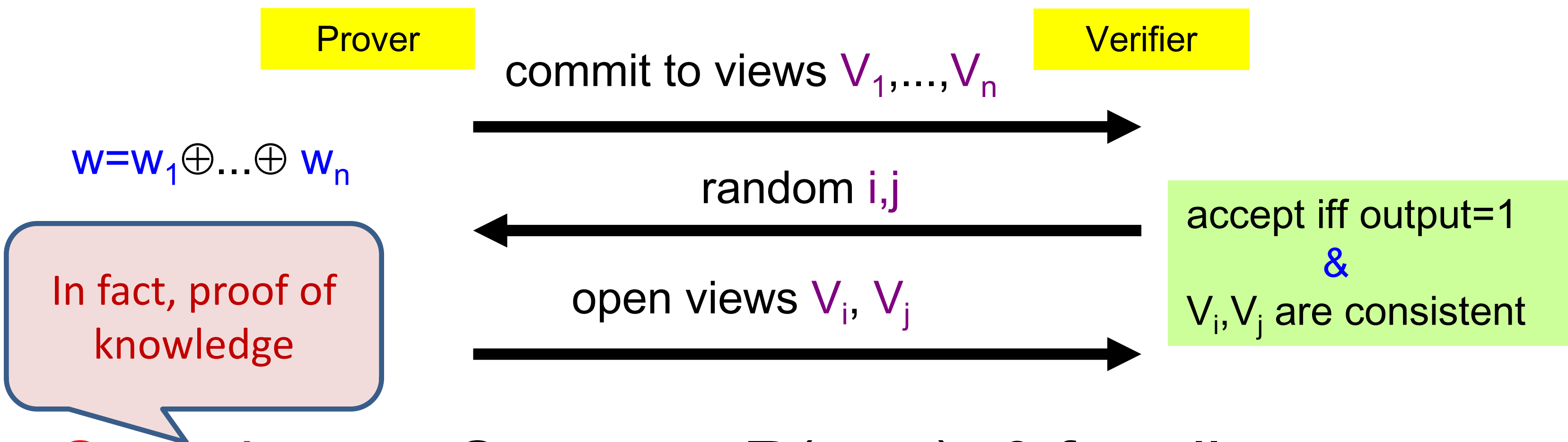
← random $i,j$

accept iff output=1
&
$V_i, V_j$ are consistent

open views $V_i$, $V_j$ →

- Completeness: √
- Zero-knowledge: by 2-security of $\pi$ and randomness of $w_i$, $w_j$.
  (Note: enough to use $w_1, w_2, w_3$ )

# Analysis




- **Soundness:** Suppose $R(x, w)=0$ for all w.
  ➔ either (1) $V_1,\ldots,V_n$ consistent with protocol $\pi$
  *or* (2) $V_1,\ldots,V_n$ not consistent with $\pi$

  (1) $\Rightarrow$ outputs=0 (perfect correctness)
  $\Rightarrow$ Verifier rejects

  (2) $\Rightarrow$ for some (i,j), $V_i,V_j$ are inconsistent.
  $\Rightarrow$ Verifier rejects with prob. $\geq 1/n^2$.

# Analysis

Prover | Verifier

$w = w_1 \oplus \ldots \oplus w_n$

commit to views $V_1, \ldots, V_n$ →

← random $i,j$

open views $V_i$, $V_j$ →

accept iff output=1
&
$V_i$,$V_j$ are consistent

Communication complexity:

$\leq$ (comm. complexity + rand. complexity + input size) of $\pi$.

# Extensions

- Variant: Use 1-secure MPC
  - Open one view and one incident channel
- Extends to OT-based MPC
  - Simple consistency check when $t \geq 2$
  - Slightly more involved with $t=1$ [HV16,IKPSY16]
- Extends to MPC with error
- Variant: Directly get $2^{-k}$ soundness error via security in malicious model (active adversary)
  - Two clients, $n=O(k)$ servers
  - $\Omega(n)$-security with abort
  - Broadcast is "free"
- Realize Com using a one-way function

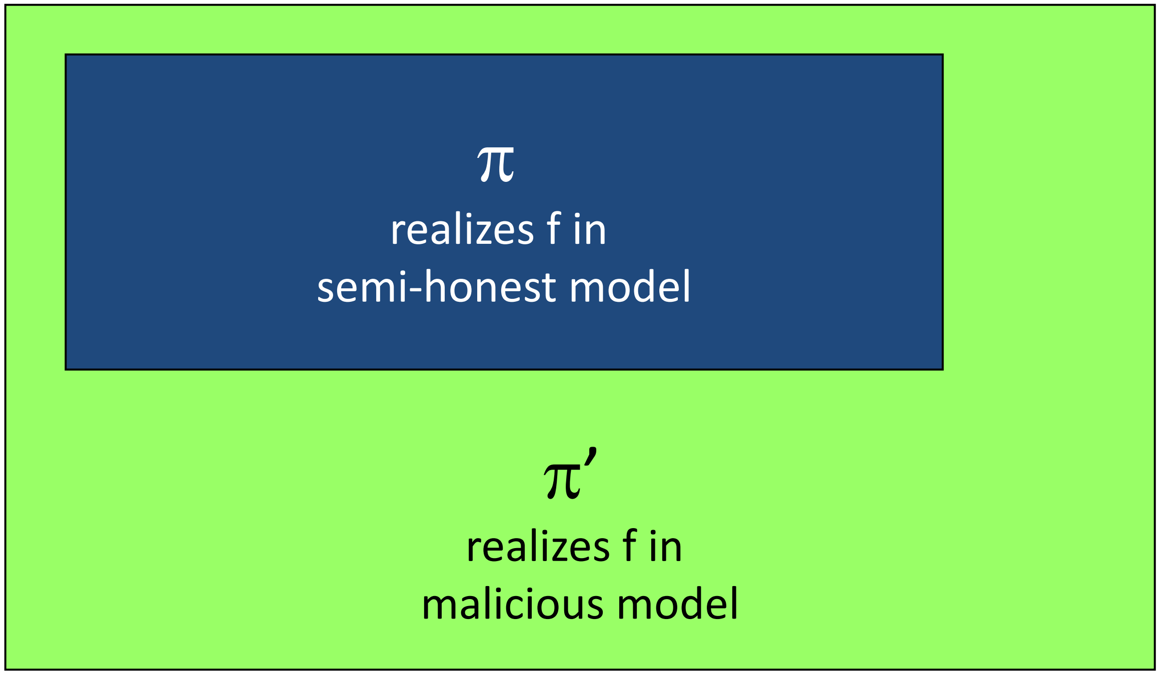
# Applications

- Simple ZK proofs using:
  - (1,3) semi-honest MPC [BGW88,CCD88] or [Mau02]
  - (2,3) or even (1,2) semi-honest $\text{MPC}^{\text{OT}}$ [GMW87,GV87,GHY87]
- Practical ZK proofs (“ZKBoo” [GMO16])
- ZK proofs with $O(|R|)$+poly(k) communication
  - Using efficient MPC + AG codes [DI06,CC06]
- Many good ZK protocols implied by MPC literature
  - ZK for linear algebra [CD01,…]

# General 2-party protocols [IPS08]

- Life is easier when everyone follows instructions…
- GMW paradigm [GMW87]:
  - semi-honest-secure $\pi$ → malicious-secure $\pi'$
  - use ZK proofs to prove “sticking to protocol”
- Non-black-box: ZK proofs in $\pi'$ involve code of $\pi$
  - Typically considered “impractical”
  - Not applicable at all when $\pi$ uses an oracle
    - Functionality oracle: OT-hybrid model
    - Crypto primitive oracle: black-box PRG
    - Arithmetic oracle: black-box field or ring
- **Is there a “black-box alternative” to GMW?**

# A dream goal



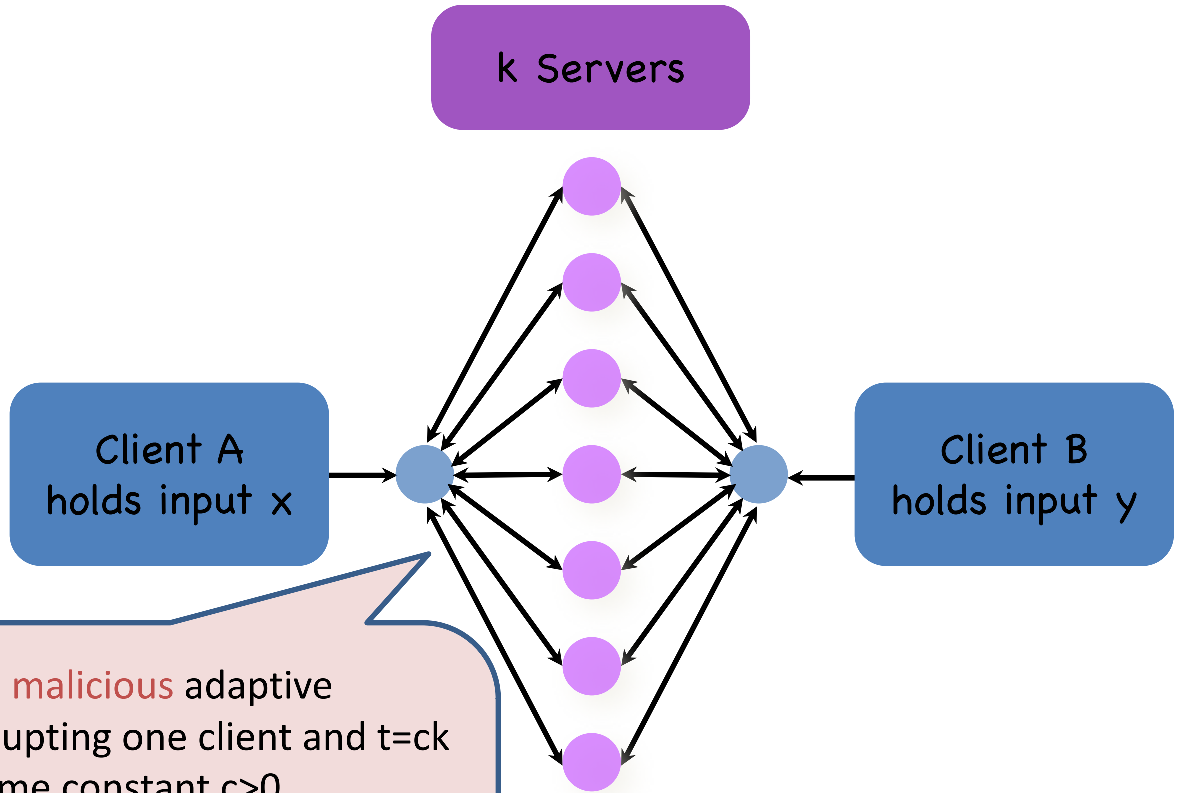

- Possible for some fixed f
  - e.g., OT [IKLP06,Hai08]
- Impossible for general f
  - e.g., ZK functionalities [IKOS07]

# Idea

- Combine two types of “easy” protocols:
  - Outer protocol:
    honest-majority MPC
  - Inner protocol:
    semi-honest 2-party protocol
    - possibly in OT-hybrid model

- Both are considerably easier than our goal
- Both can have information-theoretic security

# Outer protocol



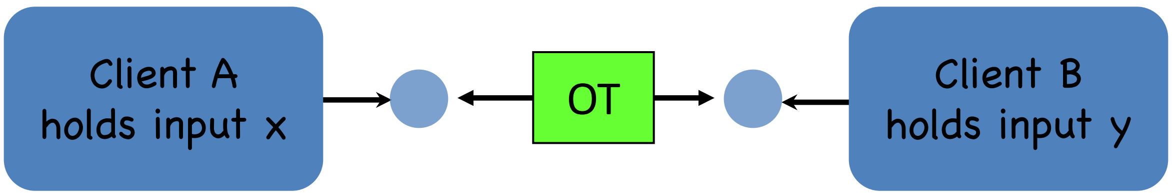

Secure against malicious adaptive adversary corrupting one client and t=ck servers, for some constant c>0.

Security with abort suffices.

Straight-line simulation.

Example: “BGW-lite”

# Inner protocol




Secure against semi-honest adversary

(Adaptive security w/erasures)

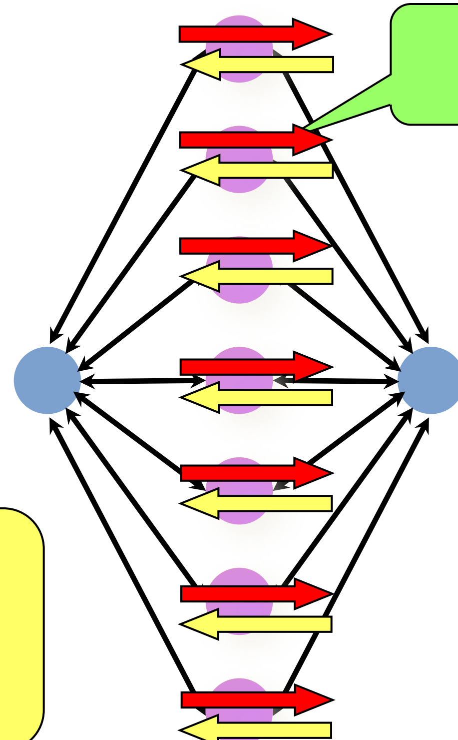
Example: “GMW-lite”

# Combining the two protocols

oblivious watch lists


doug duBois & jim goldberg NYTImes 9-22-2002

Player virtualization



OT calls by inner protocol are “risky”

outer protocol for f

# A closer look at server emulation

- Assume servers are deterministic
  - This is already the case for natural protocols
  - Can be ensured in general with small overhead
- In outer protocol, server i
  - gets messages from A and B
  - sends messages to A and B
  - may update a secret state
- Captured by reactive 2-party functionality $F_i$
  - Inputs = incoming messages
  - Outputs = outgoing messages
- Use semi-honest protocol for $F_i$
  - Distribute server between clients
  - "Local" computations do not need to be distributed.

# A closer look at watchlists

- Inner protocol can’t prevent clients from cheating by sending “bad messages”
- Watchlist mechanism ensures that cheating does not occur too often
  - Client doesn’t know which instances of inner protocol are watched
  - Two cases:
    - Client cheats in $\leq$ t instances
      ⇨ cheating is tolerated by t-security of outer protocol
    - Client cheats in >t instances
      ⇨ will be caught with overwhelming probability
- Non-interactive form of “cut-and-choose”

# Setting up the watchlists

- Each client picks n long one-time pads $R_i$
- $|R_i|$ = length of messages + randomness in execution of i-th inner protocol
  - Short PRG seed suffices for computational security
- Each client uses OT to select ~ t/2 of the other client's pads $R_i$
- Implemented via Rabin-OT for each server
  - Reduces to a constant number of (1,2) string-OTs per server for any rational probability p
  - With overwhelming probability, $p \pm 0.01$ fraction of $R_i$ are received

# Using the watchlists

- **Consider here B watching A**
  - A watches B symmetrically
- A uses sequential parts of each $R_i$ to mask her (progressive) view of the i-th inner protocol
  - If B obtained $R_i$, he has full view of i-th inner protocol
  - Can detect (and abort) as soon as A cheats
  - What about ideal OT calls in inner protocol?
    - Cheating caught w/prob ½ if OT inputs are random
    - Use OT to random-OT reduction

# Example

- Consider a "BGW-style" outer protocol
- Each server performs two types of computations:
  - Send $a_i b_i + z_i$ to A, where $a_i$ is a secret received from A and $b_i, z_i$ are secrets received from B
    - O(|C|) such computations overall
    - Can be implemented by simple inner protocols
      - unconditionally using OT [GMW87,IPS09]
      - using homomorphic encryption (e.g., Paillier)
      - using coding assumptions and OT [NP99,IPS09]
  - Send to A a public linear combination of secrets sent by B (and vice versa)
    - Can be implemented via local computation of B
- Gives efficient protocols for arithmetic computations

# Simulation (rough idea)

- Suppose A is corrupted in final protocol
- Main simulator runs outer simulator to
  - extract input of A
  - generate outer protocol messages from B
  - generate full view of inner protocols watched by A (requires corrupting ~ t/2 servers)
  - generate A's inputs and outputs in other inner protocols (communication of A with servers)
    - feed to inner simulator to generate inner protocol view
    - valid as long as A does not deviate from inner protocol
- Main simulator can observe deviation from inner protocol
  - When A cheats on i-th inner protocol, outer simulator corrupts i-th server and main simulator aborts w/prob. p

# A general protocol compiler

- **Given a $m$-party functionality F**
  - Get an honest-majority-secure outer protocol $\Pi$ for the functionality F (with $m$ clients and k servers)
  - Get a semi-honest-secure inner protocol $\rho^{OT}$ for a $m$-party functionality $G^{\Pi}$ corresponding to the servers' program in $\Pi$

    ($G^{\Pi}$ is a reactive functionality defined black-box w.r.t $\Pi$)

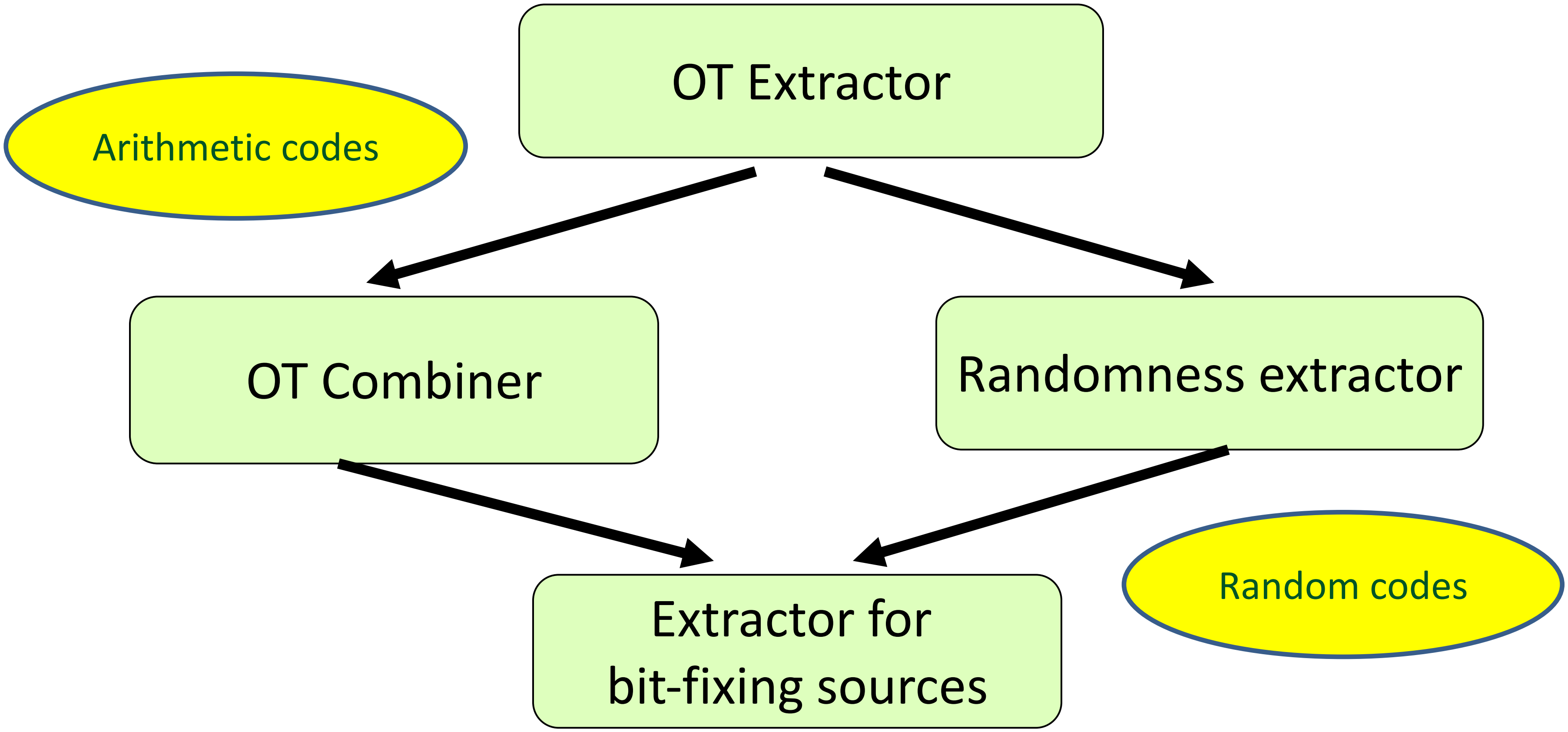
- **Our ($m$-party) protocol $\Phi^{OT}$, with black-box access to $\Pi$ and $\rho$, is a malicious-secure protocol for F.**

# Applications

- Revisiting the classics
  - BGW-lite + GMW-lite ➔ Kilian
- Efficient MPC with no honest majority
  - O(1) bits per gate in OT-hybrid model (+ additive term)
  - All crypto can be pushed to preprocessing
- Constant-round $MPC^{OT}$ ($t<n$) using black-box PRG
  - Extending 2-party "cut-and-choose" Yao
- Efficient OT extension in malicious model
- Constant-rate b.b. reduction of OT to semi-honest OT
- Secure arithmetic computation over black-box fields/rings
- Protocols making black-box use of homomorphic encryption

# More “MPC in the Head”: OT combiners and OT extractors

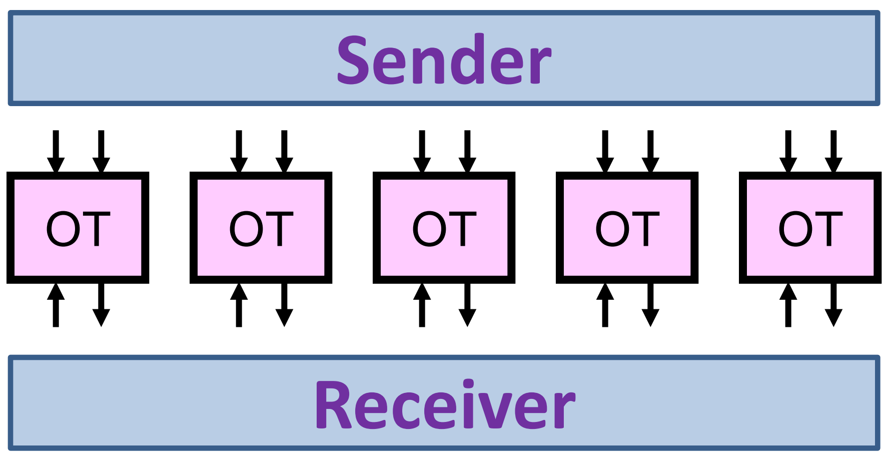
- OT combiners [HKNRR05]
  - Given n instances of OT, of which t are faulty, produce m good OTs
  - Can be obtained via honest-majority MPC [HIKN08,IPS08]
    - Outer protocol: honest-majority MPC for m OTs
    - Inner protocol: OT-based 2-party protocol for emulating MPC server
  - Used for constant-rate OT from noisy channels [HIKN08,IKOPSW11]
- OT extractors [IKOS09]
  - Generalize OT combiners by allowing global leakage
  - Construction makes an ad-hoc use of suitable “outer protocol” and “inner protocol”
  - Yield constant-rate OT protocols from imperfect noisy channels, constant-rate OT from (computational) “θ-Hiding assumption”.



OT Extractor
Arithmetic codes
OT Combiner
Randomness extractor
Random codes
Extractor for
bit-fixing sources

# More "MPC in the Head": Non-Interactive Secure Computation

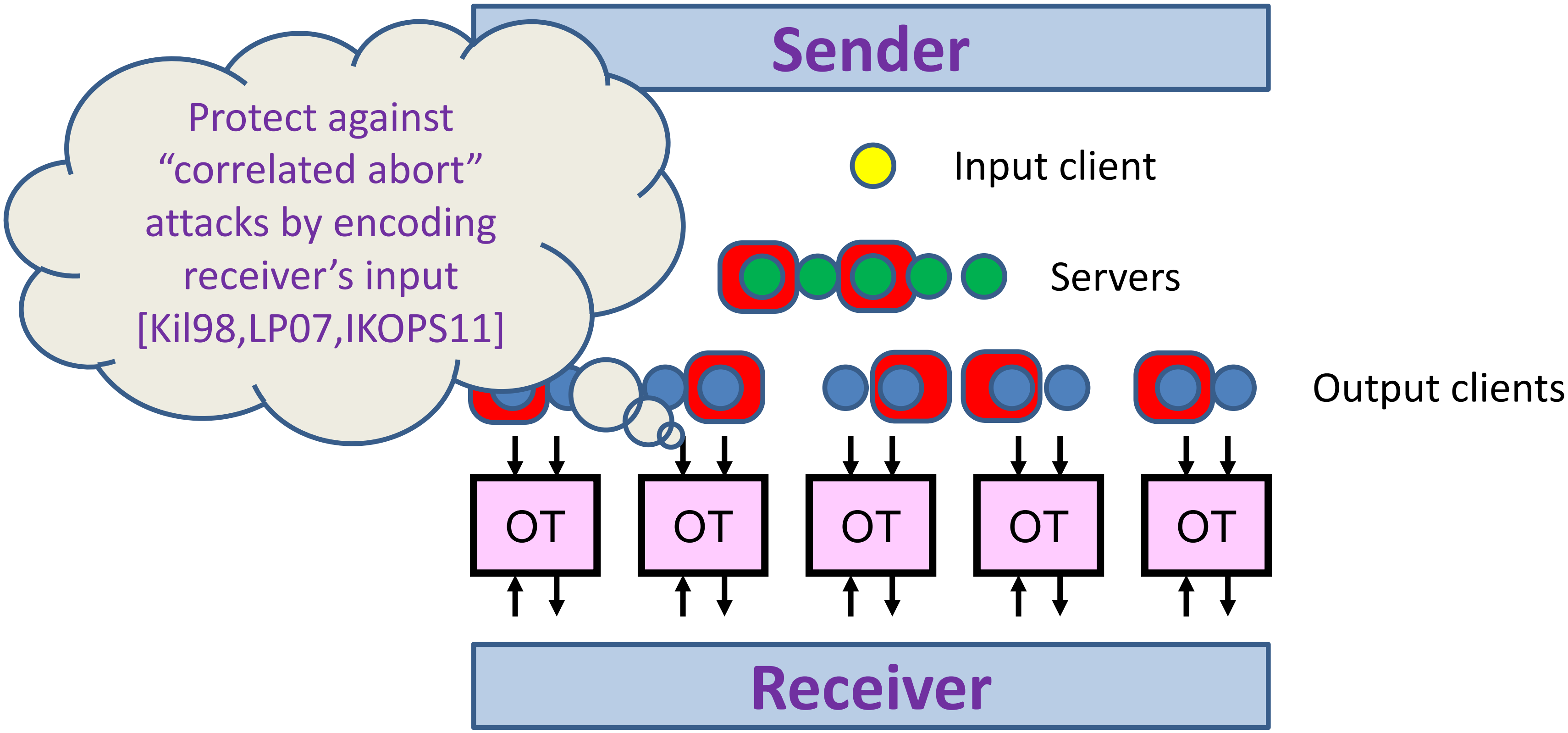
- Goal: Protect non-interactive OT-based protocols against malicious sender




- Challenge: allow Receiver to detect when Sender's OT inputs are inconsistent with protocol

# More "MPC in the Head": Non-Interactive Secure Computation

- An MPC-based approach [IKOPS11]



# Further research I

- Find other useful "black-box" connections
- Formalized via oracle game:
  - Protocol move:
    given oracle g, get (arbitrary) protocol oracle $\pi_g$
  - Build move:
    given oracle f, build oracle g
  - Goal: given oracle f, obtain a protocol $\pi_f$ in a "strong" model using only protocol moves in "weaker" model(s)
- Previous examples
  - ZK from MPC:
    build – protocol – build
  - New protocol compiler:
    protocol – build – protocol - build

# Further Research

- Other useful “black-box” connections?
  - Formalized via “MPC transformations” framework [IKPSY16]
  - Gives hope for proving negative results
- Find leaner versions of protocol compilers
  - Weaker outer protocol?
- Minimize constants in constant-rate protocols
  - Better “arithmetic codes”?
- Optimize for practical efficiency?
  - Many degrees of freedom!
  - Progress made in [LOP11]